

# Proceedings of Symposia in PURE MATHEMATICS

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Volume 91

## Probability and Statistical Physics in St. Petersburg

St. Petersburg School  
Probability and Statistical Physics  
June 18–29, 2012  
St. Petersburg State University  
St. Petersburg, Russia

V. Sidoravicius  
S. Smirnov  
Editors



**American Mathematical Society**

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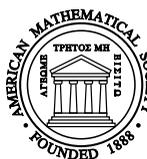
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## Preface

Among few cities in the world associated with the development of Probability Theory as a mathematical discipline, St. Petersburg holds its own very special status. The brothers D. Bernoulli and N. Bernoulli, L. Euler, V. Bunyakovsky (the author of the first course in Probability Theory in Russian), his student P. L. Chebyshev and Chebyshev's students, A. A. Markov and A. M. Lyapunov, are just a few names to mention, who laid down the fundamentals of great Russian tradition in Probability Theory. Since the nineteenth century St. Petersburg continued this tradition and has been the stronghold of Probability Theory and Statistical Physics throughout two centuries.

Spatial probability theory with its deep connections to classical Statistical Physics has made remarkable advances in the last 15 years. This involved heavy deployment (and development) of complex analysis, representation theory, random matrices, exact solvability methods, etc., which are in many ways interlinked to each other. Several larger and smaller scale meetings and schools around the globe were following this development, and it was the right time to meet in St. Petersburg during the wonderful “white nights” of 2012.

The list of speakers and spectrum of topics speak for themselves. Besides notes of the courses delivered at the school, the book benefited from the course of G. Ben Arous (NYU) and A. Fribergh (Toronto), who generously provided their lectures. We believe that readers at all levels — from advanced graduate students to seasoned researchers in these fields — will find interesting and new material in these notes.

We thank all contributors for their dedicated work during the school, preparing lecture notes and for their patience during preparation of the book.

The school was organized by the Chebyshev Laboratory at St. Petersburg State University, newly established by the grant 11.G34.31.0026 of the Government of the Russian Federation. The Laboratory aspires to involve students and young mathematicians in modern research, and has an extensive visitor program and frequent conferences and schools.

Finally, we would like to thank co-organizers of the event, Professor Dmitry Chelkak (Chebyshev Laboratory/Steklov Institute/ETH-ITS, Zurich) and Vlad Vysotsky (Chebyshev Laboratory/Steklov Institute/Arizona State), whose work made the event a success, and we also thank all of the students, researchers and administrative staff of the Chebyshev laboratory who were involved in the organization of the school since its very early stages. We are also very grateful to our colleagues from the Philosophy Department for letting two hundred mathematicians use their lecture room.

Details of the meeting and a list of participants and sponsors can be found at the event's website: <http://spsp.chebyshev.spb.ru>.

Vladas Sidoravicius and Stanislav Smirnov,  
New York, Genève and St. Petersburg,  
June 6, 2015

