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Algebraic Groups: Structure and Actions

2015 Clifford Lectures

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March 2–5, 2015

Tulane University, New Orleans, Louisiana

Mahir Bilen Can

Editor



American Mathematical Society

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Preface

The prominent (semi)group theorist Alfred Hobilitzelle Clifford (1908–1992) joined Tulane University in 1955. Since 1984, honoring his contributions, the Mathematics Department at Tulane has hosted the annual Clifford Lectures, a weeklong series of talks by a distinguished mathematician. A mini-conference is held in conjunction with each of the Clifford Lecture series. The theme of the 2015 Clifford Lecture series was Algebraic Groups: Structure and Actions, and the main speaker was Michel Brion. This volume presents the proceedings of the associated mini-conference.

The theory of algebraic groups forms a very active research area in contemporary mathematics. It has rich relations to many other areas, including algebraic geometry, number theory, and representation theory. The topics that were covered in the Clifford Lectures contributed widely to this spectrum. They included pseudo-reductive groups, structure theory for algebraic groups, groups of birational transformations, the Tschirnhaus transformations and applications, algebraic theory of quadratic forms, geometry of classifying spaces, and G -torsors, as well as operational K -theory and its applications.

The papers in this volume not only present new results on the aforementioned themes, but also provide much awaited exposés of the foundational results of algebraic group theory recast in the language of schemes at the desired generality.

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Mahir Bilen Can

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