

The Santa Cruz Conference on  
**FINITE GROUPS**

Volume 37

**PROCEEDINGS OF  
SYMPOSIA IN  
PURE MATHEMATICS**

AMERICAN MATHEMATICAL SOCIETY

**THE SANTA CRUZ CONFERENCE  
ON FINITE GROUPS**

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## PREFACE

In the last year or so there have been widespread rumors that group theory is finished, that there is nothing more to be done. It is not so.

While it is true that we are tantalizingly close to that pinnacle representing the classification of finite simple groups, one should remember that only by reaching the top can one properly look back and survey the neighboring territory. It was the task of the Santa Cruz conference not only to describe the tortuous route which brings us so close to the summit of classification, but also to chart out more accessible paths—ones which might someday be open to the general mathematical public.

A third concern was the elucidation of topics in related fields, and it is to one of these three areas that the papers in this volume are devoted.

Just a quick glance at the table of contents will reveal a wide variety of topics with which the modern group theorist must contend. Some of these, for example, the connections with the theory of modular functions, have very recent origins, but they leave us with the clear impression that, far from being dead, group theory has only just come of age.

Geoffrey Mason  
Chicago, June 1980

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