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Gaussian Measures

Vladimir I. Bogachev



American Mathematical Society

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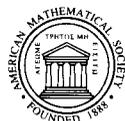
(Continued in the back of this publication)

Gaussian Measures

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Vladimir I. Bogachev



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Translated from the original Russian manuscript by Vladimir I. Bogachev.

2010 *Mathematics Subject Classification*. Primary 28C20, 60B11; Secondary 60G15, 60H07.

ABSTRACT. This book presents a systematic exposition of the modern theory of Gaussian measures. The basic properties of finite and infinite dimensional Gaussian distributions, including their linear and nonlinear transformations, are discussed. The book is intended for graduate students and researchers in probability theory, mathematical statistics, functional analysis, and mathematical physics. It contains a lot of examples and exercises. The bibliography contains 844 items; the detailed bibliographical comments and subject index are included.

Library of Congress Cataloging-in-Publication Data

Bogachev, V. I. (Vladimir Igorevich), 1961–
[Gaussovskie mery. English]
Gaussian measures / Vladimir I. Bogachev.
p. cm. — (Mathematical surveys and monographs, ISSN 0076-5376 ; v. 62)
Includes bibliographical references and index.
ISBN 0-8218-1054-5 (hc : alk. paper)
1. Gaussian measures. I. Title. II. Series: Mathematical surveys and monographs ; no. 62.

QA312.B64 1998
515'.42—dc21

98-27239
CIP

AMS softcover ISBN: 978-1-4704-1869-4

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10 9 8 7 6 5 4 3 2 1 19 18 17 16 15 14

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Preface

The modern theory of Gaussian measures lies at the intersection of the theory of random processes, functional analysis, and mathematical physics and is closely connected with diverse applications in quantum field theory, statistical physics, financial mathematics, and other areas of sciences. The study of Gaussian measures combines ideas and methods from probability theory, nonlinear analysis, geometry, linear operators, and topological vector spaces in a beautiful and nontrivial way.

The goal of this book is to present the modern theory of Gaussian measures. Chapter 1 contains basic facts about Gaussian measures on \mathbb{R}^n . In addition to the standard probabilistic facts the reader will find here a discussion of the Hermite polynomials, the Ornstein–Uhlenbeck semigroup, the Sobolev classes with Gaussian weights, the logarithmic Sobolev and Poincaré inequalities, and convexity of Gaussian measures. These analytic tools play a fundamental role in the theory. Principal results belonging to linear topological theory are discussed in Chapters 2 and 3. These include classical theorems about equivalence and singularity, zero–one laws, Cameron–Martin spaces, measurable linear functionals, and the topological properties of supports. Chapter 4 contains some inequalities and estimates related to the convexity of Gaussian measures, such as Gaussian isoperimetric inequalities, and Ehrhard’s and Anderson’s inequalities. These inequalities are applied to the study of the exponential integrability, probabilities of small balls, and large deviations. Nonlinear problems are discussed in Chapters 5 and 6, where, in particular, the Sobolev classes over Gaussian measures and Gaussian capacities are investigated. Chapter 6 deals with transformations of Gaussian measures. In addition, we give an introduction to the Malliavin calculus. In Chapter 7 we discuss some properties of finite and infinite dimensional Gaussian processes and certain related diffusion processes. These results, apart from the interest in their own right, provide a good illustration of the ideas and methods of the previous chapters.

It is worth noting here that one of the fundamental ideas in the theory of Gaussian measures is that the various centered Radon Gaussian measures are realizations of one and the same “canonical” Gaussian measure: the countable product of the standard normal Gaussian distributions on the line. This canonical measure γ is defined on the space \mathbb{R}^∞ of all real sequences. The Cameron–Martin space of γ is the classical Hilbert space l^2 . The space \mathbb{R}^∞ has a relatively poor collection of continuous linear functionals (consisting only of the functionals that depend on finitely many variables). However, the set of measurable linear functionals is much broader: it can be identified with l^2 : for any $(c_n) \in l^2$ the series $\sum_{n=1}^\infty c_n x_n$ converges γ -almost everywhere, and every measurable linear functional admits such a representation. Although the Cameron–Martin space l^2 has measure zero, every continuous linear functional on it (even every continuous linear operator) admits

a unique measurable linear extension to all of \mathbb{R}^∞ . Having in mind this basic example, one can understand better the “coordinate” interpretation of the results presented in this book. Certainly, there are problems in which the reduction to \mathbb{R}^∞ is useless. For instance, this is the case in most of the problems concerning sample path properties of Gaussian processes. Nevertheless, readers who prefer not to dwell on topological subtleties connected with infinite dimensional spaces may assume (and the isomorphism theorem from Chapter 3 gives a full justification for this) that our discussion concerns Hilbert spaces or the space \mathbb{R}^∞ . The “unique” Gaussian measure mentioned above is often encountered in the appearance of the Wiener measure on the space of continuous trajectories; in this case very interesting objects arise that have no natural analogues in the other isomorphic representations.

All auxiliary results from functional analysis and general topology used in the texts are collected in the Appendix. Prerequisites include basics of Lebesgue integration, multivariate calculus, and probability theory.

Formulas and assertions (theorems, definitions, remarks, etc.) are numbered independently of their type within each section so that the number of an assertion or a formula is preceded by the chapter number and section number.

The book contains a large number of problems (exercises). The role of the problems (in addition to the usual one) is to present a disordered collection of interesting facts and to unburden the main text from some technical details of proofs. Many problems are provided with hints; this, however, has no relation to the level of difficulty (some problems are simple exercises, but the others are really hard results borrowed from the current research literature).

The bibliography does not exhaust all the publications relating to the theory of Gaussian measures. However, together with the bibliographical comments, it enables the reader to get a sufficiently complete vision of the history of the subject as well as entertain a more thorough bibliographical research.

One can use this book as a source for several one- or two-semester courses (of different levels) for graduate students. For example, Chapter 1 can form a core for an introductory course on finite dimensional Gaussian distributions. A course on infinite dimensional Gaussian distributions can be based on Chapters 2 and 3. Chapters 5 and 6 may be helpful for lecturing on nonlinear stochastic analysis.

This book is an expanded and improved version of the Russian original based on the author’s lectures at the Department of Mechanics and Mathematics of Moscow State University. Parts of the book have been written during my visits to other universities and mathematical institutes, in particular, in Rome, Paris, Bonn, Bielefeld, Pisa, Warwick, Stockholm, Edmonton, Minneapolis, and Haifa. I am very grateful to L. Accardi, S. Albeverio, J. Baxter, G. Da Prato, G. Dell’Antonio, D. Elworthy, F. Götze, N. Jain, N. Krylov, P. Malliavin, E. Mayer-Wolf, B. Øksendal, M. Röckner, B. Schmulland, M. Zakai, and other colleagues from these institutions for the excellent working conditions. I have had many profitable discussions regarding various subjects treated here with V. Bentkus, S.G. Bobkov, N.V. Krylov, M. Ledoux, M.A. Lifshits, Yu.V. Prohorov, A.N. Shiryaev, A.V. Skorohod, O.G. Smolyanov, A.M. Stepin, V.N. Sudakov, V.V. Ulyanov, H. von Weizsäcker, A.Yu. Zaitsev, and O. Zeitouni. My special thanks are due to D.E. Aleksandrova, V. Bentkus, A.V. Kolesnikov, E.P. Krugova, N.N. Nedikov, O.V. Pugachev, T.S. Rybnikova, B. Schmulland, and N.A. Tolmachev, who read the drafts of the book and made many critical remarks.

APPENDIX A

Locally Convex Spaces, Operators, and Measures

We do not understand many matters not because our concepts are weak, but because these matters do not belong to the circle of our concepts.

Kosma Prutkov

A.1. Locally convex spaces

Basic definitions

Proofs of the facts presented below and an additional information concerning locally convex spaces can be found in [670], [220]. A nonnegative function p on a real linear space X is called a *seminorm* if $p(\lambda x) = |\lambda|p(x)$ and $p(x + y) \leq p(x) + p(y)$ for all reals λ and all vectors $x, y \in X$. A real linear space X is called a *locally convex space* if it is equipped with a family of seminorms $\mathcal{P} = (p_\alpha)_{\alpha \in A}$ on X separating the points (i.e., for every nonzero element $x \in X$ there exists an index $\alpha \in A$ such that $p_\alpha(x) > 0$). The topology on X generated by the family \mathcal{P} consists of the open sets which are arbitrary unions of the basis neighborhoods of the form

$$\left\{ x: p_{\alpha_i}(x - a) < \varepsilon_i, i = 1, \dots, n \right\}, \quad \alpha_i \in A, a \in X, n \in \mathbb{N}.$$

Clearly, different families of seminorms can define one and the same topology. A normed space is a special case of a locally convex space. The topological dual to a locally convex space X (the space of all continuous linear functionals on X) is denoted by X^* . Sometimes we use also the algebraic dual X' which is the space of all (not necessarily continuous) linear functionals on X . However, the term *dual* is reserved for the topological dual throughout this book. Every locally convex space X has a *Hamel basis*, i.e., a collection of linearly independent vectors $\{v_\alpha\}$ such that every element in X is a finite linear combination of the vectors v_α . A mapping A between linear spaces X and Y is called *affine* if $A = L + a$, where $L: X \rightarrow Y$ is a linear mapping and $a \in Y$ is a fixed vector.

A typical example of a locally convex space arising in the theory of random processes is the space \mathbb{R}^T of all real functions on a nonempty set T equipped with the topology of pointwise convergence, or, in other words, the topology generated by the family of seminorms

$$p_t(x) = |x(t)|, \quad t \in T.$$

The space \mathbb{R}^T is called the product of T copies of \mathbb{R}^1 . In particular, if T is the set \mathbb{N} of all natural numbers, then the corresponding space is denoted by \mathbb{R}^∞ . The dual to \mathbb{R}^T coincides with the linear span of the functionals $x \mapsto x(t)$, $t \in T$ (see [670, p. 137, Theorem IV.4.3]); this is clear from the fact that a linear functional

f bounded on the neighborhood $\{x: |x(t_i)| < \varepsilon, i \leq n\}$ is a linear combination of the functionals $\delta_{t_i}: x \mapsto x(t_i), i \leq n$, since it is zero on $\bigcap_{i=1}^n \text{Ker } \delta_{t_i}$.

The linear span of a set A in a linear space is denoted by $\text{span } A$.

For any sets A and B in a linear space X and any scalar λ , we put

$$\lambda A := \{\lambda a \mid a \in A\}, \quad A + B := \{a + b \mid a \in A, b \in B\}.$$

A set A in a locally convex space X is called *bounded* if, for every neighborhood of zero V in X , there exists $\lambda > 0$ such that $A \subset \lambda V$. This is equivalent to the boundedness on A of every continuous linear functional.

A set A in a locally convex space is called *symmetric* if $A = -A$. A set A in a locally convex space is called *convex* if $\lambda a + (1 - \lambda)b \in A$ for all $\lambda \in [0, 1]$ and $a, b \in A$. A convex set A is called *absolutely convex* (or *convex balanced*) if $\lambda A \subset A$ for every scalar λ with $|\lambda| \leq 1$. Clearly, this is equivalent to the convexity and symmetry of A .

The *convex hull* of a set A is the minimal convex set (denoted by $\text{conv } A$) containing A . The *absolutely convex hull* $\text{absconv } A$ of a set A is defined analogously. The *closed absolutely convex hull* of a set A is the minimal absolutely convex closed set containing A .

We say that a locally convex space (X, τ_X) is continuously embedded into a locally convex space (Y, τ_Y) if X is a linear subspace in Y and the natural embedding $(X, \tau_X) \rightarrow (Y, \tau_Y)$ is continuous. If, in addition, X is dense in Y , then we say that X is densely embedded.

Let E be a linear space and let F be a linear subspace in the space of all linear functionals on E , separating the points in E (i.e., for every two different elements in E , there is a functional from F taking on these elements different values). Denote by $\sigma(E, F)$ the locally convex topology on E generated by the family of seminorms

$$p_f(x) = |f(x)|, \quad f \in F.$$

This is the topology of pointwise convergence on F if the elements of E are considered as functionals on F . Two typical examples: the weak topology $\sigma(X, X^*)$ on the locally convex space X and the $*$ -weak topology $\sigma(X^*, X)$ on its dual. An important property of the topology $\sigma(E, F)$ is that the dual to $(E, \sigma(E, F))$ coincides (as a linear space) with F , i.e., every linear functional that is continuous in the topology $\sigma(E, F)$ has the form $x \mapsto f(x), f \in F$. In particular, any continuous in the topology $\sigma(X^*, X)$ linear functional F on the space X^* has the form $F(f) = f(a)$ for some $a \in X$.

The Mackey topology $\tau_M(X^*, X)$ on X^* is defined by means of the seminorms

$$p_K(f) = \sup_{x \in K} |f(x)|, \quad K \in \mathcal{K},$$

where \mathcal{K} is the family of all absolutely convex $\sigma(X, X^*)$ -compact subsets of X . A proof of the following Mackey theorem can be found in [670, p. 131, Ch. IV, 3.2, Corollary 1].

A.1.1. Theorem. *Every linear functional F on X^* continuous in the Mackey topology $\tau_M(X^*, X)$ has the form $F(f) = f(a)$ for some $a \in X$.*

A topological space T is called *metrizable* if the topology of T is generated by a metric. A locally convex space is metrizable precisely when its topology is generated by a countable family of seminorms. A complete metrizable locally convex space is called a *Fréchet space*. For example, the countable product of the real lines \mathbb{R}^∞ is a

Fréchet space. Any Banach space (i.e., complete normed space) is a Fréchet space. The most typical examples of Banach spaces encountered in the theory of Gaussian measures are: the space l^∞ of all bounded sequences $x = (x_n)$ with $\|x\| = \sup_n |x_n|$, its closed subspace c_0 consisting of all sequences converging to zero, the spaces $L^p(\sigma)$, where $p \in [1, \infty]$, and the space $C[a, b]$ of all continuous functions on $[a, b]$ with the sup-norm.

Every locally convex space X is completely regular, i.e., for every point $x \in X$ and every neighborhood U of x , there exists a continuous function $f: X \rightarrow [0, 1]$ such that $f(x) = 1$ and $f = 0$ outside U (it suffices to be able to construct such a function for $x = 0$ and any neighborhood of the form $U = \{p < 1\}$, where p is a continuous seminorm; in this case one can put $f(z) = 1 - p(z)$ if $z \in U$ and $f(z) = 0$ if $z \notin U$).

A.1.2. Lemma. *Let K be a compact set in a completely regular topological space X and let U be an open set containing K . Then:*

- (i) *There exists a continuous function $f: X \rightarrow [0, 1]$ equal 1 on K and 0 outside U ;*
- (ii) *Every continuous function φ on K extends to a continuous function ψ on X such that $\sup_X |\psi| = \sup_K |\varphi|$ and $\psi = 0$ outside U .*

PROOF. A proof of (i) can be found in [220, p. 19]. For the proof of (ii) it suffices to find a continuous extension of φ to X with preservation of maximum and multiply it by the function from (i). The Stone–Weierstrass theorem implies the existence of a bounded continuous function g on X equal φ on K . Now we can replace g by the function $\theta(g)$, where $\theta(t) = t$ if $|t| \leq \sup |\varphi|$, $\theta(t) = \sup |\varphi|$ if $|t| > \sup |\varphi|$. \square

Recall that a mapping F between topological spaces is called sequentially continuous if $F(x_n) \rightarrow F(x)$ whenever $x_n \rightarrow x$.

A partially ordered set Λ is called *directed* if, for every α and β from Λ , there is $\gamma \in \Lambda$ such that $\alpha \leq \gamma$ and $\beta \leq \gamma$. A *net* of elements of the set X is a subset $\{x_\lambda\}_{\lambda \in \Lambda} \subset X$ indexed by a directed set Λ . The concept of a net generalizes that of a sequence.

A net $\{x_\lambda\}_{\lambda \in \Lambda}$ in a locally convex space X is called *fundamental* (or Cauchy) if it is fundamental with respect to every seminorm q from some family of seminorms generating the topology of X (i.e., for every $\varepsilon > 0$, there exists $\lambda_\varepsilon \in \Lambda$ such that $q(x_\alpha - x_\beta) < \varepsilon$ for all $\alpha \geq \lambda_\varepsilon, \beta \geq \lambda_\varepsilon$).

- A.1.3. Definition.**
- (i) *A locally convex space X is called sequentially complete if every Cauchy sequence in X converges.*
 - (ii) *A locally convex space X is called complete if every fundamental net in X converges.*
 - (iii) *A subset A of a locally convex space X is called sequentially closed if it contains the limit of every convergent sequence of its elements.*

In a similar manner one defines the completeness and the sequential completeness for subsets of X .

It is clear that every complete locally convex space is sequentially complete. An infinite dimensional Hilbert space with the weak topology gives an example of a sequentially complete locally convex space which is not complete (Problem A.3.25). Similarly to metric spaces, locally convex spaces possess completions.

A.1.4. Theorem. *Every locally convex space X has a completion \tilde{X} , i.e., there exist a complete locally convex space \tilde{X} , a linear subspace X_0 everywhere dense in \tilde{X} and a linear homeomorphism $h: X \rightarrow X_0$.*

The product $X \times Y$ of locally convex spaces X and Y possesses the natural structure of a locally convex space: the corresponding family of seminorms is defined by $(x, y) \mapsto p(x) + q(y)$, where p and q are representatives of the families of seminorms defining the topologies of X and Y , respectively.

Convex sets and compact sets

Let us describe a construction connected with convex sets which finds numerous applications in measure theory on linear spaces. Let A be an absolutely convex set in a locally convex space X . Denote by E_A the linear span of A . Put

$$p_A(x) = \inf\{r > 0: x \in rA\}, \quad x \in E_A.$$

The function p_A on E_A is called the *Minkowski functional* (or the gauge function) of the set A .

A.1.5. Theorem. *Let A be an absolutely convex sequentially closed bounded set in a locally convex space X . Then the function p_A is a norm on E_A , whose closed unit ball is A . In addition, the natural embedding (E_A, p_A) into X is continuous. If A is sequentially complete, then (E_A, p_A) is a Banach space.*

The proof can be found in [220, p. 444, Lemma 6.5.2].

Let us formulate a number of results about compact sets in locally convex spaces that we use in the main text.

A.1.6. Proposition. *In any complete locally convex space, the closed absolutely convex hull of a compact set is compact.*

The previous statement may fail for not necessarily complete spaces. Part (ii) of the next proposition is due to [644]. The proof below is borrowed from [90].

