Mathematical Surveys and Monographs

Volume 40.1

The Classification of the Finite Simple Groups

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American Mathematical Society

http://dx.doi.org/10.1090/surv/040.1

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The authors were supported in part by NSF grant #DMS 89-03124, by DIMACS, an NSF Science and Technology Center funded under contract STC-88-09648, and by NSA grant #MDA-904-91-H-0043.

2000 *Mathematics Subject Classification.* Primary 20D05, 20E32.

ABSTRACT. This is the first monograph in a series devoted to a revised proof of the classification of the finite simple groups.

Library of Congress Cataloging-in-Publication Data

Gorenstein, Daniel.

The classification of the finite simple groups / Daniel Gorenstein, Richard Lyons, Ronald Solomon.

p. cm. — (Mathematical surveys and monographs, v. 40, number 1–)

Includes bibliographical references and index.

ISBN 0-8218-0334-4

1. Finite simple groups. I. Lyons, Richard, 1945– . II. Solomon, Ronald. III. Title. IV. Series: Mathematical surveys and monographs; no. 40, pt. 1–.

QA177.G67 1994
512′.2—dc20 94-23001
CIP

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10 9 8 7 6 5 4 3 2 05 04 03 02 01 00

To the memory of

Pearl Solomon (1918–1978)

and

Marvin Lyons (1903–1975)

Contents

Preface

Though elated at the successful completion of the classification of finite simple groups in the early 1980's, Danny Gorenstein nevertheless immediately appreciated the urgency of a "revision" project, to which he turned without delay. Before long we had joined him in this effort, which is now into its second decade. Danny always kept the project driving forward with relentless energy, contagious optimism and his unique global vision of finite simple group theory. He inspired other collaborators—Richard Foote and Gernot Stroth—to contribute theorems designed to fit our revised strategy. Their work forms a vital part of this project. The conception of these volumes is unmistakably Danny's, and those who know his mathematics and his persuasive way of explaining it should recognize them everywhere. Since his death in 1992, we have tried to maintain his standards and we hope that whatever changes and additions we have made keep the vigorous spirit which was his trademark. Of course the responsibility for any stumbling or errors must remain with us. To accompany him on this mathematical journey was a privilege and as we continue, our debt to our teacher, colleague and loyal friend is hard to measure. Thanks, Danny.

This monograph is Number 1 of a projected dozen or so volumes, and contains two of the roughly thirty chapters which will comprise the entire project. Not all of the chapters are completely written at this juncture, but we anticipate that the publication process, now begun, will continue at a steady pace. In the first section of Chapter 1, we discuss the current status of the work in some more detail; later in that chapter we delineate the background results which form the foundation for all the mathematics in the subsequent volumes.

When Danny began, there was already a well-established tradition of "revisionism" in finite group theory. Indeed, beginning in the late 1960's, Helmut Bender produced a series of "revisions" whose beauty and depth profoundly influenced finite group theory. During the final decade of the classification proof, several more group-theorists showed how to develop deep mathematics while re-addressing some of the fundamental theorems in the theory of simple groups. While it is sometimes difficult to draw a distinct line between revisionist and other mathematics, the clear successes of Bender, Michel Enguehard, George Glauberman, Koichiro Harada, Thomas Peterfalvi, and Bernd Stellmacher in various revisionist projects have inspired us.

We are extremely grateful to Michael Aschbacher, Walter Feit, George Glauberman, Gary Seitz, Stephen Smith, and John Thompson for their enthusiastic support and interest in our endeavor, and specifically for having read some of the manuscripts and made valuable comments. To our collaborators Richard Foote and Gernot Stroth, our special thanks. The ideas and comments of many colleagues have been most useful and will appear in some form in the volumes to

come. Indeed, this work has ingredients of several types: there is new mathematics; there is exposition of unpublished work of our colleagues, ranging from short arguments to entire case analyses which they have generously shared with us, and which we shall acknowledge specifically as we go along; and finally there are reworkings of published papers. For now, in addition to all the people already mentioned, we must also thank Jonathan Alperin, Michel Broué, Andrew Chermak, Michael Collins, K. M. Das, Alberto Delgado, Paul Fong, Robert Gilman, David Goldschmidt, Kensaku Gomi, Robert Griess, Robert Guralnick, Jonathan Hall, William Kantor, Martin Liebeck, Ulrich Meierfrankenfeld, Michael O'Nan, Lluis Puig, Geoffrey Robinson, Jan Saxl, Ernie Shult, Franz Timmesfeld, Jacques Tits, John Walter, Richard Weiss, and Sia K. Wong.

