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HUNGARIAN
PROBLEM BOOK I

based on the
Eötvös Competitions
1898–1905

TRANSLATED BY
ELVIRA RAPAPORT

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NEW MATHEMATICAL LIBRARY

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HUNGARIAN
PROBLEM BOOK I

BASED ON THE EÖTVÖS COMPETITIONS, 1894–1905

REVISED AND EDITED BY

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HUNGARIAN PROBLEM BOOK I

BASED ON THE EÖTVÖS COMPETITIONS, 1894–1905

From the Hungarian Prefaces

The first of these contests was held in 1894 by the Mathematical and Physical Society (of Hungary) in honor of its founder and president, the distinguished physicist Baron Loránd Eötvös, who became minister of education that year. To commemorate the event, the contests are given every fall and are open to high school graduates of that year. The contestants work in classrooms under supervision; the Society selects the two best papers, and the awards—a first and second Eötvös Prize—are given to the winners by the president himself at the next session of the Society.

The present volume, appearing on the tenth anniversary of Eötvös' death, contains the contests held to date. While it utilizes the winners' work, the solutions are in general not those found by the students....

The names of the winners are listed; their papers appeared in full in *Matematikai és Fizikai Lapok*, the Journal of the Society; here, however, the solutions were changed to suit the didactic aim of the book.

Some of my notes give definitions and proofs of theorems used in the solutions. Others serve to point out the connection between problems and famous results in literature. In some instances I was able to give a glimpse of the essence of an entire subject matter; in others the mere statement of a general theorem had to suffice....

There are few prerequisites. A person who has learned to solve quadratic equations and knows plane geometry can solve many of the problems. If he also knows trigonometry, he can solve most of them. So little of the material taught in the last two years of high school (in Hungary) is needed here that a younger student can easily learn it from books.

However, this book is meant not only for students and teachers. Anyone who retained an interest in mathematics in his adult life can find things of note and value here and will be gratified to see how much can be achieved with the elementary material to which high schools must restrict themselves.

How should the reader use this book? All I can say is: without frenzy. With a serious interest and perseverence, everyone will find the way best suited *to him* in order to benefit from the varied material contained in it....

József Kürschák

Budapest, April 9th, 1929

Problems of the Mathematics Contests, edited by József Kürschák, was published originally in 1929. The first edition was quickly sold out and the Ministry of Education commissioned a new edition. We are undertaking this work with pleasure and in the hope of contributing to the attainment of Kürschák's goal. This volume will soon be followed by another containing a similar treatment of the contests held since.

The new edition required certain changes. By and large, Kürschák's approach was retained, especially his notes which were meant to widen the reader's horizon. We augmented these notes here and there, changing them only to conform to present-day high school curricula. For example, we added permutations, the binomial theorem, half-angle formulas, etc. We added a few new notes. We included some new solutions that seemed strikingly simple or ingenious, and so we deviated from the contestants' work more than the first edition did....

We made a few technical changes to facilitate handling. Thus, we repeated the problems before giving the solution and we unified the notation, thereby necessitating alterations in and additions to the figures....

We sincerely hope to have come up to expectation and that this work will give pleasure and profit to many.

György Hajós
Gyula Neukomm
János Surányi

Budapest, September, 1955

Preface to the American Edition

In recent times much effort has been devoted to the improvement of mathematics teaching on all levels. Thus it is only natural that we search for further stimulants and improvements in this direction, as well as for new means of discovering and developing the dormant abilities which may exist in our society. The Monograph Project of the School Mathematics Study Group has such a purpose. The present translation of the Hungarian problem collection of the Eötvös Competition serves this goal. Since I am one of the few still existing links between the present mathematical generation and an older one that witnessed the first phase of the interesting development of this competition, I was asked to write a few introductory words to the English version of this collection.