A.1.7. Proposition. (i) *The metrizability of a compact space K is equivalent to the existence of a sequence of continuous functions separating the points in K . A compact set K in a locally convex space X is metrizable if and only if there exists a sequence $\{l_n\} \subset X^*$ separating the points of K .*

(ii) *The closed absolutely convex hull \tilde{K} of any metrizable compact set K in a locally convex space X is metrizable; if X is sequentially complete, then \tilde{K} is a metrizable compact space.*

PROOF. (i) Clearly, on any metrizable compact set there is a sequence of continuous functions separating the points. Recall that if on a set K one has two Hausdorff topologies τ_1 and τ_2 with respect to which K is compact and the natural embedding $(K, \tau_1) \rightarrow (K, \tau_2)$ is continuous, then this mapping is a homeomorphism. Therefore, if continuous functions f_n separate the points of a compact set K , the metric

$$\varrho(x, y) = \sum_{n=1}^{\infty} 2^{-n} \frac{|f_n(x) - f_n(y)|}{1 + |f_n(x) - f_n(y)|}$$

generates the initial topology of K . This simple observation implies also that on a compact set K in a locally convex space X the weak topology coincides with the initial one and, in addition, that the weak topology coincides with every topology

on K generated by any family of continuous linear functionals separating the points in K . Therefore, in the case where there is a countable family with this property, the corresponding topology is defined by the aforementioned metric.

Conversely, if a compact set K in a locally convex space X is metrizable, then the weak topology on K has a countable base of the form

$$\left\{x: |l_j^a(x - a)| < k^{-1}, j = 1, \dots, n(a)\right\}, \quad a \in A, l_j^a \in X^*, k \in \mathbb{N},$$

where $A \subset K$ is an at most countable set. Therefore, there exists an at most countable family of continuous linear functionals separating the points in K .

(ii) Let K be a metrizable compact set in a locally convex space X . Assume first that X is complete. According to the Riesz theorem, the dual to $C(K)$ is identified with the space of signed Borel measures on K . By the Banach–Alaoglu theorem, the closed unit ball U in $C(K)^*$ is compact in the $*$ -weak topology. Since the space $C(K)$ is separable (see Problem A.3.23), then, by virtue of (i), U is compact metrizable in the weak topology. Let us consider the mapping

$$I: U \rightarrow X, \quad I(m) = \int_K x m(dx),$$

where the integral is understood in the sense of Pettis (see Section A.3 below), and its existence follows from the completeness of X (see the proof of Lemma A.3.20 below). It is easy to see that this mapping is continuous if U is equipped with the $*$ -weak topology and X is given the weak topology. Therefore, the absolutely convex set $I(U)$ is weakly compact in X . Moreover, by the metrizability of U , this set is metrizable (see [231, Theorem 4.4.15]). Clearly, $I(U)$ contains the closed absolutely convex hull of K , since $K \subset I(U)$ by virtue of the equality $k = I(\delta_k)$, where δ_k is the probability measure at the point k . Therefore, the closure of the absolutely convex hull of K is a metrizable compact set as a closed subset of a metrizable compact space (in fact, as can be easily shown, $I(U)$ coincides with the closed absolutely convex hull of K). It remains to note that the first claim from (ii) follows now from the existence of a completion of X . \square

A.2. Linear operators

Bounded operators

Recall some well-known facts from the theory of linear operators. We consider below only real spaces.

The range of a linear operator A on a space X is denoted by $A(X)$. $\text{Ker } A$ stands for the kernel of the operator A (the preimage of zero). Denote by $\mathcal{L}(X, Y)$ the space of all continuous linear operators from a locally convex space X to a locally convex space Y . Let $\mathcal{L}(X) := \mathcal{L}(X, X)$. If X and Y are normed spaces, then $\mathcal{L}(X, Y)$ is equipped with the operator norm $\|\cdot\|_{\mathcal{L}(X, Y)}$.

An operator A on a normed space is called *compact* if it takes the unit ball to a precompact set. The space of all compact operators from X to Y is denoted by $\mathcal{K}(X, Y)$. Put $\mathcal{K}(X) := \mathcal{K}(X, X)$.

The following useful result is called the *closed graph theorem* (see [670, p. 78, Theorem III.2.3]).

A.2.1. Theorem. *Let X and Y be two Fréchet spaces (e.g., Banach spaces). A linear mapping $A: X \rightarrow Y$ is continuous if and only if its graph $\{(x, Ax), x \in X\}$*

is closed in $X \times Y$. In particular, if Banach spaces X and Y are continuously linearly embedded into a locally convex space Z and $X \subset Y$, then the natural embedding $X \rightarrow Y$ is continuous.

Let H be a Hilbert space. In the definitions and statements below for the sake of simplicity of formulations we use the notation which means implicitly that the spaces in question are infinite dimensional; clearly, in the finite dimensional case we have in mind finite bases, etc.

A.2.2. Definition. An operator $A \in \mathcal{L}(H)$ is called symmetric if $(Ax, y) = (x, Ay)$ for all $x, y \in H$. A symmetric operator $A \in \mathcal{L}(H)$ is called nonnegative if $(Ax, x) \geq 0$ for all $x \in H$.

Note that in real spaces (unlike the complex ones) the positivity of the quadratic form (Ax, x) does not imply the symmetry of A . For every nonnegative operator $B \in \mathcal{L}(H)$, there exists a unique nonnegative operator $C \in \mathcal{L}(H)$ denoted by \sqrt{B} such that $C^2 = B$. For $A \in \mathcal{L}(H)$ we put

$$|A| := \sqrt{A^*A}.$$

Note that for any $h \in H$ one has

$$(|A|x, |A|x) = (A^*Ax, x) = (Ax, Ax).$$

An operator $K \in \mathcal{L}(H)$ is compact precisely when so is the operator $|K|$. According to the Hilbert–Schmidt theorem, for any compact symmetric linear operator A on a separable Hilbert space, there exists an orthonormal basis $\{e_n\}$ such that $Ae_n = \alpha_n e_n$, where the eigenvalues α_n tend to zero.

A.2.3. Definition. An operator on a Hilbert space which preserves the inner product is called isometric (or an isometry). A linear operator is called orthogonal if it is invertible and preserves the inner product.

For example, the operator $x \mapsto (0, x_1, x_2, \dots)$ on l^2 is isometric, but not orthogonal.

The polar decomposition of an operator A is the representation

$$A = U|A|,$$

where $U \in \mathcal{L}(H)$ is a linear isometry on the closure of $|A|(H)$ given by $U(|A|x) = Ax$ and zero on the orthogonal complement of $|A|(H)$. Note that U is well-defined, which follows from the fact that if $|A|v = 0$, then $Av = 0$. The operator U is called a partial isometry. If an operator A is injective (i.e., has zero kernel) and has the dense range, then U is an orthogonal operator.

The polar decomposition can be written also in the form

$$A = \sqrt{AA^*}V,$$

where V is the operator adjoint to the partial isometry from the polar decomposition for A^* . This representation yields the following simple, but useful fact: for every operator $A \in \mathcal{L}(H)$ with dense range, one can find an injective nonnegative symmetric operator B such that $B(H) = A(H)$. Indeed, by the factorization by the kernel of A this claim reduces to the case where A is injective. The range of the symmetric nonnegative operator $B = \sqrt{AA^*}$ is dense as well, hence, as one can easily verify, this operator is injective. In addition, $B(H) = A(H)$, which follows from the formula above, since V is orthogonal.

A.2.4. Proposition. *Let E be also a Hilbert space. Then $\mathcal{K}(H, E)$ coincides with the closure of the class of the finite dimensional continuous operators with respect to the operator norm.*

A.2.5. Definition. *Let H and E be two Hilbert spaces. An operator $A \in \mathcal{L}(H, E)$ is called a Hilbert–Schmidt operator if the series*

$$\sum_{\alpha} \|Ae_{\alpha}\|_E^2 \tag{A.2.1}$$

converges for some orthonormal basis $\{e_{\alpha}\}$ in H .

If the space H is nonseparable, then the membership of A in the class of Hilbert–Schmidt operators means that A is zero on the orthogonal complement of some separable subspace $H_0 \subset H$ and

$$\sum_{n=1}^{\infty} \|Ae_n\|_E^2 < \infty$$

for some orthonormal basis $\{e_n\}$ in H_0 .

A.2.6. Proposition. (i) *If the series in (A.2.1) converges for some orthonormal basis in H , then it also does for every orthonormal basis in H and its sum does not depend on a basis.*

(ii) *A symmetric operator $A \in \mathcal{L}(H)$ is a Hilbert–Schmidt operator precisely when there exists an orthonormal basis $\{e_{\alpha}\}$ in H consisting of the eigenvectors corresponding to the eigenvalues a_{α} among which there exist at most countably many nonzero values a_n such that $\sum_{n=1}^{\infty} a_n^2 < \infty$.*

(iii) *Let $A = U|A|$ be the polar decomposition of an operator $A \in \mathcal{L}(H)$. Then A is a Hilbert–Schmidt operator precisely when so is $|A|$.*

PROOF. It suffices to prove these statements for separable H . Let $\{e_n\}$, $\{\varphi_i\}$ and $\{\psi_j\}$ be three orthonormal bases in H . Then

$$\sum_{n=1}^{\infty} \|Ae_n\|^2 = \sum_{n=1}^{\infty} \sum_{i=1}^{\infty} (Ae_n, \varphi_i)^2 = \sum_{n=1}^{\infty} \sum_{i=1}^{\infty} (e_n, A^* \varphi_i)^2 = \sum_{i=1}^{\infty} \|A^* \varphi_i\|^2.$$

Since $(A^*)^* = A$, we get $\sum_{n=1}^{\infty} \|Ae_n\|^2 = \sum_{j=1}^{\infty} \|A\psi_j\|^2$. Sufficiency of the condition in

(ii) is clear. Note that A is compact by the estimate $\|\sum_{i=n}^{\infty} x_i Ae_i\|^2 \leq \sum_{i=n}^{\infty} \|Ae_i\|^2 \|x\|^2$,

which yields the convergence of the finite dimensional operators $\sum_{i=1}^n x_i Ae_i$ in the operator norm. By the Hilbert–Schmidt theorem, A has an eigenbasis, whence the necessity of the condition in (ii). Statement (iii) is obvious. \square

The class of all Hilbert–Schmidt operators from H to E is denoted by $\mathcal{H}(H, E)$. We put

$$\mathcal{H} := \mathcal{H}(H) := \mathcal{H}(H, H).$$

A.2.7. Definition. *Let H and E be separable Hilbert spaces. A continuous linear mapping $A: H^k \rightarrow E$ is called a k -linear Hilbert–Schmidt mapping on H if*

for some orthonormal basis $\{e_n\}$ in H one has

$$\sum_{i_1, \dots, i_k=1}^{\infty} \|A(e_{i_1}, \dots, e_{i_k})\|_E^2 < \infty.$$

In this case the corresponding sum is finite for every orthonormal basis in H and is independent of bases.

Denote by $\mathcal{H}_k(H, E)$ the space of all k -linear Hilbert–Schmidt mappings from H to E . Put

$$\mathcal{H}_k := \mathcal{H}_k(H, \mathbb{R}^1).$$

The space \mathcal{H}_k is naturally isomorphic with the space $\mathcal{H}(H, \mathcal{H}_{k-1})$. Indeed, for every $\Psi \in \mathcal{H}_k$, the operator

$$T: H \rightarrow \mathcal{H}_{k-1}, \quad T(h)(a_1, \dots, a_{k-1}) = \Psi(a_1, \dots, a_{k-1}, h),$$

is Hilbert–Schmidt; conversely, for any $T \in \mathcal{H}(H, \mathcal{H}_{k-1})$, the k -linear form

$$(h_1, \dots, h_k) \mapsto T(h_k)(h_1, \dots, h_{k-1})$$

is Hilbert–Schmidt.

Note that the space $\mathcal{H}_k(H, E)$ with the inner product

$$(A, B)_{\mathcal{H}_k} = \sum_{i_1, \dots, i_k=1}^{\infty} \left(A(e_{i_1}, \dots, e_{i_k}), B(e_{i_1}, \dots, e_{i_k}) \right)_E$$

is separable Hilbert. In particular, the set of all Hilbert–Schmidt operators on H equipped with the inner product

$$(A, B)_{\mathcal{H}} = \sum_{n=1}^{\infty} (Ae_n, Be_n)_H$$

is a separable Hilbert space.

A.2.8. Definition. Let H be a separable Hilbert space. An operator $A \in \mathcal{L}(H)$ is called nuclear (or a trace class operator) if $|A|$ has an orthonormal basis formed by its eigenvectors corresponding to the eigenvalues α_n such that

$$\sum_{n=1}^{\infty} \alpha_n < \infty.$$

For a proof of the next result, see, e.g., [296, Ch. III, §8].

A.2.9. Proposition. (i) For any nuclear operator A , the sum of the series $\sum_{n=1}^{\infty} (Ae_n, e_n)_H$ does not depend on an orthonormal basis $\{e_n\}$.

(ii) A symmetric operator $A \in \mathcal{L}(H)$ is nuclear precisely when for every orthonormal basis $\{e_n\}$ in H the series

$$\sum_{n=1}^{\infty} (Ae_n, e_n)_H$$

converges.

(iii) A symmetric operator $A \in \mathcal{L}(H)$ is nuclear if and only if there exists an orthonormal basis $\{e_n\}$ in H consisting of the eigenvectors A corresponding to the eigenvalues α_n such that

$$\sum_{n=1}^{\infty} |\alpha_n| < \infty.$$

Denote by $\mathcal{L}_{(1)}(H)$ the class of all nuclear operators on H . Clearly, $\mathcal{L}_{(1)}(H) \subset \mathcal{H}(H) \subset \mathcal{K}(H)$. For every $A \in \mathcal{L}_{(1)}(H)$, the sum

$$\text{trace } A := \sum_{n=1}^{\infty} (Ae_n, e_n)$$

(which is independent of an orthonormal basis $\{e_n\}$) is called the *trace* of the operator A . The function

$$\|A\|_{(1)} := \text{trace } |A|$$

is a norm on $\mathcal{L}_{(1)}(H)$, with respect to which this space is Banach.

A.2.10. Proposition. (i) *The composition of two continuous operators between Hilbert spaces is a Hilbert–Schmidt operator if so is at least one of these two operators. In addition, if $A \in \mathcal{H}(H)$ and $B \in \mathcal{L}(H)$, then $\|AB\|_{\mathcal{H}}$ and $\|BA\|_{\mathcal{H}}$ are majorized by $\|A\|_{\mathcal{H}}\|B\|_{\mathcal{L}(H)}$.*

(ii) *The composition of two Hilbert–Schmidt operators on H as well as the composition of a nuclear operator and a continuous operator on H is a nuclear operator.*

(iii) *If $A \in \mathcal{H}(H)$, then $A^* \in \mathcal{H}(H)$ and $\|A\|_{\mathcal{H}} = \|A^*\|_{\mathcal{H}}$.*

PROOF. Note that (iii) has been shown in the proof of Proposition A.2.6. It suffices to prove (i) for operators A and B acting on one and the same space H . Since $\|BAe_n\| \leq \|B\|_{\mathcal{L}(H)}\|Ae_n\|$, we have $BA \in \mathcal{H}(H)$ and $\|BA\|_{\mathcal{H}} \leq \|B\|_{\mathcal{L}(H)}\|A\|_{\mathcal{H}}$. Using that $\|B^*\|_{\mathcal{L}(H)} = \|B\|_{\mathcal{L}(H)}$, we get from (iii) that $AB = (B^*A^*)^* \in \mathcal{H}(H)$ and $\|AB\|_{\mathcal{H}} \leq \|B\|_{\mathcal{L}(H)}\|A\|_{\mathcal{H}}$. Finally, the composition of two Hilbert–Schmidt operators is nuclear by the inequality $|(ABe_i, e_i)| = |(Be_i, A^*e_i)| \leq \|Be_i\|\|A^*e_i\|$. The second claim in (ii) follows from the first one together with the obvious observation that any nuclear operator A can be written as $A = A_1A_2$, where A_1 and A_2 are Hilbert–Schmidt operators (e.g., using the polar decomposition $A = U|A|$, one can put $A_1 = U\sqrt{|A|}$, $A_2 = \sqrt{|A|}$). \square

The following theorem describes an interesting connection between nuclear operators and functionals on the space of operators. Its proof can be found in [296, Ch. III, Theorem 12.3].

A.2.11. Theorem. *Let $T \in \mathcal{L}_{(1)}(H)$. The functional $K \mapsto \text{trace}(TK)$ on $\mathcal{K}(H)$ is continuous and its norm equals $\|T\|_{(1)}$. In addition, every continuous linear functional on $\mathcal{K}(H)$ admits such a representation, i.e., the space $\mathcal{K}(H)^*$ is naturally isomorphic to $\mathcal{L}_{(1)}(H)$.*

A.2.12. Proposition. *Let H_1, H_2, E be Hilbert spaces, let $A_i \in \mathcal{L}(H_i, E)$, $i = 1, 2$, and let $A_1(H_1) \subset A_2(H_2)$. If A_2 is compact or Hilbert–Schmidt, then so is the operator A_1 . If $H_1 = H_2 = E$ and A_2 is a nuclear operator, then A_1 is nuclear as well.*

PROOF. If A_2 is injective, then the claim follows from Proposition A.2.10. Indeed, the operator $A_2^{-1}A_1$ is well-defined by virtue of the inclusion $A_1(H_1) \subset A_2(H_2)$ and, moreover, is continuous by the closed graph theorem, since the relationships $x_n \rightarrow x$, $A_2^{-1}A_1x_n \rightarrow y$ imply that $A_1x_n \rightarrow A_2y$, whence $A_1x = A_2y$, hence $A_2^{-1}A_1x = y$. In addition, $A_1 = A_2A_2^{-1}A_1$. The general case reduces to the case above by replacing A_2 by the operator $\tilde{A}_2: H_2/\text{Ker } A_2 \rightarrow E$. \square

A.2.13. Lemma. (i) Let (T, μ) be a measurable space and let K be in $L^2(T \times T, \mu \otimes \mu)$. Then the operator

$$Tx(t) = \int_T K(t, s)x(s) \mu(ds)$$

on $L^2(\mu)$ is Hilbert–Schmidt. Conversely, every Hilbert–Schmidt operator on $L^2(\mu)$ admits such a representation.

- (ii) Let H be a Hilbert space and let $A: H \rightarrow C[a, b]$ be a continuous linear mapping. Then the composition of this mapping with the natural embedding $C[a, b] \rightarrow L^2[a, b]$ is a Hilbert–Schmidt operator. The same is true when $C[a, b]$ and $L^2[a, b]$ are replaced, respectively, by $L^\infty(\Omega, \mu)$ and $L^2(\Omega, \mu)$, where (Ω, μ) is any probability space.
- (iii) Let $W^{2,1}[0, 1]$ be the Sobolev space. Then its natural embedding to $L^2[0, 1]$ is a Hilbert–Schmidt operator.

PROOF. The first claim is an exercise in functional analysis (see, e.g., [296, Ch. III, §9]). The proof of (ii) can be found in [603, Theorem 19.2.6] (the simpler case where Ω is a compact space is considered in [603, Proposition 17.3.7]). Claim (iii) follows from (ii), since $W^{2,1}[0, 1] \subset C[0, 1]$. \square

A.2.14. Definition. A continuous linear operator D on a Hilbert space H is called diagonal if there exists an orthonormal basis $\{e_n\}$ in H consisting of the eigenvectors of D .