We are happy to acknowledge the financial support of the National Science Foundation and the National Security Agency, and the support of DIMACS and the Institute for Advanced Study. We extend our sincere thanks to our skillful and patient typists Adelaide Boullé, Lisa Magretto, Ellen Scott, Pat Toci, and Dorothy Westgate.

Finally, to our wives Lisa and Myriam, to our families, and to Helen Gorenstein, we gratefully acknowledge the love and commitment by which you have endured the stresses of this long project with good humor, grace and understanding.

Richard Lyons and Ronald Solomon
June, 1994

Preface to the Second Printing

Six years have passed since the first printing of this book and sixteen since the detailed classification strategy presented in Chapter 2 was first adumbrated. At present the first four books in our series have been published and the fifth and sixth are well on their way to completion. Since the first printing, some important second-generation pieces of the classification of the finite simple groups have also appeared, in particular two volumes [**BG1,P4**] presenting the revised proofs by Bender, Glauberman and Peterfalvi of the Odd Order Theorem of Feit and Thompson. We have taken the Feit-Thompson Theorem as a Background Result, but now could therefore remove [**FT1**] from the list of Background Results (see pp. 48–49) and replace it with Peterfalvi's revision of Chapter V of that paper [**P4**]. In addition, Peterfalvi's treatment of the Bender-Suzuki Theorem (Part II of [**P1**]) has now appeared in an English version[**P5**]. We have taken the opportunity of this second printing to add these new references, and to correct some misprints and other minor slips in the first printing.

We are nearing a milestone in our series – the completion of our revised proof of the Classification Theorem for $\mathcal{K}$-proper simple groups of *odd type* following the strategy carefully outlined in this volume, and in particular in our Theorems $\mathcal{C}_2$, $\mathcal{C}_3$ and $\mathcal{C}_7$.

During these same years there has been considerable research activity related to simple groups of *even type* from the so-called "unipotent" point of view, that is, by means of the analysis of the structure of 2-constrained 2-local subgroups and their interplay with one another. With the maturing of the theory of amalgams, such notable results have emerged as Stellmacher's second-generation proofs of the classifications of N-groups and thin groups, first proved by Thompson and Aschbacher, respectively. Furthermore an ambitious program proposed by Meierfrankenfeld is currently being pursued by Meierfrankenfeld, Stellmacher, Stroth, Chermak and others; it is directed towards the classification of groups of characteristic p-type for an arbitrary prime p, excepting those satisfying uniqueness conditions akin to the existence of a strongly p-embedded subgroup. This promises to remain an area of intense activity for another decade and one can at best speculate from this distance concerning what major theorems will emerge once the dust settles. Nevertheless, since it is the amalgam approach which we have proposed for our Theorem $\mathcal{C}_4$, recent results have naturally had an impact on the appropriateness of the strategy presented in this volume.

In the seminal work of Thompson on N-groups, the parameter $e(G)$ was introduced to denote the maximum value of $m_p(H)$ as p ranges over all odd primes and H ranges over all 2-local subgroups of G. A crucial division in the set of all groups of characteristic 2-type was made between those groups G with $e(G) \leq 2$

and those with $e(G) \geq 3$, the former being dubbed *quasithin*[1] groups. This subdivision persisted in the full classification of finite simple groups of characteristic 2-type (simple groups G of 2-rank at least 3 such that $F^*(H) = O_2(H)$ for every 2-local subgroup H of G) undertaken during the 1970's. It was however discovered by Gorenstein and Lyons in the work leading up to their Memoirs volume [**GL1**] that the case $e(G) = 3$ presented unique difficulties deserving special treatment. Hence [**GL1**] treats simple groups of characteristic 2-type with $e(G) \geq 4$, and the case $e(G) = 3$ was handled independently by Aschbacher in [**A13**]. Both treatments took a "semisimple" rather than a "unipotent" approach, studying p-signalizers and p-components of p-local subgroups for primes p other than the "characteristic", in these cases for $p > 2$.

The Quasithin Case ($e(G) \leq 2$) was subdivided further into the case $e(G) = 1$ (the "Thin Case") and the case $e(G) = 2$. The former was treated by Aschbacher[**A10**], while the latter was undertaken and almost completed by G. Mason. A unified treatment of the Quasithin Case is nearing completion by Aschbacher and S. D. Smith and will be published in this AMS series. When published, the Aschbacher-Smith volumes will represent a major milestone: the completion of the first published proof of the Classification of the Finite Simple Groups. But it is also pertinent to our strategy, since Aschbacher and Smith have relaxed the hypotheses somewhat. Rather than restricting themselves to the set of all simple groups of characteristic 2-type, they have partly accommodated their work to the strategic plan outlined in this volume by having their theorem encompass the larger class of all simple groups of even type. Indeed their main theorems have the following immediate consequence, in our terminology.