The Eötvös Competition was organized in Hungary in 1894 and played a remarkable role in the development of mathematics in that small country.† The competition was open to all freshmen entering the university; the publication of the problems and the names of the winners was from the beginning a public event of first class interest. Among the winners during the period of the first decade of the competition were such men as Fejér, von Kármán, Haar, Riesz, and numerous others who later became internationally known. With some short interruptions due to wars and related conditions the competition has been carried on to the present day, though the name was changed, and the organization and scope of the competition have become much broader in recent years. The essence however has remained the same. The problems are almost all

† Before the first world war Hungary had 19 million inhabitants; at present it has about 10 million.

from high school material (no calculus is included), they are of an elementary character, but rather difficult, and their solution requires a certain degree of insight and creative ability. Any amount of aid in the form of books or notes is permitted.

Mathematics is a human activity almost as diverse as the human mind itself. Therefore it seems impossible to design absolutely certain and effective means and methods for the stimulation of mathematics on a large scale. While the competitive idea seems to be a powerful stimulant, it is interesting to observe that it was and is still almost completely absent from academic life in Germany although mathematics has flourished in that country throughout the last two hundred years. The organization of the Eötvös Competition in Hungary was probably suggested by British and French examples that had existed in those countries for a long time. We mention in particular the "Mathematical Tripos" in Cambridge, England and the "Concours" examination problems for admission to the "Grandes Écoles" in France. These early examples suggest also that some sort of preparation is essential to arouse public interest, to attract the best competitors and to give them proper recognition. In England the participation in the Tripos is preceded by systematic coaching, and in France the public schools offer facilities to prepare for the "Concours" examinations. In Hungary a similar objective was achieved by a Journal published primarily for high school students as another natural stimulant to the student's preparation for participation in the competition upon entering the university.†

The Journal was organized almost simultaneously with the competition, i.e. in 1894, by Dániel Arany; for many years it was edited by the able high school teacher László Rácz‡ and later by various other teachers of high quality. The articles were supplied partly by teachers and partly by mathematicians affiliated with the university, mostly younger persons. The Journal carried articles primarily from elementary mathematics, much triangle geometry, some projective and descriptive geometry, algebra and occasionally some number theory, later also some ventures into calculus. But the most important and most fertile part was the

† A good account of the Eötvös Competition and of the Journal is given in an article by Tibor Radó: "On mathematical life in Hungary", *American Mathematical Monthly*, vol. 39 (1932), pp. 85–90. (One slight correction has to be made on p. 87, line 6: There *was* a girl winner, first prize, 1908.)

‡ His name will go down in history for a second reason: Rácz was the teacher of J. von Neumann in high school. Cf. the Obituary Note by S. Ulam, *Bulletin of the American Mathematical Society*, vol. 64 (1958), pp. 1–49; on p. 2 the name Rácz appears in distorted spelling.

problem section; it occupied a large part of the content and was essentially written for the students and by the students. The best solution sent in was printed with the name and school of the author, and a list of the others who sent in correct solutions was given.

I remember vividly the time when I participated in this phase of the Journal (in the years between 1908 and 1912); I would wait eagerly for the arrival of the monthly issue and my first concern was to look at the problem section, almost breathlessly, and to start grappling with the problems without delay. The names of the others who were in the same business were quickly known to me and frequently I read with considerable envy how they had succeeded with some problems which I could not handle with complete success, or how they had found a better solution (that is, simpler, more elegant or wittier) than the one I had sent in. The following story may not be accurate in all details but it is certainly revealing:

"The time is about 1940, the scene is one of the infamous labor camps of fascist Hungary just at the beginning of its pathetic transformation from semi-dictatorship to the cannibalism of the Nazi pattern. These camps were populated mostly by Jewish youth forced to carry out some perfectly useless tasks. One young man (at present one of the leading mathematicians of Hungary) was in the camp; let us call him Mr. X. He was panting under the load of a heavy beam when the sergeant shouted at him in a not too complimentary manner, addressing him by his last name. The supervising officer stood nearby, just a few steps away, and said: 'Say, did I hear right, your name is X?' 'Yes,' was the answer. 'Are you by chance the same X who worked years ago in the High School Journal?' 'Yes,' was again the answer. 'You know, you solved more, and more difficult problems than any one of us and we were very envious of you.' The end of the story is that Mr. X received more lenient treatment in the camp and later even had some mathematical contact with the all-powerful officer."