Let us recall a theorem due to von Neumann (see [326, Theorem 14.13]).

A.2.15. Theorem. For every symmetric bounded linear operator A on a separable Hilbert space H and every $\varepsilon > 0$, there exist a diagonal operator D_ε and a symmetric Hilbert–Schmidt operator S_ε on H such that $A = D_\varepsilon + S_\varepsilon$ and $\|S_\varepsilon\|_{\mathcal{H}(H)} \leq \varepsilon$.

A.2.16. Lemma. Let E, H be two Hilbert spaces, $A \in \mathcal{L}(E, H)$. Suppose that H is separable and the operator A is injective. Then E is separable as well.

PROOF. The set $A^*(H^*)$ is dense in E^* by the injectivity of A . Hence the space E^* is separable, which implies the separability of E . \square

Semigroups and unbounded operators

A linear mapping A defined on a dense linear subspace $D(A)$ in a Hilbert space H and taking values in H is called a densely defined linear operator. A densely defined operator is called closed if its graph is a closed set in $H \times H$. If A is a densely defined linear operator, then the domain $D(A^*)$ of the operator A^* is defined as the set of all vectors $y \in H$ such that the functional $x \mapsto (Ax, y)$ is continuous on $D(A)$ with the norm from H . By the Riesz theorem, there exists $z \in H$ such that $(Ax, y) = (x, z)$ for all $x \in D(A)$. By definition, $A^*y = z$. Note that the set $D(A^*)$ may coincide with $\{0\}$.

We say that A is a symmetric operator in a (real) Hilbert space H if A is a linear mapping from a dense linear subspace $D(A) \subset H$ (called the domain of A) to H such that $(Ax, y) = (x, Ay)$ for all $x, y \in D(A)$. For any symmetric operator, the adjoint operator is defined at least on $D(A)$, hence is also densely defined.

A.2.17. Definition. A symmetric linear operator is called self-adjoint if it coincides with its adjoint (i.e., $D(A^*) = D(A)$ and $A^* = A$ on this domain).

Unlike the case of bounded operators, a symmetric operator may fail to be self-adjoint.

A.2.18. Definition. Let X be a Banach space. A family $(T_t)_{t \geq 0} \subset \mathcal{L}(X)$ is called a strongly continuous semigroup on X if $T_0 = I$, $T_{t+s} = T_t T_s$ for all $t, s \geq 0$, and, for every $x \in X$, the mapping $t \mapsto T_t x$ from $[0, \infty)$ to X is continuous.

One of the fundamental results in the theory of operator semigroups states that the linear subspace

$$D(L) := \left\{ h \in X : \lim_{t \rightarrow 0} \frac{T_t h - h}{t} \text{ exists in the norm of } X \right\}$$

is dense in X (see [214, p. 620, Lemma VIII.1.8]). In addition, the linear operator L defined on $D(L)$ by the equality

$$Lh = \lim_{t \rightarrow 0} \frac{T_t h - h}{t},$$

is closed. This operator is called the *generator* of the semigroup $(T_t)_{t \geq 0}$.

A.3. Measures and measurability

Measures and integrals

Concerning the facts from the Lebesgue integration theory mentioned below, see [697, Ch. II]. The term “measure” means a countably additive bounded nonnegative measure on a σ -field of sets \mathcal{M} . For two measures μ and ν on \mathcal{M} , the absolute continuity of μ with respect to ν is denoted by $\mu \ll \nu$. If $\mu \ll \nu$ and $\nu \ll \mu$, then μ and ν are called equivalent (notation: $\mu \sim \nu$). The mutual singularity of two measures is denoted by $\mu \perp \nu$. For every measure μ on \mathcal{M} , the symbol \mathcal{M}_μ denotes the Lebesgue completion of \mathcal{M} with respect to μ . The sets from \mathcal{M}_μ are called μ -measurable. The functions measurable with respect to \mathcal{M}_μ are called μ -measurable. A mapping that coincides with a given mapping F μ -a.e. is called a *modification* or *version* of F . For $p \geq 1$ by $L^p(\mu)$ we denote the Banach spaces of μ -measurable functions whose absolute values are integrable in power p . The norm in $L^p(\mu)$ is denoted by $\|\cdot\|_{L^p(\mu)}$ or by $\|\cdot\|_p$. The integral of a function f over a set A with respect to a measure μ is denoted by $\int_A f(x) \mu(dx)$ or by $\int_A f d\mu$. In the case of integrating over the whole space the limits of integration are sometimes omitted. The indicator function of a set A is denoted by I_A ($I_A(x) = 1$ if $x \in A$, $I_A(x) = 0$ if $x \notin A$). If μ is a measure and A is a μ -measurable set, then the measure $\mu|_A := I_A \cdot \mu$ (i.e., $\mu|_A(B) = \mu(A \cap B)$) is called the restriction of μ to the set A .

It is known that every signed measure m (a countably additive real function on a σ -field \mathcal{B} in a space Ω) can be written as $m = m^+ - m^-$, where m^+ and m^- are mutually singular nonnegative measures on \mathcal{B} called the positive and negative parts of m , respectively. The quantity $\|m\| := m^+(\Omega) + m^-(\Omega)$ is called the total variation of m (or shortly the variation of m). The variation is a norm on the linear space of all signed measures on \mathcal{B} making it into a Banach space. The variation distance $\|\mu - \nu\|$ between two nonnegative measures μ and ν on σ -field \mathcal{B} can be written as

$$\|\mu - \nu\| = \sup \left\{ |\mu(B) - \nu(B)| + |\mu(\Omega \setminus B) - \nu(\Omega \setminus B)|, B \in \mathcal{B} \right\}.$$

Let μ_n be probability measures defined on σ -fields \mathcal{B}_n in spaces X_n . Put $X = \prod_{n=1}^{\infty} X_n$. Let $\mathcal{B} = \otimes_{n=1}^{\infty} \mathcal{B}_n$ be the σ -field generated by all sets of the form $B = B_1 \times B_2 \times \cdots \times B_n \times X_{n+1} \times X_{n+2} \cdots$, where $B_i \in \mathcal{B}_i$. Recall that the countable product $\bigotimes_{n=1}^{\infty} \mu_n$ is a probability measure μ on \mathcal{B} (called a *product-measure*) defined by $\mu(B) = \mu_1(B_1) \cdots \mu_n(B_n)$ for the sets B of the form above. It is readily seen that this set function is well-defined. A well-known theorem in measure theory states μ is countably additive (which is not obvious) on the algebra generated by such sets, hence it uniquely extends to a measure on \mathcal{B} denoted also by $\bigotimes_{n=1}^{\infty} \mu_n$ and called the product of the μ_n 's. The construction of countable products enables one to define arbitrary products $\bigotimes_{\alpha} \mu_{\alpha}$ of probability measures on σ -fields \mathcal{B}_{α} in spaces X_{α} . To this end, it suffices to note that the σ -field $\otimes_{\alpha} \mathcal{B}_{\alpha}$, generated by the sets $\prod_{\alpha} C_{\alpha}$, where $C_{\alpha} \in \mathcal{B}_{\alpha}$ and only finitely many of the C_{α} 's differ from X_{α} , consists of the sets of the form $E = C \times Y$, where $C \in \bigotimes_{n=1}^{\infty} \mathcal{B}_{\alpha_n}$ and $Y = \prod_{\beta \neq \alpha_n} X_{\beta}$. Hence we may put $\bigotimes_{\alpha} \mu_{\alpha}(E) = \bigotimes_{n=1}^{\infty} \mu_{\alpha_n}(C)$.

Let μ be a measure on a measurable space (X, \mathcal{B}) , let Y be a space with a σ -field \mathcal{E} , and let $f: X \rightarrow (Y, \mathcal{E})$ be a μ -measurable mapping (i.e., $f^{-1}(\mathcal{E}) \subset \mathcal{B}_{\mu}$; such mappings are also called $(\mathcal{B}_{\mu}, \mathcal{E})$ -measurable). Then the measure

$$\mu \circ f^{-1}: A \mapsto \mu(f^{-1}(A))$$

on \mathcal{E} is called the image of the measure μ under the mapping f . A function φ on Y is integrable with respect to $\mu \circ f^{-1}$ precisely when the function $\varphi \circ f$ is μ -integrable on X . In this case the following identity called *the change of variables formula* holds true:

$$\int_Y \varphi(y) \mu \circ f^{-1}(dy) = \int_X \varphi(f(x)) \mu(dx). \quad (\text{A.3.1})$$

A.3.1. Definition. A family $\mathcal{F} \subset L^1(\mu)$ is called *uniformly integrable* if

$$\lim_{C \rightarrow \infty} \sup_{f \in \mathcal{F}} \int_{|f| \geq C} |f(x)| \mu(dx) = 0.$$

A sufficient condition for the uniform integrability is the estimate

$$\sup_{f \in \mathcal{F}} \int |f(x)| \log |f(x)| \mu(dx) < \infty.$$

Concerning the uniform integrability we refer the reader to Meyer's book [541, Chapter II, §2] and Shiryaev's book [697, Chapter II]. The next classical result is called the Vitali–Lebesgue theorem.

A.3.2. Theorem. Let $\{f_n\} \subset L^1(\mu)$ be a sequence convergent almost everywhere (or in measure) to a function f . If the sequence $\{f_n\}$ is uniformly integrable, then it converges to f in the norm of $L^1(\mu)$.

Let (X, \mathcal{M}, μ) be a space with measure and let $\mathcal{A} \subset \mathcal{M}$ be a one more σ -field. Recall that for every integrable function f there exists a function $E^{\mathcal{A}}f$ measurable

with respect to \mathcal{A} such that

$$\int_X \mathbb{E}^{\mathcal{A}} f(x) g(x) \mu(dx) = \int f(x) g(x) \mu(dx)$$

for every bounded function g measurable with respect to \mathcal{A} . The function $\mathbb{E}^{\mathcal{A}} f$ is called *the conditional expectation* of f with respect to \mathcal{A} .

A.3.3. Definition. Let (X, \mathcal{M}, μ) be a probability space, $T \subset \mathbb{R}^1$ and let \mathcal{A}_t , $t \in T$, be an increasing family of σ -fields contained in \mathcal{M} . The family $\{f_t\}_{t \in T}$ of μ -integrable functions is called a *martingale with respect to $\{\mathcal{A}_t\}$* if

$$\mathbb{E}^{\mathcal{A}_s} f_t = f_s, \quad \forall t, s \in T, s \leq t.$$

If in the relationship above we replace the sign “=” by “ \geq ”, then we get the definition of a *submartingale*.

A.3.4. Example. Let $f \in L^1(P)$, where (Ω, \mathcal{F}, P) is a probability space, and let $\{\mathcal{A}_t\}_{t \in T} \subset \mathcal{F}$ be an increasing family of σ -fields. Then the family $\{\mathbb{E}^{\mathcal{A}_t} f\}_{t \in T}$ is a martingale with respect to $\{\mathcal{A}_t\}_{t \in T}$.

The following two results obtained by Doob (the last statement in the next theorem is due to P. Lévy) play an important role in probability theory. Their proofs can be found in [541, Ch. V, §3] or [697, Ch. VII, §3].

A.3.5. Theorem. Let $\{f_n\}$ be a martingale on a probability space (X, \mathcal{M}, P) with respect to an increasing sequence of σ -fields $\{\mathcal{A}_n\}$ in \mathcal{M} . If the family $\{f_n\}$ is uniformly integrable, then there exists a function $f \in L^1(P)$, measurable with respect to the σ -field \mathcal{A}_∞ generated by $\{\mathcal{A}_n\}$, such that $\mathbb{E}^{\mathcal{A}_n} f = f_n$ for all n . In addition, $f_n \rightarrow f$ almost everywhere and in $L^1(P)$. If $f \in L^r(P)$, where $r > 1$, then there is the convergence in $L^r(P)$ as well. Conversely, if $f \in L^1(P)$ is measurable with respect to \mathcal{A}_∞ , then $\{\mathbb{E}^{\mathcal{A}_n} f\}$ is a uniformly integrable martingale convergent to f a.e. and in $L^1(P)$.

The next result is Doob’s inequality.

A.3.6. Theorem. Let $\{f_n\}$ be a submartingale with respect to an increasing sequence of σ -fields such that the functions f_n are nonnegative and

$$\sup_n \|f_n\|_{L^2(P)} < \infty.$$

Then $\sup_n f_n \in L^2(P)$ and $\|\sup_n f_n\|_{L^2(P)} \leq 2 \sup_n \|f_n\|_{L^2(P)}$.

σ -fields in locally convex spaces

Recall that for every family F of functions on a set X , there exists the smallest σ -field $\mathcal{E}(X, F)$ (denoted also by $\mathcal{E}(\{F\})$), with respect to which all functions from F are measurable. This σ -field is generated by the family of all sets of the form $\{x \in X : f < c\}$, where $f \in F$, $c \in \mathbb{R}^1$.

Denote by $C(X)$ the set of all continuous real functions on a topological space X and by $C_b(X)$ its subspace formed by all bounded functions.

Now we introduce three important σ -fields in locally convex spaces arising in connection with Gaussian measures. Let X be a locally convex space. Let us introduce the following notation:

$\mathcal{E}(X)$ is the cylindrical σ -field generated by X^* ;

$\mathcal{B}_0(X)$ is the Baire σ -field generated by $C(X)$;
 $\mathcal{B}(X)$ is the Borel σ -field of X generated by all open sets.

Clearly, $\mathcal{E}(X) \subset \mathcal{B}_0(X) \subset \mathcal{B}(X)$. It is readily verified that $\mathcal{B}_0(X) = \mathcal{B}(X)$ for every metric space X . The proof of this fact and the following theorem can be found in [800, Ch. I].

A.3.7. Theorem. *Suppose that X is a separable Fréchet space. Then, for every family $\Gamma \subset X^*$ separating the points in X , the following equalities hold true:*

$$\mathcal{E}(X, \Gamma) = \mathcal{E}(X) = \mathcal{B}_0(X) = \mathcal{B}(X).$$

In addition, there exists a countable family Γ with such a property.

However, in some important cases $\mathcal{B}(X)$ is strictly larger than $\mathcal{E}(X)$.

A.3.8. Example. Let T be an uncountable set and $X = \mathbb{R}^T$. Then $\mathcal{E}(X) = \mathcal{B}_0(X)$ is strictly smaller than $\mathcal{B}(X)$.

PROOF. The equality $\mathcal{E}(\mathbb{R}^T) = \mathcal{B}_0(\mathbb{R}^T)$ follows from Theorem A.3.9. It follows from Lemma 2.1.2 that $\mathcal{E}(\mathbb{R}^T)$ is not equal to $\mathcal{B}(\mathbb{R}^T)$. \square

The following deep result was proved in [219].

A.3.9. Theorem. *Let X be a locally convex space with the topology $\sigma(X, X^*)$. Then $\mathcal{E}(X) = \mathcal{B}_0(X, \sigma(X, X^*))$. Moreover, for every function f continuous in the topology $\sigma(X, X^*)$, there exist a continuous function g on \mathbb{R}^∞ and functionals $l_j \in X^*$ such that $\{f > 0\} = \{g \circ \pi > 0\}$, where $\pi(x) = (l_1(x), l_2(x), \dots)$.*

A mapping F from a topological space X to a topological space Y is called Borel if $F^{-1}(B) \in \mathcal{B}(X)$ for every $B \in \mathcal{B}(Y)$. If $Y = \mathbb{R}^1$ with the standard topology, then F is called a Borel function.

Radon measures

Let us recall some basic notions and results from measure theory on topological spaces. The proofs of the results mentioned below and further references can be found in [674] and [800].

A.3.10. Definition. *Let X be a topological space.*

- (i) *A countably additive measure on $\mathcal{B}(X)$ is called a Borel measure.*
- (ii) *A countably additive measure on $\mathcal{B}_0(X)$ is called a Baire measure.*
- (iii) *A Borel measure μ on X is called a Radon measure if, for every $B \in \mathcal{B}(X)$ and every $\varepsilon > 0$, there exists a compact set $K \subset B$ such that $\mu(B \setminus K) < \varepsilon$.*

A measure μ is called *tight* if condition (iii) is satisfied for $B = X$.

A.3.11. Theorem. *Every Borel measure on any complete separable metric space is Radon.*

If μ is a Borel (e.g., Radon) measure on a topological space X , then by μ -measurable sets we always mean the elements of $\mathcal{B}(X)_\mu$ (the Lebesgue completion of $\mathcal{B}(X)$ with respect to μ).

Any Radon measure on a locally convex space E is uniquely determined by its values on $\mathcal{E}(X)$.

A.3.12. Proposition. *Suppose that μ is a Radon measure on a locally convex space X . Then, for every μ -measurable set A , there is a set $B \in \mathcal{E}(X)$ such that*

$$\mu(A \triangle B) = 0.$$

Moreover, if $G \subset X^$ is an arbitrary linear subspace separating the points in X , then such a set B can be chosen in $\mathcal{E}(X, G)$.*

PROOF. Let $\varepsilon > 0$. Let us find compact sets $K \subset A$ and $S \subset X$ such that $\mu(K) > \mu(A) - \varepsilon$ and $\mu(S) > \mu(X) - \varepsilon$. We may assume that $K \subset S$. There exists an open set $U \supset K$ with $\mu(U) < \mu(K) + \varepsilon$. Since on the compact set S the initial topology coincides with the weak topology, there is a set V open in the weak topology such that $V \cap S = U \cap S$. By the compactness of K one can find a set W which is a finite union of open cylindrical sets such that $K \subset W \subset V$. Then we have $W \in \mathcal{E}(X)$ and

$$\mu(W \triangle A) \leq \mu(W \triangle K) + \varepsilon \leq \mu(V \setminus K) + \varepsilon \leq 3\varepsilon,$$

whence our claim. The same proof works for G replacing X^* , since the topology $\sigma(X, G)$ coincides with $\sigma(X, X^*)$ on S . \square

A.3.13. Corollary. *Let μ be a Radon measure on a locally convex space X . Then the collection of all bounded cylindrical functions on X is dense in $L^p(\mu)$ for every $p \in [1, \infty)$. In addition, the linear space T generated by the functions of the form $\exp(if)$, $f \in X^*$, is dense in the complex spaces $L^p(\mu)$. Moreover, both claims remain valid if X^* is replaced by any linear subspace $G \subset X^*$ separating the points in the space X .*

A.3.14. Definition. *Let μ be a Borel measure on a topological space X . A closed set $S_\mu \subset X$ is said to be the topological support of μ if $\mu(X \setminus S_\mu) = 0$ and there is no smaller closed set with this property.*

Every Radon measure has the topological support (see Problem A.3.35).