THE $e(G) \leq 2$ THEOREM. *Let G be a finite $\mathcal{K}$-proper simple group of even type. Suppose that every 2-local subgroup H of G has p-rank at most 2 for every odd prime p. Then one of the following conclusions holds:*

(a) *$G \in \mathcal{C}hev(2)$, and G is of twisted Lie rank at most 2, but G is not isomorphic to $U_5(q)$ for any $q > 4$; or*

(b) *$G \cong L_4(2)$, $L_5(2)$, $Sp_6(2)$, A_9, $L_4(3)$, $U_4(3)$, $G_2(3)$, M_{12}, M_{22}, M_{23}, M_{24}, J_1, J_2, J_3, J_4, HS, He, or Ru.*

However it must be noted that the Aschbacher-Smith Theorem will not complete the classification of "quasithin" groups of even type as this term is defined in the current volume (p. 82), the reason being that during the 1980's we chose to redefine "quasithin" to include the $e(G) = 3$ case for groups of even type. This was motivated both by the peculiar difficulties of the $e(G) = 3$ case noted above and by conversations concerning the Amalgam Method which suggested to us that treating groups with $e(G) \leq 3$ via the Amalgam Method would not be substantially more difficult than treating groups with $e(G) \leq 2$.

In any event the recent work of Aschbacher and Smith has now reopened the possibility of restoring "quasithin" more or less to its original meaning, or more precisely to the hypotheses of the $e(G) \leq 2$ Theorem above. The groups of even type with $e(G) = 3$ then could be treated by a "semisimple" strategy akin to the one for the proof of Theorem $\mathcal{C}_6$ discussed briefly in Section 24 of Chapter 2.

Such a change in strategy would entail a different case division defining rows 4, 5 and 6 of the classification grid (p. 85), i.e., altered hypotheses for Theorems

[1]The word "quasithin" has been used in a number of distinct senses by various authors. In this paragraph and the next, we stick to the original meaning $e(G) \leq 2$.

$\mathcal{C}_4$, $\mathcal{C}_5$ and $\mathcal{C}_6$ (pp. 105–106). Namely the $e(G) \leq 2$ Theorem would take the place of Theorem $\mathcal{C}_4$, and in rough terms the dichotomy $m_p(M) \geq 4$ or $m_p(M) \leq 3$ for certain 2-local subgroups would be replaced throughout by the dichotomy $m_p(M) \geq 3$ or $m_p(M) \leq 2$. Consequently there would be changes in the sets $\mathcal{K}_4$, $\mathcal{K}_5$ and $\mathcal{K}_6$ of target groups, the first one shrinking and the other two expanding (and all would be nonempty). In addition the uniqueness theorems $\mathcal{M}(S)$ and $U(\sigma)$ would have to be strengthened to accommodate weakened hypotheses on the p-ranks of 2-local subgroups of G. The authors are now contemplating and investigating the possibility of adopting this different approach.

Richard Lyons and Ronald Solomon
August, 2000

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Reference [P4] has been added in the Second Printing. It can be used as a substitute Background Reference for [FT1].

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GLOSSARY

PAGE	SYMBOL
90	$\sim$
97	$\succeq$
4–5	$[\mathrm{I}_1]$, $[\mathrm{I}_2]$, $[\mathrm{I}_A]$, $[\mathrm{I}_G]$, $[\mathrm{II}_A]$, $[\mathrm{II}_G]$, $[\mathrm{II}_P]$, $[\mathrm{II}_S]$
60, 101	$2A_n$, $(2)A_n$
101	$2D_n(q)$
101	$3U_4(3)$
101	$4L_3(4)$
101	$[A_1 \times A_2]\overline{K}$ (central extension)
101	$A\overline{K}$ (central extension)
81	$\mathcal{A}lt$
6, 8	A_n
7, 8	$A_n(q)$, ${}^2A_n(q)$, $A_n^+(q)$, $A_n^-(q)$
130	$\mathcal{A}(T)$
60, 104	$\mathcal{B}_2$, $\mathcal{B}_2$-group
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24	$B_{p'}(X)$
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9, 11	Co_1, Co_2, Co_3
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102	$\tilde{\mathcal{C}}_p$
100	$\mathcal{C}_{p'}$, $\mathcal{C}_{p'}$-group
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ISBN 0-8218-0960-1

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SURV/40.1.S