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Mathematical Foundations
of Quantum Field Theory
and Perturbative String Theory

Hisham Sati
Urs Schreiber
Editors



American Mathematical Society

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Preface

Conceptual progress in fundamental theoretical physics is linked with the search for suitable mathematical structures that model the physics in question. There are a number of indications that today we are in a period where the fundamental mathematical nature of quantum field theory (QFT) and of the worldvolume aspects of string theory is being identified. It is not unlikely that future generations will think of the turn of the millennium and the beginning of the 21st century as the time when it was fully established that QFT in general and worldvolume theories in particular are precisely the representations of higher categories of cobordisms with structure or, dually, encoded by copresheaves of local algebras of observables, vertex operator algebras, factorization algebras and their siblings.

While significant insights on these matters have been gained in the last several years, their full impact has possibly not yet received due attention, notably not among most of the theoretical but pure physicists for whom it should be of utmost relevance. At the same time, those who do appreciate the mathematical structures involved may wonder how it all fits into the big physical picture of quantum field and string theory.

This volume is aimed at trying to improve on this situation by collecting original presentations as well as reviews and surveys of recent and substantial progress in the unravelling of mathematical structures underlying the very nature of quantum field and worldvolume string theory. All contributions have been carefully refereed.

It is reassuring that some of the conferences on fundamental and mathematical physics these days begin to witness a new, more substantial interaction between theoretical physicists and mathematicians, where the latter no longer just extract the isolated remarkable conjectures that the black box *string theory* has been producing over the decades, but finally hold in their hands a workable axiom system that allows one to genuinely consider core aspects of QFT in a formal manner. This book has grown out of the experience of such meetings.

The editors express their thanks to the authors who kindly made their work available for this volume. We also acknowledge the hard work of the referees. We thank Sergei Gelfand, Christine Thivierge, and the dedicated staff at the American Mathematical Society for their effort in publishing this volume. We also thank Arthur Greenspoon for carefully proofreading the papers and for his input on the volume as a whole.

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