In measure theory an important role is played by *Souslin sets* defined as the images of complete separable metric spaces under continuous mappings to Hausdorff topological spaces. Hausdorff topological spaces that are continuous images of complete separable metric spaces are called *Souslin spaces*. Non-Borel sets of this kind were discovered by M. Ya. Souslin. For example, the orthogonal projection of a Borel set in \mathbb{R}^2 to \mathbb{R}^1 may fail to be Borel, but it is a Souslin set. It is known (see [185]) that there exist an infinitely differentiable function $f: \mathbb{R}^1 \rightarrow \mathbb{R}^1$ and a Borel set $B \subset \mathbb{R}^1$ such that $f(B)$ is not Borel. N. N. Lusin established the measurability of Souslin sets. In the next theorem we have collected the most important properties of Souslin sets frequently used in measure theory. Their proofs can be found in [674, p. 124, Chapter II, Corollary 1; p. 95, Theorem 2; p. 103, Corollary 3; p. 107, Corollary 16] or in [344].

A.3.15. Theorem. *Suppose that X and Y are Hausdorff topological spaces and $f: X \rightarrow Y$ is a mapping. Then:*

- (i) *Every Souslin set in Y is measurable with respect to every Radon measure on Y .*
- (ii) *If X is a complete separable metric space and f is continuous, then $f(B)$ is a Souslin set in Y for every Borel set $B \subset X$. If, in addition, f is injective, then $f(B)$ is Borel in Y .*

- (iii) If X and Y are Souslin spaces and f is a Borel mapping, then the images and preimages of Souslin sets are Souslin. If f is injective, then $f(B)$ is Borel in Y for every Borel set B in X .

It is known that every Borel measure on any Souslin space is Radon (see [674, p. 122]). On every Souslin space X , there exists a countable set of continuous functions separating the points. Therefore, all compact sets in Souslin spaces are metrizable. Hence every Borel measure on any Souslin space is concentrated on a countable union of metrizable compact sets. Note also that all Borel subsets of Souslin spaces are Souslin. It is worth mentioning that a space which is a countable union of its Souslin subspaces is Souslin itself. However, the complement of a Souslin set may fail to be Souslin; moreover, if the complement of a Souslin set is Souslin, then this set is Borel (see [674, Corollary 1, p. 101]). A Souslin space may be nonmetrizable (for example, the space l^2 with the weak topology). However, sequentially closed sets in Souslin spaces are Borel (see [674, Corollary 1, p. 109]). In particular, any sequentially continuous function on a Souslin space is Borel. A proof of the following result can be found in [674, Lemma 18, p. 108].

A.3.16. Proposition. *Let X be a Souslin space. Then $\mathcal{B}(X)$ is generated by some countable family of sets. In addition, $\mathcal{B}(X) = \mathcal{E}(X, \{f_n\})$ for every sequence of Borel functions f_n separating the points in X . Finally, if X is a Souslin locally convex space, then such a sequence can be chosen in any set $G \subset X^*$ separating the points in X .*

A very important object connected with a measure on locally convex space is its Fourier transform.

A.3.17. Definition. *Let X be a locally convex space and let μ be a measure on $\mathcal{E}(X)$. The Fourier transform $\tilde{\mu}$ of the measure μ is defined by the formula*

$$\tilde{\mu}: X^* \rightarrow \mathbb{C}, \quad \tilde{\mu}(f) = \int_X \exp(if(x)) \mu(dx). \quad (\text{A.3.2})$$

A.3.18. Proposition. *Any two measures on $\mathcal{E}(X)$ with equal Fourier transforms coincide.*

According to Corollary A.3.13, any two Radon measures with equal Fourier transforms are equal.

The following theorem may be useful for constructing Radon measures.

A.3.19. Theorem. *Let X be a locally convex space, let G be a linear subspace in X^* separating the points in X , and let μ be an additive nonnegative function on the algebra \mathcal{R}_G of all cylindrical sets generated by G . Suppose that μ has the following property: for any ε , there exists a compact set $K_\varepsilon \subset X$ such that $\mu(C) < \varepsilon$ for every cylindrical set $C \in \mathcal{R}_G$ which is disjoint with K_ε . Then μ uniquely extends to a Radon measure on X .*

Let X be a locally convex space and let μ and ν be two measures on $\mathcal{E}(X)$. Then the measure $\mu \otimes \nu$ is defined on $\mathcal{E}(X \times X)$ (note that $\mathcal{E}(X \times X) = \mathcal{E}(X) \otimes \mathcal{E}(X)$, which can be easily deduced from the equality $(X \times X)^* = X^* \times X^*$). The image of this measure under the mapping $X \times X \rightarrow X$, $(x, y) \mapsto x + y$, is called the *convolution* of the measures μ and ν and is denoted by $\mu * \nu$. It is easily verified that, letting $\lambda = \mu * \nu$, one has $\tilde{\lambda} = \tilde{\mu} \tilde{\nu}$. With the aid of Theorem A.3.19 one readily proves that the product

of Radon measures μ_i , $i = 1, \dots, n$, on a locally convex spaces X_i is uniquely extended to a Radon measure μ on $X_1 \times \dots \times X_n$. More generally, if μ_n are Radon probability measures on locally convex spaces X_n , then the product-measure $\bigotimes_{n=1}^{\infty} \mu_n$ extends uniquely to a Radon measure on $X = \prod_{n=1}^{\infty} X_n$. By the product of Radon measures we always mean the result of this extension. Certainly, for reasonable spaces (e.g., separable metric or Souslin), there is no need to consider extensions, since the product-measure is defined on the Borel σ -field of the product space from the very beginning. It is known (see [800, p. 60, Ch. I, Theorem 4.1]) that if μ is the aforementioned product of two Radon measures μ_1 and μ_2 , then for every $B \in \mathcal{B}(X_1 \times X_2)$, the function $x_2 \mapsto \mu_1(B_{x_2})$, where $B_{x_2} = \{x_1 \in X_1 : (x_1, x_2) \in B\}$, is Borel and

$$\mu(B) = \int_{X_2} \mu_1(B_{x_2}) \mu_2(dx_2).$$

In particular, if μ and ν are Radon measures on a locally convex space X , then their convolution $\mu * \nu$ extends uniquely to a Radon measure (again denoted by $\mu * \nu$). By the convolution of Radon measures we always mean the result of this extension. In this case, according to [800, p. 64, Ch. I, Proposition 4.4]), for every $B \in \mathcal{B}(X)$, the function $x \mapsto \mu(B - x)$ is Borel and the following equality holds true:

$$\mu * \nu(B) = \int_X \mu(B - x) \nu(dx).$$

Pettis integral

Let $f: (X, \mathcal{B}(X)_\mu) \rightarrow (X, \mathcal{E}(X))$ be a measurable mapping on a locally convex space X with a Radon measure μ . The element $m \in X$ is called the *Pettis integral* of the mapping f if for every l from X^* the function $l(f)$ is integrable with respect to μ and its integral equals $l(m)$. Put

$$\int_X f(x) \mu(dx) := m.$$

A.3.20. Lemma. *Let μ be a Radon probability measure on a sequentially complete locally convex space X concentrated on a metrizable compact set K . Then any sequentially continuous linear mapping $A: X \rightarrow X$ has Pettis integral which is an element of the closed convex hull of the compact set $A(K)$.*

PROOF. According to Problem A.3.36, there exists a sequence of probability measures μ_n with finite supports in K that converges weakly to the measure μ . For the measures μ_n , the Pettis integrals $I_n := \int_K Ax \mu_n(dx)$ obviously exist and are elements of the convex hull Q of the compact set $A(K)$. Since for any $l \in X^*$ the function $l \circ A$ is continuous on the metrizable compact set K (being sequentially continuous), by construction, the sequence $l(I_n)$ converges to $\int_K l(Ax) \mu(dx)$. Hence the sequence $\{I_n\}$ is a Cauchy sequence in the weak topology. Note that if X is complete, then the closure of Q is compact, and the initial topology coincides with the weak one on this closure. Hence $\{I_n\}$ converges to some point $m \in X$. Clearly, m is the Pettis integral of A .

Suppose now that X is only sequentially complete. The compact set $A(K)$ is metrizable (as a continuous image of a metrizable compact space, see [231,

Theorem 4.4.15]). By virtue of Proposition A.1.7, the closure of Q is a metrizable compact set as well. Therefore, we conclude again that the sequence $\{I_n\}$ converges to some point m , which is the Pettis integral of A . \square

If X is a separable Banach space and (Ω, μ) is a space with measure, then for measurable mappings $f: \Omega \rightarrow X$ satisfying the condition $\|f\|_X \in L^1(\mu)$, the notion of the Bochner integral is defined by the analogy with the Lebesgue integral for scalar functions, see [214, Ch. III]). In this case the Pettis integral exists as well and coincides with the Bochner one. Let us denote by $L^p(\mu, X)$ the Banach space of all μ -measurable X -valued mappings f such that

$$\|f\|_{L^p(\mu, X)} := \left\{ \int_{\Omega} \|f(x)\|_X^p \mu(dx) \right\}^{1/p} < \infty.$$

This notation is also used in the case when X is a normed space, but then it is additionally required that f be Bochner integrable.

Random vectors

Let (Ω, \mathcal{F}, P) be a probability space, X a locally convex space. The symbol $\mathbb{E}\xi$ is used to denote the expectation of a random variable ξ on Ω (i.e., $\mathbb{E}\xi$ is the Lebesgue integral of the measurable function ξ). A measurable mapping $\xi: \Omega \rightarrow (X, \mathcal{E}(X))$ is called a *random vector* in X . The measure $P_{\xi}(C) = P(\xi^{-1}(C))$ is called the distribution (or the law) of ξ . Clearly, every probability measure on $\mathcal{E}(X)$ can be obtained in such a form (with the identity mapping $\xi(x) = x$). If we have a family of probability measures μ_n on X , then there is a family of independent random vectors ξ_n on one and the same probability space Ω such that $P_{\xi_n} = \mu_n$ (take $\Omega = \prod_{n=1}^{\infty} X_n$, $X_n = X$, $P = \bigotimes_{n=1}^{\infty} \mu_n$, $\xi_n(\omega) = \omega_n$). A random process $\xi = (\xi_t)_{t \in T}$ is, by definition, a collection of random variables on a probability space (Ω, \mathcal{F}, P) . In this case

$$C_{t_1, \dots, t_n, B} = \left\{ \omega: (\xi_{t_1}(\omega), \dots, \xi_{t_n}(\omega)) \in B \right\} \in \mathcal{F} \quad (\text{A.3.3})$$

for every Borel set $B \in \mathcal{B}(\mathbb{R}^n)$ and any $t_1, \dots, t_n \in T$. Therefore, we can define a measure on the algebra $\mathcal{R}(\mathbb{R}^T)$ of the cylindrical sets of the form (A.3.3) by the formula

$$\mu^{\xi}(C_{t_1, \dots, t_n, B}) = P\left((\xi_{t_1}, \dots, \xi_{t_n}) \in B\right).$$

This measure is automatically countably additive, hence it uniquely extends to a countably additive measure on $\mathcal{E}(\mathbb{R}^T)$ denoted by μ^{ξ} and called the distribution of the process ξ in the functional space (or the measure generated by ξ). Conversely, any probability measure μ on $\mathcal{E}(\mathbb{R}^T)$ is the distribution of the random process $\xi_t(\omega) = \omega(t)$ if we take $\Omega = \mathbb{R}^T$ and $P = \mu$.

Note that for any finite collection $t_1, \dots, t_n \in T$, the formula above defines a probability measure P_{t_1, \dots, t_n} on \mathbb{R}^n called the finite dimensional distribution of ξ . It is clear that if $\{s_1, \dots, s_k\} \subset \{t_1, \dots, t_n\}$, i.e., $s_i = t_{j_i}$, $i = 1, \dots, k$, then the image of P_{t_1, \dots, t_n} under the mapping

$$(x_1, \dots, x_n) \mapsto (x_{j_1}, \dots, x_{j_k}), \quad \mathbb{R}^n \rightarrow \mathbb{R}^k,$$

coincides with P_{s_1, \dots, s_k} (i.e., the projections are consistent). The following result is a celebrated theorem due to Kolmogorov [421] (see its proof also in [822, Ch. 5]).

A.3.21. Theorem. *Suppose that for every finite set of points $t_1, \dots, t_n \in T$, a probability measure P_{t_1, \dots, t_n} on \mathbb{R}^n is given such that the aforementioned consistency property is satisfied. Then there exists a probability measure P whose finite dimensional projections are exactly P_{t_1, \dots, t_n} .*

Another Kolmogorov's result enables us to construct measures on the space $C[a, b]$ (its proof can be found in [822, Ch. 5]).

A.3.22. Theorem. *Let $\xi_t, t \in [a, b]$, be a random process such that for some $\alpha > 1, C \geq 0$, and $\varepsilon > 0$, one has*

$$\mathbb{E}|\xi_t - \xi_s|^\alpha \leq C|t - s|^{1+\varepsilon}, \quad \forall t, s \in [a, b].$$

Then there exists a random process $\eta_t, t \in [a, b]$, with continuous trajectories such that for each t one has $\eta_t = \xi_t$ a.s. In particular, the process η_t has the same finite dimensional distributions as ξ_t (hence $\mu^\eta = \mu^\xi$). In addition, $(\mu^\xi)^(C[a, b]) = 1$. Moreover, $(\mu^\xi)^*(H^\delta[a, b]) = 1$ for every $\delta \in (0, \varepsilon/\alpha)$, where $H^\delta[a, b]$ is the set of all functions satisfying the Hölder condition of order δ .*

Note that the same is true for the processes with values in separable Banach spaces (see [289, Ch. 3, §5]).

Problems

A.3.23. (i) Let K be a metrizable compact space. Show that the space $C(K)$ with the sup-norm is separable. (ii) Show that the product I^c of the continuum of segments is separable, but the Banach space $C(I^c)$ is not.

A.3.24. Prove that if X a separable normed space, then there exists a countable family of continuous linear functionals separating the points in X , hence X^* is separable in the *-weak topology. The converse is not true.

A.3.25. (i) Show that any reflexive Banach space is sequentially complete in the weak topology. (ii) Let X be an infinite dimensional normed space. Show that X is not complete in the weak topology (i.e., there exists a net which is Cauchy in the weak topology, but has no limit in the weak topology). In addition, the space X^* is not complete in the *-weak topology.

A.3.26. Construct an example of a locally convex space X such that there exists a sequentially continuous linear functional on X that is not continuous.

A.3.27. Let K be a convex compact set in a separable metrizable locally convex space X . Show that K is the intersection of a sequence of closed half-spaces; if K absolutely convex, then p_K has the form $p_K(x) = \sup_i l_i(x)$ for some sequence $\{l_i\} \subset X^*$.

A.3.28. Let X be a Banach space. Prove that if a linear mapping $A: X \rightarrow X$ is continuous from the weak topology to the norm topology, then the range of A is finite dimensional.

A.3.29. Show that there exists no continuous norm on the space \mathbb{R}^∞ with its natural topology.

A.3.30. Let X be a Hilbert space, let Y be a locally convex space, and let $L \in \mathcal{L}(X, Y)$. (i) Show that L takes the closed unit ball to a closed set. (ii) Show that if $K \in \mathcal{K}(X, Y)$, then K maps the closed unit ball to a compact set.

A.3.31. (i) Give an example of two nonnegative nuclear operators on a Hilbert space which have dense ranges intersecting only at zero. (ii) Let K be a compact operator on an infinite dimensional separable Hilbert space H . Prove that there exists a compact

operator S such that its range is dense in H , but intersects the range of K only at zero.
 (iii) Let X be a Banach space and $A \in \mathcal{L}(X)$. Show that if the set $A(X)$ is dense and does not coincide with X , then there exists an operator $B \in \mathcal{L}(X)$ such that the set $B(X)$ is dense and intersects $B(X)$ only at zero. Hint: see [682].

A.3.32. Let K be a compact set in a Hilbert space H . Show that K is contained in a compact ellipsoid of the form $A(U)$, where A is a symmetric compact operator on H and U is the unit ball of H . Hint: it suffices to consider separable H with an orthonormal basis $\{e_n\}$; construct an increasing sequence of natural numbers $k(n)$, $n \in \mathbb{N}$, such that $\sum_{j=k(n-1)+1}^{k(n)} (x, e_j)^2 \leq 2^{-n}$ for all $x \in K$ and $n > 1$. Let $Ae_j = \alpha_j e_j$, where $\alpha_j = 1$ if $j \leq k(1)$ and $\alpha_j = n^{-1}$ if $k(n-1) < j \leq k(n)$.

A.3.33. Let X be a Banach space of cardinality greater than c . Show that $\mathcal{E}(X)$ is strictly smaller than $\mathcal{B}_0(X) = \mathcal{B}(X)$. Hint: see [800, Ch. I].

A.3.34. Let (Ω, \mathcal{B}) be a measurable space and let $\{f_n\}$ be a sequence of \mathcal{B} -measurable functions. Show that the following sets are in \mathcal{B} :

$$\Lambda = \left\{ \omega \in \Omega : \exists \lim_{n \rightarrow \infty} f_n(\omega) \right\}, \quad \Xi = \left\{ \omega \in \Omega : \sup_n f_n(\omega) < \infty \right\}.$$

A.3.35. Show that every Radon measure has the topological support.

A.3.36. Let K be a compact metric space and let $Q \subset K$ be a countable set dense in K . Prove that for every Borel measure μ on K , there exists a sequence of linear combinations of Dirac's measures δ_q , $q \in Q$, convergent weakly to μ .

A.3.37. Let X and Y be Souslin spaces and let $f: X \times Y \rightarrow \mathbb{R}^1$ be a bounded Borel function. Show that the function $g(x) = \sup_{y \in Y} f(x, y)$ is measurable with respect to every Borel measure on X .

A.3.38. Let $\{\xi_n\}$ be a martingale with values in a separable normed space E and let q be a continuous norm on E . Prove that $\{q(\xi_n)\}$ is a submartingale. Hint: using the Hahn–Banach theorem represent q as the supremum of a sequence of continuous linear functionals.

A.3.39. Let E be a separable Hilbert space, let (Ω, P) be a probability space, and let $S \subset L^2(P, E)$ be a compact set. Show that there exists a nonnegative compact operator K in E with dense range such that $F(\omega) \in K(E)$ a.e. for every $F \in S$ and

$$\sup_{F \in S} \int \|K^{-1}F(\omega)\|_E^2 P(d\omega) < \infty.$$

A.3.40. Let μ be the Radon extension of the product of the continuum of Lebesgue measures λ on $[0, 1]$ (μ is defined on the compact space $X = [0, 1]^c$). Prove that $\mu(K) = 0$ for every metrizable compact set $K \subset [0, 1]^c$. Hint: consider the base \mathcal{U} of the topology in $[0, 1]^c$ formed by the products $U_T = \prod_t J_t$, where, for some finite set T , $J_t = [0, 1]$ if $t \notin T$ and $J_t = (a, b)$ with rational a, b such that $|b - a| \leq 1/2$ if $t \in T$; show that there is a set $U \in \mathcal{U}$ such that $U \cap K$ belongs to an uncountable number of the sets U_T .