The profound interest which these young men took in the Journal was decisive in many of their lives. The intensive preoccupation with interesting problems of simple and elementary character and the effort of finding clear and complete answers gave them a new experience, the taste of creative intellectual adventure. Thus they were bound finally and unalterably to the jealous mistress that mathematics is. There remained still the question of what special studies to undertake, whether it should be mathematics or physics or engineering; but this was after all a secondary matter; the main road was charted for life. We may think of the adage of Kronecker who compares mathematicians with lotus eaters: "Wer einmal von dieser Kost etwas zu sich genommen hat, kann nie mehr davon lassen." (He who has once tasted of this fruit can never more forswear it.)

And a final observation. We should not forget that the solution of any worth-while problem very rarely comes to us easily and without hard work; it is rather the result of intellectual effort of days or weeks or months. Why should the young mind be willing to make this supreme effort? The explanation is probably the instinctive preference for certain values, that is, the attitude which rates intellectual effort and spiritual achievement higher than material advantage. Such a valuation can only be the result of a long cultural development of environment and public spirit which is difficult to accelerate by governmental aid or even by more intensive training in mathematics. The most effective means may consist of transmitting to the young mind the beauty of intellectual work and the feeling of satisfaction following a great and successful mental effort. The hope is justified that the present book might aid exactly in this respect and that it represents a good step in the right direction.

Gábor Szegő

Stanford University, February, 1961

List of Explanatory Notes

List of Biographical Notes

List of Winners

1894	Mihály Seidner	Pál Pap
1895	Emil Riesz	Ignác Pilczer
1896	Aladár Visnya	Győző Zemplén
1897	Bernát Fazekas	Lipót Fejér
1898	Tivadar Kármán	Gábor Gróffits
1899	Ödön Kornis	Ödön Spiczer
1900	Iréneusz Juvantz	Kázmér Szmodics
1901	Gyula Póka	Ernő Baranyó
1902	Dénes Kőnig	Hildegárd Szmodics
1903	Alfréd Haar	Béla Horvay
1904	Marcel Riesz	István Fuchs
1905	Gyula Ujj	Constantin Neubauer

The Eötvös Contests in elementary mathematics have been open to Hungarian students in their last year of high school ever since 1894. They are famous for the simplicity of the concepts employed, the mathematical depth reached, and the diversity of elementary mathematical fields touched. But perhaps their most remarkable feature is the influence that they, together with a mathematics journal for students, seem to have had on the young people of that small country. Among the winners of the first eleven contests (i.e. those contained in the present volume) many turned into scientists of international fame; e.g. L. Fejér, T. von Kármán, D. Kőnig, M. Riesz. Among the winners of the next twenty contests (i.e. those contained in volume 12) are G. Szegő, T. Radó, E. Teller; all three are well known in the United States, where they now reside. This translation of the Eötvös Contest Problems from 1894-1928 is based on the revised Hungarian edition of J. Kürschák's original compilation. Kürschák combined his excellence in mathematics with his interest in education when he supplied the elegant solutions and illuminating explanations.

JÓZSEF KÜRSCHÁK (1864-1933) was born and educated in Hungary. He was professor of mathematics at the Polytechnic University in Budapest, member of the Hungarian Academy and permanent member of the Examination Board for prospective high school teachers of mathematics.

His many contributions to mathematics include work in the calculus of variations, in algebra and in number theory. He used his great pedagogical skill in developing and teaching an exceptionally good mathematics course for beginning engineering students. He also gave courses for future high school teachers, mainly in elementary geometry and in geometrical constructions. Several of his papers deal with the teaching and popularization of mathematics. His devotion to intelligently guided problem solving is illustrated by the famous problem book which forms the basis of the present volume.

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