Sherlock Holmes in Babylon
and Other Tales of Mathematical History

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Marlow Anderson,
Victor Katz,
Robin Wilson
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Introduction

For the past one hundred years, the Mathematical Association of America has been publishing high-quality articles on the history of mathematics, some written by distinguished historians such as Florian Cajori, Julian Lowell Coolidge, Max Dehn, David Eugene Smith, Carl Boyer, and others. Many well-known historians of the present day also contribute to the MAA’s journals. Some years ago, Robin Wilson and Marlow Anderson, along with the late John Fauvel, a distinguished and sorely missed historian of mathematics, decided that it would be useful to reprint a selection of these papers and to set them in the context of modern historical research, so that current mathematicians can continue to enjoy them and so that newer articles can be easily compared with older ones. After John’s untimely death, Victor Katz was asked to fill in and help bring this project to completion.

A careful reading of some of the older papers in particular shows that although modern research has introduced some new information or has fostered some new interpretations, in large measure they are neither dated nor obsolete. Nevertheless, we have sometimes decided to include two or more papers on a single topic, written years apart, to show the progress in the history of mathematics.

The editors hope that you will enjoy this collection covering nearly four thousand years of history, from ancient Babylonia up to the time of Euler in the eighteenth century. We wish to thank Don Albers, Director of Publication at the MAA, and Gerald Alexanderson, chair of the publications committee of the MAA, for their support for the history of mathematics at the MAA in general, and for this project in particular. We also want to thank Beverly Ruedi for her technical expertise in preparing this volume for publication.
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About the Editors

Marlow Anderson is a professor of mathematics at The Colorado College, in Colorado Springs; he has been a member of the mathematics department there since 1982. He was born in Seattle, and received his undergraduate degree from Whitman College. He studied partially ordered algebra at the University of Kansas and received his PhD in 1978. He has written over 20 research papers. In addition, he is co-author of a book on lattice-ordered groups, and also an undergraduate textbook on abstract algebra.

Victor Katz is currently Professor of Mathematics at the University of the District of Columbia. He has long been interested in the history of mathematics and its use in teaching. The first edition of his textbook: *A History of Mathematics: An Introduction* was published in 1993, with a second edition in 1998 and a shorter version to appear in 2004. He has directed three major NSF-supported and MAA-administered grant projects dealing with the history of mathematics, collectively titled the *Institute in the History of Mathematics and Its Use in Teaching* (IHMT). Under these projects, over a hundred college faculty (and thirty-five high school teachers) studied the history of mathematics, including how to teach courses in the subject and how to use it in teaching mathematics courses. In the third of the projects, the *Historical Modules Project*, eleven modules were developed for teaching topics in the secondary mathematics curriculum via the use of history. These are available now on a CD.

Robin Wilson is currently Head of the Pure Mathematics Department at the Open University, U.K., and Fellow in Mathematics at Keble College, Oxford University. He was Visiting Professor in the History of Mathematics at Gresham College, London, in 2001–02 and is a frequent visiting professor at Colorado College. He has written and edited about 25 books, in topics ranging from graph theory and combinatorics, via philately and the Gilbert & Sullivan operas, to the history of mathematics. In 1975 he was awarded a Lester Ford award by the MAA for “outstanding expository writing.” He is well known for his bright clothes and atrocious puns.
Covering a span of almost 4000 years, from the ancient Babylonians to the eighteenth century, this collection chronicles the enormous changes in mathematical thinking over this time as viewed by distinguished historians of mathematics from the past and the present. Each of the four sections of the book (Ancient Mathematics, Medieval and Renaissance Mathematics, The Seventeenth Century, The Eighteenth Century) is preceded by a Foreword, in which the articles are put into historical context, and followed by an Afterword, in which they are reviewed in the light of current historical scholarship. In more than one case, two articles on the same topic are included to show how knowledge and views about the topic changed over the years. This book will be enjoyed by anyone interested in mathematics and its history—and, in particular, by mathematics teachers at secondary, college, and university levels.