A.3.41. Let A be an absolutely convex set in a locally convex space X and let E_A be its linear span. (a) Show that if A is a Borel set, then its Minkowski functional p_A is a Borel function on E_A . (b) Show that if all the sets tA , $t \in \mathbb{R}^1$, are measurable with respect to some measure μ on $\mathcal{E}(X)$ (or with respect to a Borel measure μ), then p_A is μ -measurable. Hint: write the set $\{p_A < c\}$ as the countable union of the sets $r_n A$ over all rational numbers $r_n < c$.

Bibliographical Comments

О, пожелтевшие листы
В стенах вечерних библиотек,
Когда раздумья так чисты,
А пыль пьянее, чем наркотик!
Н. Гумилев. В библиотеке

You will see there a lot of what I am telling you (although, sometimes this is said there in an indirect way, as an alternative, etc.), because the authors are professionals and most of the real information they have incorporated in their texts.

D. V. Deopik. Biblical Archaeology

To Chapter 1

Normal density was first used by A. Moivre [551] in the central limit theorem, and was later considered by P. Laplace. Two dimensional normal distributions appeared in the works of Adrain [7] (1809), Laplace [463] (1810), Plana [617] (1813), Gauss [280] (1823) (initially random vectors were assumed to have independent coordinates) and were further studied by Bravais [110] and other researchers. Some historical information and references can be found in [4], [533], [591], [593], [722], [723], [730]. The term “normal distribution” was suggested in the second half of the 19th century (F. Galton, K. Pearson, C. S. Peirce). In particular, in [594] the term “normal” applies to the curve of errors. See historical notes in [591]. For quite a long time normal distributions were also called “Laplace distributions”, but for the past 40 years this name became obsolete (cf. the polemical remarks in [262]). Now the terms “normal random variable”, “normal distribution”, “Gaussian random variable”, “Gaussian distribution”, “Gaussian measure” are generally accepted. Some additional information about Gaussian measures on finite dimensional spaces can be found in the books [18], [413], [545], [591], [769], [770]. See also the bibliography [321]. Calculation of normal distribution density is discussed in [528]. The polynomials H_k are often called the Chebyshev–Hermite polynomials (see, e.g., [591], [737]), since they were investigated independently by P. Chebyshev and Ch. Hermite. We use a shorten name in order to avoid confusion with other Chebyshev polynomials that are encountered more often in the approximation theory.

Identity (1.2.4) was noted by many authors, see, e.g., [616]. Mehler’s formula goes back to Mehler’s work [540]. Logarithmic Sobolev inequality was obtained by Gross [317]. The hypercontractivity of the Ornstein–Uhlenbeck semigroup was established by Nelson [561]. A probabilistic proof can be found in [564]. An important contribution to this direction is due to Bakry, Emery, and Ledoux [33], [34], [35], [467], [469], [471].

Theorem 1.7.1 is essentially due to B. Maurey and G. Pisier (see [605]). Inequality (1.8.1) was proved in [628] (see also [475]) and generalized in [109], whence our proof is borrowed. Anderson’s inequality was obtained in [17] for a considerably larger class of

measures (see [119]). Lemma 1.7.7 is a classical result in the theory of information (see [443, Ch. 2] and the references therein).

To Chapter 2

Bachelier [24] was the first to consider the Brownian motion from the mathematical point of view (in the framework of the analytical machinery of that time). From the modern point of view, Bachelier used the Brownian process to describe stock prices in some options of that epoch in France. The Gaussian property of the increments of the Brownian motion was noted in the physical works by A. Einstein and M. Smoluchowski in 1905–1906. Note also Langevin's work [462], where the equation bearing now his name was introduced. Historical comments can be found in [113]. The first rigorous construction of the *integral over the Wiener measure* (in the form of the Daniell integral on the space of continuous functions) was realized by Wiener [823], [824], [825], [826]. There are several different ways of proving the existence of the Wiener measure. One of them is based on the Fourier series with random coefficients (see, e.g., [390], [430]). One can also use the Haar functions on this way (see [170]). A second proof (given in the text) is based on two Kolmogorov's theorems. A third possibility comes with Gross's theory of abstract Wiener spaces. There exist proofs (see [603]) using the concept of a radonifying operator. In addition, there are modifications of those approaches (see, e.g., [800]). Instead of Kolmogorov's theorem one can use approximations of the Brownian motion by discrete random walks (see, e.g., [64]) and then the arguments based on the weak compactness. Certainly, not all possibilities to prove the existence of the Wiener measure are mentioned here. A large series of papers on the classical Wiener measure was published in the 40–50s by Cameron, Martin, and their collaborators (see [133] — [144] and the survey [425]). Uhlenbeck and Ornstein [782] soon after Wiener's works suggested another model of a diffusion corresponding to a Gaussian random process bearing now their names; this type of diffusion was investigated further in [204] and [816]. The first general definition of a Gaussian measure on an infinite dimensional space is due to Kolmogorov [422]. Intensive investigations of Gaussian measures on Banach spaces and the distributions of Gaussian processes in functional spaces started in the 50s (see, in particular, [261], [262], [267], [311], [335]), [480], [556], [630]. In the 60s the general theory of Gaussian measures on linear spaces was developed in [159], [240], [278], [285], [312] — [315], [371], [396], [432], [563], [638], [658], [696], [662], [688], [810] — [813]. The books by Shilov and Fan Dyk Tin' [696], Neveu [563], and Rozanov [658] were the first monographs devoted to the theory of Gaussian measures. An extensive bibliography is contained in [563].

Most of the results of the linear theory presented in this book were already known that time (although, not always in the general form known now). In subsequent years, a significant progress was achieved in the unification of the theory; connections between Gaussian distributions in the function spaces and Gaussian measures on Banach and locally convex spaces were clarified (see also comments to Chapter 3. Finally, note that large parts of the theory of Gaussian measures are presented in the books [178], [189], [289], [338], [359], [445], [491], [563], [658], [696], [832], [835], and surveys [27], [28], [81]. Kuo's book [445] has been one of the most widely cited references in the theory of Gaussian measures over the past 20 years.

Theorem 2.2.12 is taken from [753]. Additional results concerning characterizations of Gaussian measures on infinite dimensional spaces can be found in [330], [405], [631].

The fact that the integral of a Gaussian process over the parametric set in \mathbb{R}^1 is a Gaussian random variable was noted by Pitcher [609] (this fact follows directly from [206, Ch. II, Theorem 2.8]). In the general case the assertion in Example 2.3.16 was proved by Rajput [634]. Another proof is found in [504]. The simplified argument given in the text was suggested by Vakhania [797].

The Cameron–Martin formula was proved originally in [137] for the classical Wiener space and the absolutely continuous functions $h \in C[0, 1]$ such that h' has bounded variation. Later, in the works [529], [136], and [745], it was noticed independently that instead

of the boundedness of variation of the derivative it suffices to require that $h' \in L^2[0, 1]$ (the necessity of this condition was noted in [677]). The conditions for the absolute continuity and formulas for the density of a shift of a general Gaussian measure were obtained in [311]. The Cameron–Martin space is often called the reproducing kernel Hilbert space of a Gaussian measure; this term is used also for the Hilbert space of all measurable linear functionals by analogy with the Hilbert space obtained as the completion of the pre-Hilbert space of functions on the set T with respect to the inner product generated by a kernel $K(\cdot, \cdot)$ on T^2 (such a construction was considered by Aronszajn [21]).

The zero–one law for Gaussian measures follows directly from the classical zero–one law of Kolmogorov [421] (and in this sense can be regarded as Kolmogorov’s result); Cameron and Graves [136] proved the zero–one law for the Wiener measure. More general formulations were found in [397]. The zero–one law for subgroups was proved in [380], [397], [372], [31], [477]. An important event in the theory of Gaussian measures was the discovery by Hajek [324], [325] and Feldman [238] of the dichotomy “equivalence or singularity”. Related results were obtained in the works in mathematical physics [677], [683]. Further progress was achieved in Rozanov’s papers [653] — [658]. In the 60s and 70s those results were generalized and specified for various special classes of Gaussian processes by many authors (see [288], [342], [400], [590], [658], [687], [688], [805], [808], [815], [830]). It was noticed in [478] that the dichotomy “equivalence or singularity” can be deduced from the zero–one law. The proof given in the text is due to Talagrand [753]. Concerning zero–one laws and equivalence/singularity, see also [132], [218], [244], [363], [664]; [212] and [242] give short proofs of the zero–one law for stable measures.

For alternate proofs of the Itô–Nisio theorem, see [243], [491]. Related results are obtained in [581], [582]. Natural modifications were studied by Tsirelson [774], [775], who obtained Theorem 2.6.4.

Results related to Fernique’s theorem were obtained in [709], [461]; [364] and [128] study the integrability of seminorms of Gaussian vectors in non locally convex topological vector spaces.

Measurable linear functionals on the classical Wiener space are described in [136], [310]. More general situations were considered in [810], [696]. See [720] concerning generalizations of the large numbers law for measurable linear functionals. The stochastic integral of a non-random function against a Brownian path (the Paley–Wiener–Zygmund stochastic integral) was introduced in [586] (see also [585]).

Infinite dimensional Gaussian distributions are actively used in the financial mathematics (see [698]).

To Chapter 3

Many results in the theory of Radon Gaussian measures were obtained originally in more special cases (for example, for the Wiener measure or for the product-measures). For this reason, it is sometimes embarrassing to attribute the priorities. One of the first detailed expositions of the theory of Radon Gaussian measures was Borell’s article [96], which influenced considerably this area. The main tool of transferring the classical results to the general locally spaces setting are theorems 3.4.1, 3.4.4, and 3.5.1 which are essentially due to Tsirelson [774], [775]. Modifications of the original Tsirelson proofs were suggested in [663], [752]. In addition to the works already cited, Gaussian measures on locally convex spaces were investigated in [196], [240], [577], [633], [635], [666], [772]. The first correct proof of the separability of the spaces $L^2(\gamma)$ for Radon Gaussian measures was published in [667] (independently, but later, this fact was proved in [774], [96]); the proof given in the text follows [667] (Lemma 1.8.8 used in this proof was noted in [667] and later in [487]).

Convergence of series and sequences of Gaussian vectors was investigated in [97], [117], [127], [149], [151], [371], [374], [434], [460], [583], [774], [775], [796], [800], [815], and in the papers cited therein. A detailed exposition of the related problems has

been given by Yurinsky [835]. Expansions of Gaussian processes in the series with respect to the eigenfunctions of integral operators go back to Karhunen's and Loève's works [407], [506]. The works of Itô and Nisio [371] and Tsirelson [774], [775] were of particular importance for this direction. Representation (3.5.2) for general separable Banach spaces was obtained in [374], [434] and for separable Fréchet spaces in [633]. Concerning Gaussian function series, see [55], [117], [390], [523], [774], [775], and historical notes in [392]. Theorem 3.5.7 was obtained in [149]; Theorem 3.5.10, answering a question posed in [445], is due to [457].

Supports of Gaussian measures were studied in [367], [583], [633]. Theorem 3.6.5 for Banach spaces is due to [118]; for Fréchet spaces it was proved (by the argument presented in the text) in [73]. The first example of a Gaussian measure on a separable Banach space without Hilbert support was constructed by Dudley [209]. It was shown in [319] and [716] that the classical Wiener measure on $C[0, 1]$ has no Hilbert support ([319] mentions also an unpublished proof given earlier by S. Kwapien). The short proof of a more general fact given in the text is borrowed from [93].

Measurable linear functionals and operators were investigated (in addition to the works mentioned in the comments to Chapter 2) in [96], [254], [578], [665]. Theorem 3.7.6 and Proposition 3.7.10(i) go back to Gross's works [312], [314]; Assertion (i) in Proposition 3.7.10 is generalization of [445, Corollary I.4.4]; assertion (ii) in this proposition extends a theorem of Goodman (see [445, Theorem I.4.6]).

The weak convergence of Gaussian measures was considered in [41], [151], [161], [162], [163], [247], [303], [801].

Abstract Wiener spaces were introduced by Gross [314] and investigated by many authors in the 60s and 70s (see [25], [29], [30], [43], [211], [213], [316], [398], [432] — [435], [445], [662]). The fact that any nondegenerate Gaussian measure on a separable Banach space can be represented as an abstract Wiener measure was proved by Sato [662], Jain and Kallianpur [374], and Kuelbs [433]. An analogue of the filtration $\sigma(w_s, s \leq t)$ on the classical Wiener space can be introduced for abstract Wiener spaces (see [793]).

See [265], [753] for interesting examples exhibiting various set-theoretical pathologies arising for non Radon Gaussian measures on Banach spaces (such as l^∞).

There are two directions, which link the material of Chapter 3 with the theory of locally convex and Banach spaces, but are not discussed in this book. The first of them is concerned with various notions of a radonifying operator. Let X and Y be two locally convex spaces. A continuous linear operator $T: X \rightarrow Y$ is called γ -radonifying if, for every cylindrical Gaussian measure ν on X with the continuous Fourier transform, the measure $\nu \circ T^{-1}$ is tight. In the case, where X and Y are Hilbert spaces, this class is precisely the class of all Hilbert–Schmidt operators. An additional information can be found in [557], [603], [800]. The second direction deals with the characterization of the quadratic forms on X^* which are Gaussian covariances. In the case of a Hilbert space this class coincides with the family of all nonnegative forms generated by nuclear operators, hence, coincides with the class of covariances of all probability measures μ with $\int \|x\|^2 \mu(dx) < \infty$. The situation is different in general Banach spaces; see Remark 3.11.24 and remarks in section 7.5. Further references can be found in [164], [499], [557], [750], [796], [799], [800].

Various estimates connected with Gaussian measures can be found in [39], [293], [302], [304], [587], [760], [837]. Functions separating Gaussian measures are discussed in [482], [495].

Gaussian measures on non locally convex linear spaces are considered in [124], [125], [126], [127], [243], and in the references given therein. Gaussian measures on projective spaces are discussed in [652]. There exist analogues of Gaussian measures on more general spaces (for example, on groups, on the spaces over p -adic numbers, in the noncommutative analysis); see [232], [328], [334], [418], [419], [814]. Evaluation of Gaussian functional integrals is discussed in [221]. Applications of Gaussian measures in the quantum field theory can be found in [60], [266], [291], [520], [703], [704]. Concerning applications

in statistics, see, e.g., [122], [359], [360], [389], [399], [443], [504], [521], [607], [637], [777]. Applications of Gaussian measures in the theory of complexity of algorithms are discussed in [773]. Concerning Kolmogorov's widths and related objects on Gaussian spaces, see [510], [773], and the references therein.

To Chapter 4

The logarithmic concavity of a broad class of measures including all Gaussian measures was proved in [628] (for convex sets) and in [94]. The arguments based on the Brunn–Minkowski inequality were applied in [840] for the proof of certain special cases of the logarithmic inequality and the logarithmic concavity of the function $x \mapsto \gamma_n(A + x)$ for the absolutely convex sets $A \subset \mathbb{R}^n$. The isoperimetric inequality for Gaussian measures on infinite dimensional spaces was proved by Sudakov and Tsirelson [736] and Borell [95]. The exponential integrability of H -Lipschitzian functions was first proved in [172]. Ehrhard's works [222] — [225] became a considerable contribution to this direction; we follow these works in our exposition. Additional results connected with the isoperimetric inequalities and Brunn–Minkowski type inequalities can be found in [38], [102], [471], [472]. Corollary 4.4.2 was obtained (with a different proof) in [776]; for more details about the distribution of the maximum of a Gaussian process, see [189], [197], [198], [490], [491], [734], [754], [756]. Proposition 4.4.3 is taken from [172].

The proof of Theorem 4.6.1 given in the text is borrowed from [671]. Clearly, this result follows immediately from Šidak's inequality (Corollary 4.6.2) proved by Šidak [699], [700], and Khatri [416] (see comments given in [701] concerning the reasoning in [675] and several other papers). This inequality was conjectured by Dunn [215], who also proved some special cases.

Onsager and Machlup [579] investigated the behavior on balls of the measure μ generated by a diffusion process ξ_t (with a constant diffusion coefficient). Their work became a starting point of intensive investigations (see [152], [361]). The first general results on the existence of the Onsager–Machlup functions were obtained in [579], [747], [264] for the Wiener measure on $C[0, 1]$ with the sup-norm and for Gaussian measures on Hilbert spaces with the usual norm. A stronger result — the existence of the limit in (4.7.1) for any bounded absolutely convex set V in a locally convex space and $V_\varepsilon = \varepsilon V$ — was obtained by Borell [98]. Later Dudley, Hoffmann–Jørgensen, and Shepp [345] proved that the Onsager–Machlup function exists for the product γ of the standard Gaussian measures on the real line, provided that the γ -measurable norm q has the form $q(x) = \sup q(x_1, \dots, x_n, 0, 0, \dots)$. In [689], Shepp and Zeitouni considered the case, where γ is the Wiener measure on $C[0, 1]$ and q is a norm satisfying certain special conditions; they proved that under their conditions the limit in (4.7.1) exists for all $h \in H(\gamma)$. All these results were improved in [80]. Various results related to the Onsager–Machlup functions are obtained in [98], [302] — [304], [438], [496], [500], [509].

An important problem of the theory of random processes concerns estimates and asymptotics of the distributions of small values of Gaussian processes. A closely related to that is the investigation of Gaussian measures of small balls in Banach spaces. The distribution of the absolute value of the Wiener process and the asymptotics of the Wiener measure on small centered balls are special cases of more general results obtained by Kolmogorov and Leontovich [423] and Petrovskii [597]. The asymptotic behavior of Gaussian measures of small centered balls in Hilbert spaces was described by Zolotarev [842]. His result was generalized in [343] and precised in different directions by many authors. Concerning Gaussian measures of small balls and related problems as well as additional references, see [37], [61], [166], [193], [356], [437], [441], [442], [484], [486], [488], [492], [493], [498], [550], [553], [559], [569], [684], [746], [748], [762], [783], [817], [843].

The large deviations principle for Gaussian measures appeared with Schilder's theorem [672] for the Wiener measure. Its extension to general separable Banach spaces is due to Donsker and Varadhan (see [203]). Borell [96] considered the case of a general locally convex space. Ben Arous and Ledoux [54] gave formulations in non-topological

terms. Our exposition follows [54] (although the space X was assumed to be separable Banach in [54], exactly the same proofs are applicable to general locally convex spaces). Closely related to large deviations, Laplace's method of estimating Gaussian integrals is discussed in detail in [53], [612], [643]. In connection with the material of this section, see also [111], [112], [264], [402], [439], [489], [534], [558], [836].

To Chapter 5

Differentiability of measures on infinite dimensional spaces was considered first by Pitcher [608] and later in a series of Fomin's papers (see [260] and the surveys [23], [82], [93]). At present the theory of differentiable measures is a large area of infinite dimensional analysis with interesting applications in stochastic analysis and mathematical physics (see [82], [93], [178]). Useful integration by parts formulas for Gaussian measures were found in [180] — [182]. More general integration by parts formulas were obtained in [414] (see also [76], [82], [93]). The directional derivatives of measures were introduced by Fomin, who considered also the differentiability of Gaussian measures. Directional logarithmic derivatives of measures appeared in [23]. Logarithmic gradients were introduced in [14]. The analyticity of Gaussian measures was established in [14], [56], [645].

Sobolev classes over infinite dimensional spaces were defined first by N. N. Frolov (see [268] — [271]), who studied also their embeddings. Later such classes were considered by many other authors; see [26], [251], [361], [426] — [428], [464], [542] — [544], [596], [695], [738], [739], [819], [821]. The principal results about the equivalence of the different definitions of Sobolev classes belong to Meyer and Sugita. A proof of Meyer's equivalence shorter than the original argument of Meyer is due to Pisier [606]. Riesz transforms of Gaussian measures on \mathbb{R}^n are considered in [322]. Sobolev classes of mappings with values in certain special Banach spaces are considered in [518] in connection with the study of stochastic flows. Non Gaussian Sobolev classes are discussed, e.g., in [59], [82], [87], [88]. A different approach to the Sobolev classes over Gaussian measures is given in the book [570]. Gaussian analysis in terms of the Fock spaces (\mathcal{X}_k in our notation) is presented in [381].

Divergences of vector fields and derivatives of measures along vector fields were studied in [180], [181], [426] — [428], [512], [636], [711], and in many works, connected with the Malliavin calculus (see the surveys [82], [93], [177]). In the general case (for non Gaussian measures on manifolds) these concepts were introduced in [179]. Relations with extended stochastic integrals are investigated in [283], [567]. Divergence is one of the central objects in the Malliavin calculus discussed in Chapter 6 (see, e.g., [517]).

Extension of the logarithmic Sobolev and hypercontractivity to infinite dimensions is straightforward due to their dimensionless character. A more detailed information about hypercontractive semigroups can be found in [318]. In addition to the works already cited, the Ornstein–Uhlenbeck semigroups were discussed in [474], [600], [784]. In our exposition, we used the papers [469], [471].

The development of the Malliavin calculus motivated a considerable interest in Gaussian capacities. Important results are obtained in [273], [274], [275], [294], [341], [404], [448], [513], [740], [751] (see also [147]). Lemma 5.9.8 was proved in [12]. Tightness of Gaussian capacities (giving, in particular, their topological invariance) was established in [448], [740], [12] for Banach spaces and in [252], [89], [90] for locally convex spaces. The most general result presented in the text is due to [90]. The question about the topological invariance of Gaussian capacities was raised by Itô and Malliavin. More general Gaussian capacities associated with the operator semigroups $(T_t)_{t \geq 0}$ are discussed in [82], [404], [412]. Many classical results which are concerned with “almost everywhere properties” can be obtained in a sharper form as “quasi-everywhere properties” (see [397], [515]). Large deviations estimates in terms of capacities can be found in [102], [833].

The exponential integrability of Sobolev functions, the logarithmic Sobolev inequality, and the Poincaré inequality are studied in [9], [10], [47], [272], where one can find also non-Gaussian generalizations.

A discussion of nonlinear Wiener functionals, which are determined by their restrictions to the Cameron–Martin space, can be found in [152], [575], [741], [742], [743], [838]. Various problems connected with functions from Sobolev classes and approximations by such functions are discussed in [75], [207], [277], [300], [601]. In [571], [573], [785], [786], one is concerned with the characterization of independence of random variables from $W^\infty(\gamma)$ by means of their derivatives. In particular, [786, Theorem 6.4] gives the converse to Problem 5.12.40 (Craig’s theorem; see [413, 15.13]).

Distributions on spaces with Gaussian measures, in particular, Sobolev classes with negative orders of differentiability are considered in [339], [446], [576], [819], [821].

Concerning holomorphic functions on abstract Wiener spaces, see [235], [236], [517], [519], [743], [744].

Measurable polynomials (in the form of multiple stochastic integrals or in an abstract form) appeared in the works of Wiener [827], Cameron and Martin [143], Itô [366], Segal [676], and were further studied in [810], [715], [673], and many other works; see, e.g., [99], [178], [187], [201], [207], [289], [337], [351], [352], [353], [382], [399], [460], [466], [511], [531], [534], [572], [710], [728]. Abstract measurable polylinear mappings were introduced by Smolyanov [715], where, in particular, their relation to measurable polynomials was investigated. Various problems related to multiple stochastic integrals (limit theorems, estimates of distributions) are investigated in [199], [200], [305], [509], [584], [763], [764]. Second order measurable polynomials and double stochastic integrals were considered by many authors; see, e.g., [336], [527], [604], [619], [620], [649], [651], [681], [696], [807], [809], [810]. For quadratic forms, Proposition 5.10.9 was obtained in [2]. Proposition 5.10.10 extends the results from [329] and [650]. Part of the proof of Proposition 5.10.7 and Corollary 5.12.15 are due to Borell [99]. Closely related results are found in [103], [460], [466]. Zero–one laws for polynomials presented in Section 5.10 generalize earlier results obtained in [2], [329], [650]. An extension of the zero–one law to certain quasi–analytic functions is given in [449, Theorem 6.1]; note that Proposition 5.10.10 for real functions follows from the theorem cited, but can be extended to quasi–analytic mappings in the spirit of [449].

The Wick products of Gaussian variables and polynomials in Gaussian random functions are discussed in [201], [381], [520], [703].

It was suggested in [315] and [383] to take for universally null those sets that are zero with respect to all homotetic images of a fixed Gaussian measure. Clearly, the corresponding class of sets is larger than that of all Gaussian null sets.

To Chapter 6

The principal results presented in this chapter concerning linear transformations of Gaussian measures have been known for a long time (sometimes in a slightly less general form, but with very similar proofs); see, e.g., Segal’s work [677], and later works [312], [563], [566], [658], [679], [680], [696], [810], [812], [828]. Related problems were discussed in detail in the books [696], [563], [658], [320]. Linear transformations of Gaussian measures, in particular, of the Wiener measure, were investigated also in [28], [589], [806]. Another aspect of linear transformations is considered in [702].

There is a lot of works devoted to the equivalence conditions for the measures generated by Gaussian processes and fields from various special classes (clearly, the straightforward application of the general theorem may face insuperable difficulties of computational character). For a related discussion, see [16], [123], [153], [167], [289], [298], [299], [337], [338], [342], [359], [393], [394], [401], [548], [658], [661], [719], [802] — [804], [808], [829], [831], and the references therein.

The description of Gaussian measures equivalent to the Wiener measure (see Example 6.5.1) is due to Shepp [687] (see also [298]) who gave a different proof (a partial result was obtained in [806]). The result in Example 6.5.2 is also borrowed from [687].

The results presented in Section 6.6 go back to the pioneering works [137], [139], [145], [135], where shifts, linear transformations, and then more general nonlinear transformations $T\omega = \omega + F(\omega)$ were studied consecutively in the case of the classical Wiener space. One of the basic assumptions in this circle of problems is that F takes values in the Cameron–Martin space. Further progress was due to Maruyama, Girsanov, and Skorohod. The main result in Maruyama’s paper [530] is the proof of the existence of the transition density of any nondegenerate one dimensional diffusion. In order to get that statement, it was proved in [530] that the distribution of a one dimensional diffusion with the unit diffusion coefficient and drift f is equivalent to the Wiener measure (under mild integrability conditions on f , e.g., for bounded f) and its Radon–Nikodym density is the function Λ given by the equality $\Lambda(w_\bullet) = \exp\left(\int_0^1 f(w_s) dw_s - \frac{1}{2} \int_0^1 f(w_s)^2 ds\right)$. However, for quite a long time Maruyama’s paper was unknown to most of the researchers in the field. The aforementioned absolute continuity result was obtained independently in [630]. It was proved by Girsanov [290] and Skorohod [706] that the distributions of two multidimensional diffusion processes (note that the multidimensional case is more complicated and cannot be handled by Maruyama’s arguments) with one and the same diffusion coefficient A are equivalent under very broad assumptions on drift f (in particular, if $A = I$, they are equivalent to the Wiener measure). In addition, Girsanov’s paper contains a more general result which in the case $A = I$ states that the process $\xi_t = w_t - \int_0^t f(s, \omega) ds$ is Wiener with respect to the measure $Q = \Lambda \cdot P$ provided $f(t, \omega)$ is an adapted square-integrable process and $\mathbb{E}\Lambda = 1$. As shown in [290], the condition $\mathbb{E}\Lambda = 1$ is satisfied in many important cases. In the framework of abstract Wiener spaces (or Hilbert spaces with Gaussian measures), related results were obtained in [312], [32], [660], [708], [444], [712], and other papers. In this setting, the progressive measurability condition is replaced by certain smoothness of F . Ramer’s result [636] became a decisive step in this direction. Further improvements are due to Kusuoka [447], whose method was used in our exposition. Theorem 6.6.7, which generalizes Kusuoka’s result, is obtained by Üstünel and Zakai [789]; their proof (reproduced in the text) is a modification of an argument from [447]. Buckdahn’s works (see [116]) gave an impetus to active investigations of Girsanov’s type transformations without nonanticipativity conditions. The distributions of the sequences $\xi_n + \eta_n$, where the ξ_n ’s are independent standard Gaussian random variables and the η_n ’s are some random variables on the same probability space, are investigated in [388]; this is related to the transformations of \mathbb{R}^∞ of the form $I + F$, where $F: \mathbb{R}^\infty \rightarrow l^2$ satisfies the nonanticipativity condition of Example 6.7.4 considered in that paper. Interesting additional results can be found in [27], [229], [284], [287], [452], [453], [570], [787] — [791], [794], [839], [841]. Analytic transformations of \mathbb{R}^n preserving the standard Gaussian measure are considered in [226], [503]. Applications of the results about equivalent transformations of Gaussian measures to quasiinvariant measures and diffusions on infinite dimensional manifolds are described in [44], [208], [228], [415], [562], [685], [729].

The Malliavin calculus [512] appeared as a new method of proving the smoothness of the transition probabilities of multidimensional diffusion processes with possibly degenerate diffusion coefficients. One of its first impressive applications was the proof of an important special case of the celebrated Hörmander theorem about hypoelliptic second order differential operators. It was soon realized that Malliavin’s method is a beautiful and efficient tool in the study of nonlinear transformations of measures (not necessarily Gaussian) on infinite dimensional spaces. The Malliavin calculus has a lot of interesting links with functional analysis, stochastic analysis, and the topology of manifolds. The introduction into this calculus presented in this book has only aimed at describing the basic ideas conformably to the Gaussian case. Various interpretations of Malliavin’s method can be found in the works of many authors following [512]; see, e.g., [51], [65], [82], [93],

[177], [189], [361], [516], [517], [570], [574], [693], [727], [819], and the extensive bibliography in [82], [93], [517], [570]. In all such modifications the essence of the method remains invariable, though; they differ rather by the terminology (for example, derivatives along vector fields are called “differential operators”, “operators carré du champ”, etc.) However, the concrete situations to which the general method applies are most diverse. Additional results concerning smoothness of the distributions of various functionals can be found in the works cited and in [77], [281], [514], [555], [621]. Applications of the Malliavin calculus to the study of asymptotics are given in [454], [820]. See [362] for applications to occupation densities. The first general results about the absolute continuity of the distributions of Wiener functionals were obtained by Shigekawa [692], [693] with the aid of methods of the Malliavin calculus. However, in many cases for this purpose it is more efficient to apply the standard facts from the geometric measure theory to the conditional measures on the finite dimensional subspaces. The idea of using conditional measures for the study of nonlinear (typically, infinite dimensional) images of measures was employed systematically in [289] and [710]. For mappings to \mathbb{R}^n , the same idea was used later, e.g., in [74], [93], [106], [107], [187], [188]. Certain refinements of this method are discussed in [189]. The absolute continuity of the finite dimensional images of Gaussian measures was investigated also in [449], [450], [690]. In particular, Corollary 6.8.6 was proved in [449] and [690] for the expansions in Hermite polynomials; note that [449, Proposition 7.1] extends this result to quasi-analytic functions discussed in [449]. See [537] concerning the absolute continuity of infinite dimensional images. Lemma 6.9.7 was suggested by Uglanov [780].

The construction of the surface measures described in this chapter follows Airault and Malliavin [11]. Different approaches were suggested earlier in [445], [710], [778] — [780]. As a principal difference (cf. [778]) note that in the case of a Banach space, when defining the ε -neighborhood of a surface, instead of the Cameron–Martin space norm one can use the norm of the space itself (e.g., in the case of a Hilbert space X to use the unit normal vectors with respect to the norm of X). Clearly, this leads to a different, although close, theory. However, as shown in [75], Malliavin’s method is efficient on this way as well. Green’s and Stokes formulas are discussed in [721], [301], and in the works cited above. See [253] concerning surface measures and Hausdorff measures. Gaussian spherical measures are investigated in [331], [332], [333].

Supports of measures induced by sufficiently regular mappings are discussed in [8]; in particular, it is interesting to know when does the support of the measure $\gamma \circ F^{-1}$ coincide with the closure of $F(H(\gamma))$. The connectivity of the support of the measure $\gamma \circ F^{-1}$ for any $F \in W^\infty(\gamma, \mathbb{R}^d)$ was shown in [233] by the aid of the Malliavin calculus (for the functions from $W^{2,1}(\gamma)$ this result was given later in [207]).

The boundedness and smoothness of the distribution density of the norm of a Gaussian vector have been studied by many authors; see [42], [189], [436], [438], [440], [491], [497], [592], [780], and the references therein. The case of Hilbert spaces has been investigated especially well; see [355], [356], [500], [535], [588], [640], [807], [842], where, in particular, some asymptotic expansions can be found.

As we have seen, the smoothness of the finite dimensional images of measures can be verified by means of appropriate estimates of their Fourier transforms. This leads naturally to the investigation of the infinite dimensional oscillatory integrals; see [53], [74], [77], [82], [93], [514], [517], [519], and the references therein.

To Chapter 7

Many results and constructions of the abstract theory of Gaussian measures originated in the study of the trajectories of Gaussian random processes. Kolmogorov’s sufficient condition of continuity of the sample paths (Theorem A.3.22) was first published in Slutsky’s work [714]. Important early results in this direction are due to Hunt [354] and Belyaev [48], [49], [50]. In the 60s the two principal directions in the study of Gaussian processes were the equivalence problem (commented above) and the continuity and boundedness of

sample paths. The latter problem was investigated in [131], [192], [239], [279], [376], [424], [523] — [525], [629]. Various geometric characteristics generated by the covariance functions of Gaussian processes (such as the metric entropy) have deep connections with supports of Gaussian measures. An outstanding contribution to this area is due to Dudley [209], [210], Sudakov [731] — [733], Fernique [243], and Talagrand [755], [758], [759], [761]. Various aspects of the sample path theory (including additional references) are presented in the books [5], [6], [359], [471], [472], [481], [491], [611], [829]. Conditions for a Gaussian process to have sample paths of bounded p -variation are found in [379]; analogous problems for the usual variation and decompositions of Gaussian martingales are considered in [378]. Numerous interesting properties of the trajectories of the classical Brownian motion and some related processes and fields are discussed in [40], [104], [115], [369], [390], [406], [481], [560], [639]. The papers [115], [171], [357], [358] give some information about Sobolev and Besov classes containing Brownian and other Gaussian processes sample paths. Properties of the fractional Brownian motion are discussed in [522]. Various problems of the theory of Gaussian processes are treated in [216], [610]. For applications of the wavelet decompositions to the study of Gaussian processes, see [55], [817]. A survey of subgaussian processes is given in [377]. Markov properties of Gaussian random processes and fields and connections with Gibbs distributions are discussed in [202], [286], [346], [552], [613], [647], [648], [657], [659].

There exists an extensive literature devoted to infinite dimensional Wiener processes and more general diffusions; concerning the problems discussed in this chapter and further references, see [14], [15], [19], [79], [90], [92], [176], [178], [248], [249], [250], [255], [259], [365], [368], [445], [546]. An analogue of the Ornstein-Uhlenbeck process corresponding to the so called Lévy Laplacian (defined as a certain limit of $n^{-1} \sum_{i=1}^n \partial_{e_i}^2$) is discussed in [1]; although this process has compact state space, it preserves many Gaussian features.

Some additional information about logarithmic gradients of measures is found in [82]. The fact that a probability measure with the logarithmic derivative $\beta(x) = -x$ is Gaussian (see Proposition 7.3.9) follows from a result in [178] as noticed in [76]; this simple fact was proved in another way in [647] in the case of the Wiener measure; see also [568], [648], [91] (the latter paper contains a more general result). Expression (7.4.2) for the logarithmic gradient of an H -spherically symmetric measure was derived in [568]; the fact that this relationship implies that μ is H -spherically symmetric is due to [82]. A characterization of H -spherically symmetric probability measures as the measures that are ergodic with respect to the group of rotations of H is given in [330]. In relation to the symmetry properties of Gaussian measures note that the standard Gaussian product-measure on \mathbb{R}^∞ can be represented as a certain limit of normalized surface measures on finite dimensional spheres (see, e.g., [340], [538]).

Our exposition in Section 7.5 follows [78], [91]. Analogous results for nonconstant diffusion coefficients have been obtained in [85]. The differentiability of the transition probabilities of infinite dimensional diffusions is investigated in [52], [176], [282], [445], [554], [602]. Boundary value problems in abstract Wiener spaces are studied in [178], [417], [445], [505]. See [258] and [646] concerning Martin boundaries on abstract Wiener spaces.

A much more detailed discussion of the limit theorems for infinite dimensional random elements related to Gaussian measures can be found in [20], [58], [117], [429], [472], [592], [669], [800]. A good account of various results connected with normal approximations and asymptotic expansions in finite dimensions is given in [63].

References

- [1] Accardi L., Bogachev V.I., *The Ornstein–Uhlenbeck process associated with the Lévy Laplacian and its Dirichlet form*, Probab. Math. Stat. **17** (1997), no. 1, 95–114.
- [2] de Acosta A., *Quadratic zero-one laws for Gaussian measures and the distribution of quadratic forms*, Proc. Amer. Math. Soc. **54** (1976), 319–325.
- [3] Adams R.A., *Sobolev Spaces*, Academic Press, New York, 1975.
- [4] Adams W.J., *The Life and Times of the Central Limit Theorem*, Kaedmon Publ. Co., New York, 1974.
- [5] Adler R.J., *The Geometry of Random Fields*, Wiley, New York, 1981.
- [6] Adler R.J., *An introduction to Continuity, Extrema, and Related Topics for General Gaussian Processes*, Inst. of Math. Stat. Lect. Notes, Monograph Ser., 12. Inst. of Math. Stat., Hayward, CA, 1990.
- [7] Adrain R., *Research concerning the probabilities of the errors which happen in making observations*, The Analyst, or Math. Museum, Philadelphia, **1** (1808–1809), no. 4, 93–109.
- [8] Aida S., Kusuoka S., Stroock D., *On the support of Wiener functionals*, Pitman Research Notes in Math. Sci., Vol. 284, pp. 3–34. D. Elworthy and N. Ikeda eds., Longman, 1993.
- [9] Aida S., Masuda T., Shigekawa I., *Logarithmic Sobolev inequalities and exponential integrability*, J. Funct. Anal. **126** (1994), no. 1, 83–101.
- [10] Aida S., Stroock D., *Moment estimates derived from Poincaré and logarithmic Sobolev inequalities*, Math. Research Letters **1** (1994), no. 1, 75–86.
- [11] Airault H., Malliavin P., *Intégration géométrique sur l'espace de Wiener*, Bull. Sci. Math. **112** (1988), no. 1, 3–52.
- [12] Albeverio S., Fukushima M., Hansen W., Ma Z.M., Röckner M., *An invariance result for capacities on Wiener space*, J. Funct. Anal. **106** (1992), 35–49.
- [13] Albeverio S., Hoegh-Krohn R., *Mathematical Theory of Feynman Path Integrals*, Lecture Notes in Math. **523**, Springer, Berlin — New York, 1976.
- [14] Albeverio S., Hoegh-Krohn R., *Dirichlet forms and diffusion processes on rigged Hilbert spaces*, Z. Wahrscheinlichkeitstheorie verw. Geb. **40** (1977), no. 1, 1–57.
- [15] Albeverio S., Röckner M., *Stochastic differential equations in infinite dimensions: solutions via Dirichlet forms*, Probab. Theory Relat. Fields **89** (1991), 347–386.
- [16] Alekseev V.G., *Sufficient conditions of equivalence and orthogonality of Gaussian measures*, Izv. Akad. Nauk SSSR **28** (1964), no. 5, 1083–1090 (in Russian).
- [17] Anderson T.W., *The integral of a symmetric unimodal function over a symmetric convex set and some probability inequalities*, Proc. Amer. Math. Soc. **6** (1955), no. 2, 170–176.
- [18] Anderson T.W., *An Introduction to Multivariate Statistical Analysis*, 2nd ed., Wiley, New York, 1984.
- [19] Antoniadis A., Carmona R., *Eigenfunction expansions for infinite dimensional Ornstein–Uhlenbeck processes*, Probab. Theory Relat. Fields **74** (1987), 31–54.
- [20] Araujo A., Gine E., *The Central Limit Theorem for Real and Banach Valued Random Variables*, John Wiley and Sons, New York, 1980.

- [21] Aronszajn N., *Theory of reproducing kernels*, Trans. Amer. Math. Soc. **68** (1950), 337–404.
- [22] Aronszajn N., *Differentiability of Lipchitzian mappings between Banach spaces*, Studia Math. **57** (1976), no. 2, 147–190.
- [23] Averbukh V.I., Smolyanov O.G., Fomin S.V., *Generalized functions and differential equations in linear spaces*, Trudy Moskovsk. Matem. Ob. **24** (1971), 133–174 (in Russian); English transl.: Trans. Moscow Math. Soc. **24** (1971), 140–184.
- [24] Bachelier L., *Théorie de la speculation*, Ann. Sci. École Norm. Sup. **3** (1900), 21–86.
- [25] Badrikian A., *Séminaire sur les fonctions aléatoire linéaires et les mesures cylindriques*, Lecture Notes in Math. **139** (1970), 1–221.
- [26] Badrikian A., *Calcul stochastique anticipatif par rapport à une mesure gaussienne*, Sémin. d'Anal. Moderne, 21, Univ. de Sherbrooke, Dep. de Math. Sherbrooke, PQ, 1988.
- [27] Badrikian A., *Transformation of Gaussian measures*, Ann. Math. Blaise Pascal (1996), Numéro special, 13–58.
- [28] Badrikian A., *Measurable linear mappings from a Wiener space*, Ann. Math. Blaise Pascal (1996), Numéro Spécial, 59–113.
- [29] Badrikian A., Chevet S., *Mesures cylindriques, espaces de Wiener et fonctions aléatoires gaussiennes*, Lecture Notes in Math. **379** (1974), 1–383.
- [30] Badrikian A., Chevet S., *Questions liées à la théorie des espaces de Wiener*, Ann. Inst. Fourier **24** (1974), no. 2, 1–25.
- [31] Baker C.R., *Zero-one laws for Gaussian measures on Banach spaces*, Trans. Amer. Math. Soc. **186** (1973), 291–308.
- [32] Baklan V.V., Shatashvili A.D., *Transformations of Gaussian measures by non linear mappings in Hilbert space*, Dopovidi Akad. Nauk. Ukrain. RSR **9** (1965), 1115–1117 (in Russian).
- [33] Bakry D., *L'hypercontractivité et son utilisation en théorie des semigroupes*, Lecture Notes in Math. **1581** (1994), 1–114.
- [34] Bakry D., Emery M., *Diffusions hypercontractives*, Lecture Notes in Math. **1123** (1985), 177–206.
- [35] Bakry M., Ledoux M., *Lévy–Gromov's isoperimetric inequality for an infinite dimensional diffusion generator*, Invent. Math. **123** (1996), 259–281.
- [36] Bakry M., Michel D., *Sur les inégalités FKG*, Lecture Notes in Math. **1526** (1992), 170–188.
- [37] Baldi P., Roynette B., *Some exact equivalents for the Brownian motion in Hölder norm*, Probab. Theory Relat. Fields **93** (1992), no. 4, 457–484.
- [38] Ball K., *The reverse isoperimetric problem for Gaussian measure*, Discrete Comput. Geom. **10** (1993), no. 4, 411–420.
- [39] Barsov S.S., Ulyanov V.V., *Estimates for the closeness of Gaussian measures*, Dokl. Akad. Nauk SSSR **34** (1986), 273–277 (in Russian); English transl.: Soviet Math. Dokl. **34** (1986), 462–466.
- [40] Bass R., *Probability estimates for multiparameter Brownian processes*, Ann. Probab. **16** (1988), 251–264.
- [41] Baushev A.N., *On the weak convergence of Gaussian measures*, Teor. Veroyatn. i Primenen. **32** (1987), no. 4, 734–742 (in Russian); English transl.: Theory Probab. Appl. **32** (1987), no. 4, 670–677.
- [42] Baushev A.N., *On the boundedness of the distribution density of the norm of a Gaussian vector*, Teor. Veroyatn. i Primenen. **41** (1996), no. 2, 403–409 (in Russian); English transl.: Theory Probab. Appl. **41** (1996), no. 2, 334–340.
- [43] Baxendale P., *Gaussian measures on function spaces*, Amer. J. Math. **98** (1976), no. 4, 891–952.
- [44] Baxendale P., *Wiener processes on manifolds of maps*, Proc. Roy. Soc. Edinburgh, Ser. A. **87** (1980), no. 1–2, 127–152.

- [45] Baxter G., *A strong limit theorem for Gaussian processes*, Proc. Amer. Math. Soc. **7** (1956), 522–527.
- [46] Beckner W., *Inequalities in Fourier analysis*, Ann. Math. **102** (1975), 159–182.
- [47] Beckner W., *A generalized Poincaré inequality for Gaussian measures*, Proc. Amer. Math. Soc. **105** (1989), no. 2, 397–400.
- [48] Belayev Yu.K., *Continuity and Hölder's conditions for sample functions of stationary Gaussian processes*, In: Proc. 4th Berkeley Symp. Math. Statist. and Probab., 1960, Vol. 2, pp. 23–33. University of California Press, Berkeley — Los Angeles, 1961.
- [49] Belyaev Yu.K., *Local properties of the sample functions of a stationary Gaussian process*, Teor. Veroyatn. i Primenen. **5** (1960), no. 1, 128–131 (in Russian); English transl.: Theory Probab. Appl. **5** (1960), 117–120.
- [50] Belyaev Yu.K., *On the continuity and differentiability of realizations of Gaussian processes*, Teor. Veroyatn. i Primenen. **6** (1961), no. 3, 372–375 (in Russian); English transl.: Theory Probab. Appl. **6** (1961), no. 3, 340–342.
- [51] Bell D., *The Malliavin Calculus*, Wiley and Sons, New York, 1987.
- [52] Belopolskaya A.Ya., Dalecky Yu.L., *Stochastic Equations and Differential Geometry*, Vischa Shkola, Kiev, 1989 (in Russian); English transl.: Kluwer Academic Publ., 1990.
- [53] Ben Arous G., *Methodes de Laplace et de la phase stationnaire sur l'espace de Wiener*, Stochastics **25** (1988), 125–153.
- [54] Ben Arous G., Ledoux M., *Schilder's large deviation principle without topology*, Pitman Research Notes in Math. **284** (1993), 107–121.
- [55] Benassi A., Jaffard S., *Wavelet decomposition of one and several dimensional Gaussian processes*, Recent Advances in Wavelet Analysis (L.L. Schumaker and G. Webb, eds.), pp. 119–154. Academic Press, New York, 1993.
- [56] Bentkus V., *Analyticity of Gaussian measures*, Teor. Veroyatn. i Primenen. **27** (1982), no. 1, 147–154 (in Russian); English transl.: Theory Probab. Appl. **27** (1982), 155–161.
- [57] Bentkus V., Götze F., *Uniform rates of convergence in the CLT for quadratic forms in multidimensional spaces*, Probab. Theory Relat. Fields **109** (1997), 367–416.
- [58] Bentkus V., Götze F., Paulauskas V., Rachkauskas A., *The accuracy of Gaussian approximations in Banach spaces*, Itogi Nauki i Tekhn., Teor. Veroyantostei–6, pp. 39–139. VINITI, Moscow, 1991 (in Russian).
- [59] Berezansky Yu.M., Kondratiev Yu.G., *Spectral Methods in Infinite Dimensional Analysis*, Nauk. Dumka, Kiev, 1988 (in Russian); English transl.: Kluwer Academic Publ., 1993.
- [60] Berezin F.A., *The Method of Second Quantization*, Nauka, Moscow, 1965 (in Russian); English transl.: Academic Press, New York, 1966.
- [61] Berman S.M., Kôno N., *The maximum of a Gaussian process with nonconstant variance: a sharp bound for the distribution tail*, Ann. Probab. **17** (1989), no. 2, 632–650.
- [62] Bernstein S., *On a property characterizing Gauss's law*, Trudy Leningrad. Politehn. Inst. **3** (1941), 21–22 (in Russian).
- [63] Bhattacharya R.N., Ranga Rao R., *Normal Approximation and Asymptotic Expansions*, John Wiley and Sons, New York, 1976.
- [64] Billingsley P., *Convergence of Probability Measures*, John Wiley and Sons, New York, 1968.
- [65] Bismut J.M., *Large Deviations and the Malliavin Calculus*, Progress in Math., Vol. 45, Birkhäuser, 1984.
- [66] Bobkov S., *A functional form of the isoperimetric inequality for the Gaussian measure*, J. Funct. Anal. **135** (1996), 39–49.
- [67] Bobkov S.G., Götze F., *Exponential integrability and transportation cost related to logarithmic Sobolev inequalities*, J. Funct. Anal. (1998).

- [68] Bochnak J., Siciak J., *Polynomials and multilinear mappings in topological vector spaces*, *Studia Math.* **39** (1971), no. 1, 59–76.
- [69] Bogachev V.I., *Negligible sets and differentiable measures in Banach spaces*, *Vestnik Mosk. Univ. Ser. Mat. Mekh.* (1982), no. 3, 47–52 (in Russian); English transl.: *Moscow Univ. Math. Bull.* **37** (1982), no. 2, 54–59.
- [70] Bogachev V.I., *Three problems of Aronszajn from measure theory*, *Funk. Anal. i Pril.* **18** (1984), 75–76 (in Russian); English transl.: *Funct. Anal. Appl.* **18** (1984), 242–244.
- [71] Bogachev V.I., *Negligible sets in locally convex spaces*, *Matem. Zamet.* **36** (1984), no. 1, 51–64 (in Russian); English transl.: *Math. Notes* **36** (1984), 519–526.
- [72] Bogachev V.I., *Some results on differentiable measures*, *Matem. Sborn.* **127** (1985), no. 3, 336–351 (in Russian); English transl.: *Math. USSR Sbornik* **55** (1986), no. 2, 335–349.
- [73] Bogachev V.I., *Locally convex spaces with the CLT property and supports of measures*, *Vestnik Mosk. Univ. Ser. Mat. Mekh.* (1986), no. 6, 16–20 (in Russian); English transl.: *Moscow Univ. Math. Bull.* **41** (1986), no. 6, 19–23.
- [74] Bogachev V.I., *Differential properties of measures on infinite dimensional spaces and the Malliavin calculus*, *Acta Univ. Carol., Math. et Phys.*, **30** (1989), no. 2, 9–30.
- [75] Bogachev V.I., *Smooth measures, the Malliavin calculus and approximation in infinite dimensional spaces*, *Acta Univ. Carol., Math. et Phys.*, **31** (1990), no. 2, 9–23.
- [76] Bogachev V.I., *Infinite dimensional integration by parts and related problems*, Preprint no. 235, SFB 256, Bonn Univ. (1992), 1–37.
- [77] Bogachev V.I., *Functionals of random processes and infinite-dimensional oscillatory integrals connected with them*, *Izvest. Akad. Nauk SSSR* **156** (1992), no. 2, 243–278 (in Russian); English transl.: *Russian Sci. Izv. Math.* **40** (1993), no. 2, 235–266.
- [78] Bogachev V.I., *Remarks on invariant measures and reversibility of infinite dimensional diffusions*, In: *Probability Theory and Mathematical Statistics (Proc. Conf. on Stochastic Anal., Euler Math. Inst., St.-Petersburg, 1993)*, I.A. Ibragimov et al., eds., pp. 119–132. Gordon and Breach Publ., Amsterdam, 1996.
- [79] Bogachev V.I., *Deterministic and stochastic differential equations in infinite dimensional spaces*, *Acta Appl. Math.* **40** (1995), 25–93.
- [80] Bogachev V.I., *The Onsager–Machlup functions for Gaussian measures*, *Dokl. Rossiiskoi Akad. Nauk* **344** (1995), no. 4, 439–441 (in Russian); English transl.: *Russian Acad. Math. Dokl.* **52** (1995), no. 2, 216–218.
- [81] Bogachev V.I., *Gaussian measures on linear spaces*, *J. Math. Sci.* **79** (1996), no. 2, 933–1034.
- [82] Bogachev V.I., *Differentiable measures and the Malliavin calculus*, *J. Math. Sci.* **87** (1997), no. 5, 3577–3731.
- [83] Bogachev V.I., *Gaussian Measures*, Fizmatlit, Moscow, 1997 (in Russian).
- [84] Bogachev V.I., *On the small balls problem for equivalent Gaussian measures*, *Matem. Sbornik* **189** (1998), no. 5, 47–68 (in Russian); English transl.: *Sbornik Math.* (1998).
- [85] Bogachev V.I., Krylov N.V., Röckner M., *Regularity of invariant measures: the case of non-constant diffusion part*, *J. Funct. Anal.* **138** (1996), 223–242.
- [86] Bogachev V.I., Mayer-Wolf E., *Some remarks on Rademacher’s theorem in infinite dimensions*, *Potential Anal.* **5** (1996), no. 1, 23–30.
- [87] Bogachev V.I., Mayer-Wolf E., *Absolutely continuous flows generated by Sobolev class vector fields in finite and infinite dimensions*, Preprint SFB 343 Univ. Bielefeld (1996), no. 3, 1–49.
- [88] Bogachev V.I., Mayer-Wolf E., *Flows generated by Sobolev type vector fields and the corresponding transformations of probability measures*, *Dokl. Russian Akad. Sci.* **358** (1998), no. 4, 442–446 (in Russian); English transl.: *Russian Acad. Math. Dokl.* (1998).

- [89] Bogachev V.I., Röckner M., *Les capacités gaussiennes sont portées par des compacts metrisables*, C. R. Acad. Sci. Paris **315** (1992), 197–202.
- [90] Bogachev V.I., Röckner M., *Mehler formula and capacities for infinite dimensional Ornstein–Uhlenbeck processes with general linear drift*, Osaka J. Math. **32** (1995), no. 2, 237–274.
- [91] Bogachev V.I., Röckner M., *Regularity of invariant measures in finite- and infinite dimensional spaces and applications*, J. Funct. Anal. **133** (1995), 168–223.
- [92] Bogachev V.I., Röckner M., Schmuland B., *Generalized Mehler semigroups and applications*, Probab. Theory Relat. Fields **105** (1996), 193–225.
- [93] Bogachev V.I., Smolyanov O.G., *Analytic properties of infinite dimensional distributions*, Uspehi Matem. Nauk **45** (1990), no. 3, 3–83 (in Russian); English transl.: Russian Math. Surveys **45** (1990), no. 3, 1–104.
- [94] Borell C., *Convex measures on locally convex spaces*, Ark. Math. **12** (1974), no. 2, 239–252.
- [95] Borell C., *The Brunn–Minkowski inequality in Gauss space*, Invent. Math. **30** (1975), no. 2, 207–216.
- [96] Borell C., *Gaussian Radon measures on locally convex spaces*, Math. Scand. **38** (1976), no. 2, 265–284.
- [97] Borell C., *Approximation on locally convex spaces*, Invent. Math. **34** (1976), no. 3, 215–229.
- [98] Borell C., *A note on Gauss measures which agree on small balls*, Ann. Inst. H. Poincaré **B 13** (1977), no. 3, 231–238.
- [99] Borell C., *Tail probabilities in Gauss space*, Lecture Notes in Math. **644** (1978), 73–82.
- [100] Borell C., *Convexity in Gauss space*, In: Statistical and physical aspects of Gaussian processes (Saint-Flour, 1980), pp. 27–37, Colloq. Internat. CNRS, **307**, CNRS, Paris, 1981.
- [101] Borell C., *Gaussian correlation inequalities for certain bodies in R^n* , Math. Ann. **256** (1981), no. 4, 569–573.
- [102] Borell C., *Capacitary inequalities of the Brunn–Minkowski type*, Math. Ann. **263** (1983), no. 2, 179–184.
- [103] Borell C., *On polynomial chaos and integrability*, Probab. Math. Stat. **3** (1984), 191–203.
- [104] Borodin A.N., Salminen P., *Handbook of Brownian Motion — Facts and Formulae*, Birkhäuser Verlag, Basel — Boston — Berlin, 1996.
- [105] Borovkov A.A., Utev S.A., *On an inequality and a related characterization of the normal distribution*, Teor. Veroyatn. i Primenen. **28** (1984), 209–218 (in Russian); English transl.: Theory Probab. Appl. **28** (1984), 219–228.
- [106] Bouleau N., Hirsch F., *Propriétés d’absolue continuité dans les espaces de Dirichlet et applications aux équations différentielles stochastiques*, Lecture Notes in Math. **1204** (1986), 131–161.
- [107] Bouleau N., Hirsch F., *Dirichlet Forms and Analysis on Wiener Space*, Walter de Gruyter, Berlin — New York, 1991.
- [108] Brascamp H., Lieb E.H., *Some inequalities for Gaussian measure*, In: Functional integration and its applications (A. M. Arthurs ed.), pp. 1–14. Oxford Univ. Press (Clarendon), London — New York, 1975.
- [109] Brascamp H., Lieb E.H., *On extensions of the Brunn–Minkowski and Prékopa–Leindler theorems, including inequalities for log concave functions, and with an application to the diffusion equation*, J. Funct. Anal. **22** (1976), 366–389.
- [110] Bravais A., *Sur les probabilités des erreurs de situation d’un point*, Mémoires Acad. Roy. Sci. Inst. France **9** (1846), 255–332.
- [111] Breitung K., *Asymptotic Approximations for Probability Integrals*, Lecture Notes in Math. **1592**, Springer, Berlin, 1994.

- [112] Breitung K., Richter W.-D., *A geometric approach to an asymptotic expansion for large deviation probabilities of Gaussian random vectors*, J. Multivar. Anal. **58** (1996), 1–20.
- [113] Brush S.G., *A history of random processes. I. Brownian movement from Brown to Perrin*, Archive for the History of the exact sciences. **5** (1968), 1–36; reprinted in [730].
- [114] Bryc W., *The Normal Distribution. Characterizations with Applications*, Lecture Notes in Statistics **100**, Springer-Verlag, New York, 1995.
- [115] Brzeźniak Z., *On Sobolev and Besov spaces regularity of Brownian paths*, Stochastics and Stoch. Reports **56** (1996), 1–15.
- [116] Buckdahn R., *Anticipative Girsanov transformations*, Probab. Theory Relat. Fields **89** (1991), no. 2, 211–238.
- [117] Buldygin V.V., *The Convergence of Random Elements in Topological Spaces*, Naukova Dumka, Kiev, 1980 (in Russian).
- [118] Buldygin V.V., *Supports of probability measures in separable Banach spaces*, Teor. Veroyatn. i Primenen. **29** (1984), no. 3, 528–532 (in Russian); English transl.: Theory Probab. Appl. **29** (1984), no. 3, 546–549.
- [119] Buldygin V.V., Kharazishvili A.B., *Brunn–Minkowski Inequality and its Applications*, Naukova Dumka, Kiev, 1985 (in Russian).
- [120] Burago D.M., Zalgaller V.A., *Geometric Inequalities*, Nauka, Moscow, 1980 (in Russian); English transl.: Springer-Verlag, Berlin — New York, 1988.
- [121] Burkholder D.L., *Martingales and Fourier analysis in Banach spaces*, Lecture Notes in Math. **1206** (1985), 61–108.
- [122] Burnashev M.V., *Discrimination of hypotheses for Gaussian measures and a geometrical characterization of Gaussian distribution*, Mat. Zamet. **32** (1982), no. 4, 549–556 (in Russian); English transl.: Math. Notes **32** (1982), no. 4, 754–761.
- [123] Butov A.A., *The equivalence of measures corresponding to canonical Gaussian processes*, Uspehi Matem. Nauk **37** (1982), no. 5, 169–170 (in Russian); English transl.: Russian Math. Surveys **37** (1982), no. 5, 162–163.
- [124] Byczkowski T., *Gaussian measures on L_p spaces, $0 \leq p < \infty$* , Studia Math. **59** (1977), 249–261.
- [125] Byczkowski T., *Norm convergent expansion for L_Φ -valued Gaussian random elements*, Studia Math. **64** (1979), 87–95.
- [126] Byczkowski T., *RKHS for Gaussian measures on metric vector spaces*, Bull. Polish Acad. Sci. Math. **35** (1987), no. 1–2, 93–103.
- [127] Byczkowski T., Inglot T., *Gaussian random series on metric vector spaces*, Math. Z. **196** (1987), no. 1, 39–50.
- [128] Byczkowski T., Zak T., *On the integrability of Gaussian random vectors*, Lecture Notes in Math. **828** (1980), 21–29.
- [129] Cacoullos Th., *On upper and lower bounds for the variance of a function of a random variable*, Ann. Probab. **10** (1982), no. 3, 799–809.
- [130] Cacoullos T., Papathanasiou V., Utev S.A., *Another characterization of the normal law and a proof of the central limit theorem*, Teor. Veroyatn. i Primenen. **37** (1992), no. 4, 648–657 (in Russian); English transl.: Theory Probab. Appl. **37** (1992), no. 4, 581–588.
- [131] Cambanis S., *On some continuity and differentiability properties of paths of Gaussian processes*, J. Multivariate Anal. **3** (1973), 420–433.
- [132] Cambanis S., Rajput B., *Some zero-one laws for Gaussian processes*, Ann. Probab. **1** (1973), 304–312.
- [133] Cameron R., *The translation pathology of Wiener space*, Duke Math. J. **21** (1954), no. 4, 623–627.
- [134] Cameron R., *A family of integrals serving to connect the Wiener and Feynman integrals*, J. Math. Phys. **39** (1960), no. 2, 126–140.

- [135] Cameron R.H., Fagen R.E., *Nonlinear transformations of Volterra type in Wiener space*, Trans. Amer. Math. Soc. **7** (1953), no. 3, 552–575.
- [136] Cameron R.H., Graves R.E., *Additive functionals on a space of continuous functions. I*, Trans. Amer. Math. Soc. **70** (1951), 160–176.
- [137] Cameron R.H., Martin W.T., *Transformation of Wiener integral under translation*, Ann. Math. **45** (1944), 386–396.
- [138] Cameron R.H., Martin W.T., *The Wiener measure of Hilbert neighbourhoods in the space of real continuous functions*, J. Math. Phys. **23** (1944), no. 4, 195–209.
- [139] Cameron R.H., Martin W.T., *Transformations of Wiener integrals under a general class of linear transformations*, Trans. Amer. Math. Soc. **58** (1945), 184–219.
- [140] Cameron R.H., Martin W.T., *Evaluation of various Wiener integrals by use of certain Sturm–Liouville differential equations*, Bull. Amer. Math. Soc. **51** (1945), no. 2, 73–90.
- [141] Cameron R.H., Martin W.T., *Fourier–Wiener transforms of analytical functionals*, Duke Math. J. **12** (1945), 489–507.
- [142] Cameron R.H., Martin W.T., *Fourier–Wiener transforms of functionals belonging to L_2 over the space C* , Duke Math. J. **14** (1947), 99–107.
- [143] Cameron R.H., Martin W.T., *The orthogonal development of non linear functionals in series of Fourier–Hermite polynomials*, Ann. Math. **48** (1947), 385–392.
- [144] Cameron R.H., Martin W.T., *The behaviour of measure and measurability under change of scale in Wiener space*, Bull. Amer. Math. Soc. **53** (1947), no. 2, 130–137.
- [145] Cameron R.H., Martin W.T., *The transformation of Wiener integrals by nonlinear transformations*, Trans. Amer. Math. Soc. **66** (1949), 253–283.
- [146] Cameron R.H., Storvick D.A., *Two related integrals over spaces of continuous functions*, Pacif. J. Math. **55** (1974), no. 1, 19–37.
- [147] Caraman P., *Module and p -module in an abstract Wiener space*, Rev. Roum. Math. Pures et Appl. **26** (1982), no. 5, 551–599.
- [148] Carlen E., *Superadditivity of Fisher’s information and logarithmic Sobolev inequalities*, J. Funct. Anal. **101** (1991), 194–211.
- [149] Carmona R., *Measurable norms and some Banach space valued Gaussian processes*, Duke Math. J. **44** (1977), no. 1, 109–127.
- [150] Carmona R., *Tensor product of Gaussian measures*, Lecture Notes in Math. **644** (1978), 96–124.
- [151] Carmona R., Kono N., *Convergence en loi et lois du logarithme itéré pour les vecteurs gaussiens*, Z. Wahrscheinlichkeitstheorie verw. Geb. **36** (1976), 241–267.
- [152] Carmona R., Nualart D., *Traces of random variables on Wiener space and the Onsager–Machlup functional*, J. Funct. Anal. **107** (1992), 402–438.
- [153] Chatterji S.D., Mandrekar V., *Equivalence and singularity of Gaussian measures and applications*, In: Probab. Anal. and Related Topics, Vol. 1, pp. 169–199. Academic Press, 1978.
- [154] Chatterji S.D., Ramaswamy S., *Mesures gaussiennes et mesures produits*, Lecture Notes in Math. **920** (1982), 570–589.
- [155] Chen L.H.Y., *An inequality for the multivariate normal distribution*, J. Multivariate Anal. **12** (1982), 306–315.
- [156] Chen L.H.Y., Lou J.H., *Characterization of probability distributions by Poincaré-type inequalities*, Ann. Inst. H. Poincaré. **23** (1987), no. 1, 91–110.
- [157] Chentsov N.N., *Wiener random fields of several parameters*, Dokl. Akad. Nauk SSSR **106** (1956), no. 4, 607–609 (in Russian).
- [158] Chernoff H., *A note on an inequality involving the normal distribution*, Ann. Probab. **9** (1981), 533–535.
- [159] Chevet S., *p -ellipsoides de l^q , exposant d’entropie, mesures cylindriques gaussiennes*, C. R. Acad. Sci. Paris **A269** (1969), 658–660.

- [160] Chevet S., *Un résultat sur les mesures gaussiennes*, C. R. Acad. Sci. Paris. **A284** (1977), 441–443.
- [161] Chevet S., *Sur les suites de mesures gaussiennes étroitement convergentes*, C. R. Acad. Sci. Paris **296** (1983), no. 4, 227–230.
- [162] Chevet S., *Compacité dans l'espace des probabilités de Radon gaussiennes sur un Banach*, C. R. Acad. Sci. Paris **296** (1983), 275–278.
- [163] Chevet S., *Gaussian measures and large deviations*, Lecture Notes in Math. **990** (1983), 30–46.
- [164] Chobanjan S.A., Tarieladze V.I., *Gaussian characterizations of certain Banach spaces*, J. Multivar. Anal. **7** (1977), 183–203.
- [165] Christensen J.P.R., *Topology and Borel Structure*, North-Holland, Amsterdam, 1974.
- [166] Christoph G., Prohorov Yu.V., Ulyanov V., *On distribution of quadratic forms in Gaussian random variables*, Teor. Veroyatn. i Primenen. **40** (1995), no. 2, 301–312 (in Russian); English transl.: Theory Probab. Appl. **40** (1995), 250–260.
- [167] Chung D.M., Rajput B.S., *Equivalent Gaussian measure whose R - N derivative is the exponential of a diagonal form*, J. Math. Anal. Appl. **81** (1981), no. 1, 219–233.
- [168] Chung K.L., Erdős P., Sirao T., *On the Lipschitz's condition for Brownian motions*, J. Math. Soc. Japan **11** (1959), 263–274.
- [169] Chuprunov A.N., *On measurability of linear functionals*, Matem. Zamet. **33** (1983), no. 6, 943–948 (in Russian); English transl.: Math. Notes. **33** (1983), no. 6, 483–486.
- [170] Ciesielski Z., *Hölder condition for realization of Gaussian processes*, Trans. Amer. Math. Soc. **99** (1961), no. 3, 403–413.
- [171] Ciesielski Z., Kerkyacharian G., Roynette B., *Quelques espaces fonctionnels associés à des processus gaussiens*, Studia Math. **107** (1993), 171–204.
- [172] Cirelson B.S., Ibragimov I.A., Sudakov V.N., *Norms of Gaussian sample functions*, Lecture Notes in Math. **550** (1976), 20–41.
- [173] Cramer H., *Über eine Eigenschaft der normalen Verteilungsfunktion*, Math. Z. **41** (1936), 405–411.
- [174] Cruzeiro A.-B., *Équations différentielles sur l'espace de Wiener et formules de Cameron–Martin non-linéaires*, J. Funct. Anal. **54** (1983), no. 2, 206–227.
- [175] Da Prato G., Malliavin P., Nualart D., *Compact families of Wiener functionals*, C. R. Acad. Sci. Paris **315** (1992), 1287–1291.
- [176] Da Prato G., Zabszyk J., *Stochastic Equations in Infinite Dimensions*, Cambridge University Press, Cambridge, 1992.
- [177] Daletskii Yu.L., *Stochastic differential geometry*, Uspehi Matem. Nauk **38** (1983), no. 3, 87–111 (in Russian); English transl.: Russian Math. Surveys **38** (1983), no. 3, 97–125.
- [178] Daletskii Yu.L., Fomin S.V., *Measures and Differential Equations in Infinite Dimensional Spaces*, Nauka, Moscow, 1983 (in Russian); English transl.: Kluwer Academic Publ., 1993.
- [179] Daletskii Yu.L., Maryanin B.D., *Smooth measures on infinite-dimensional manifolds*, Dokl. Akad. Nauk SSSR **285** (1985), no. 6, 1297–1300 (in Russian); English transl.: Soviet Math. Dokl. **32** (1985), 863–866.
- [180] Daletskii Yu.L., Paramonova S.N., *Stochastic integrals with respect to a normally distributed additive set function*, Dokl. Akad. Nauk SSSR **208** (1973), 512–515 (in Russian); English transl.: Soviet Math. Dokl. **14** (1973), 96–99.
- [181] Daletskii Yu.L., Paramonova S.N., *A certain formula of the theory of Gaussian measures and the estimation of stochastic integrals*, Teor. Veroyatn. i Primenen. **19** (1975), 844–849 (in Russian); English transl.: Theory Probab. Appl. **19** (1975), 812–817.

- [182] Daletskii Yu.L., Paramonova S.N., *Integration by parts with respect to measures in function space. I*, Teor. Veroyatn. i Matem. Statist. **17** (1977), 51–61 (in Russian); English transl.: Theory Probab. Math. Statist. **17** (1979), 55–68.
- [183] Danzer L., Grünbaum B., Klee V., *Helly's theorem and its relatives*, In: Convexity. Proceedings of Symp. Pure Math., Vol. 7, Amer. Math. Soc., Providence, Rhode Island, 1963.
- [184] Darmois G., *Sur une propriété caractéristique de la loi de probabilité de Laplace*, C. R. Acad. Sci. Paris **232** (1951), 1999–2000.
- [185] Darst R.B., *C^∞ -functions need not be bimeasurable*, Proc. Amer. Math. Soc. **27** (1971), 128–132.
- [186] Das Gupta S., Eaton M.L., Olkin I., Perlman M., Savage L.J., Sobel M., *Inequalities on the probability content of convex regions for elliptically contoured distributions*, In: Proc. 6th Berkeley Symp. Math. Statist. Probab., Vol. 2, pp. 241–267. University of California Press, Berkeley, 1972.
- [187] Davydov Yu.A., *On distributions of multiple Wiener–Ito integrals*, Teor. Veroyatn. i Primenen. **35** (1990), no. 1, 51–62 (in Russian); English transl.: Theory Probab. Appl. **35** (1990), 27–37.
- [188] Davydov Yu.A., Lifshits M.A., *The fibering method in some probability problems*, Itogi Nauki i Tekhniki Akad. Nauk SSSR VINITI. Teor. Veroyatn., Mathem. Statist. i Teor. Kibern., Vol. 22 (1984), 61–157; English transl.: J. Soviet Math. **31** (1985), no. 2, 2796–2858.
- [189] Davydov Yu.A., Lifshits M.A., Smorodina N.V., *Local Properties of Distributions of Stochastic Functionals*, Fizmatlit, Moscow, 1995 (in Russian); English transl.: Amer. Math. Soc., Providence, Rhode Island, 1998.
- [190] Daw R.H., Pearson E.S., *Abraham De Moivre's 1733 derivation of the normal curve: a bibliographical note*, Biometrika **59** (1972), 677–680.
- [191] Deheuvels P., Lifshits M., *Strassen-type functional laws for strong topologies*, Probab. Theory Relat. Fields **97** (1993), 151–167.
- [192] Delporte L., *Fonctions aléatoires presque sûrement continues sur un intervalle fermé*, Ann. Inst. H. Poincaré **B1** (1964), 111–215.
- [193] Dembo A., Mayer-Wolf E., Zeitouni O., *Exact behavior of Gaussian seminorms*, Statist. Probab. Lett. **23** (1995), no. 3, 275–280.
- [194] Deville R., Godefroy G., Zizler V., *Smoothness and Renormings in Banach Spaces*, Longmann Scientific, 1993.
- [195] Diestel J., *Geometry of Banach Spaces*, Lecture Notes in Math. **485**, Springer, Berlin, 1975.
- [196] Dineen S., Noverraz Ph., *Gaussian measures and polar sets in locally convex spaces*, Ark. Mat. **17** (1979), no. 2, 217–223.
- [197] Dmitrovskii V.A., *A boundedness condition and estimates of the distribution of the maximum of random fields on arbitrary sets*, Dokl. Akad. Nauk SSSR **253** (1980), no. 2, 271–274 (in Russian); English transl.: Soviet Math. Dokl. **22** (1981), 59–62.
- [198] Dmitrovskii V.A., *On the integrability of the maximum and conditions of continuity and local properties of Gaussian fields*, In: Probability Theory and Mathematical Statistics, Proc. Fifth Vilnius Conf., Vol. 1 (B. Grigelionis et als., eds.), pp. 271–284. VSP BV/Mokslas, Utrecht, 1990.
- [199] Dobrushin R.L., *Gaussian and their subordinated self-similar random fields*, Ann. Probab. **7** (1979), no. 1, 1–28.
- [200] Dobrushin R.L., Major P., *Non-central limit theorems for nonlinear functionals of Gaussian fields*, Z. Wahrscheinlichkeitstheorie verw. Geb. **50** (1979), no. 1, 27–52.
- [201] Dobrushin R.L., Minlos R.A., *Polynomials in linear random functions*, Uspehi Matem. Nauk **32** (1977), no. 2, 67–122 (in Russian); English transl.: Russian Math. Surveys **32** (1977), no. 2, 71–127.

- [202] Dobrushin L.R., Minlos R.A., *An investigation of the properties of generalized Gaussian random fields*, *Selecta Math. Sov.* **1** (1981), 215–263.
- [203] Donsker M.D., Varadhan S.R.S., *Asymptotic evaluation of certain Markov process expectations for large time. I*, *Commun. Pure and Appl. Math.* **28** (1975), no. 1, 1–47.
- [204] Doob J.L., *The Brownian movement and stochastic equations*, *Ann. Math.* **43** (1942), no. 2, 351–369.
- [205] Doob J.L., *The elementary Gaussian processes*, *Ann. Math. Stat.* **15** (1944), 229–282.
- [206] Doob J.L., *Stochastic Processes*, Wiley, New York, 1953.
- [207] Dorogovtsev A.A., *Stochastic Analysis and Random Linear Maps in Hilbert Space*, Naukova Dumka, Kiev, 1992 (in Russian); English transl.: VSP, Utrecht, 1994.
- [208] Driver B., *A Cameron–Martin type quasi-invariance theorem for the Brownian motion on a compact manifold*, *J. Funct. Anal.* **110** (1992), 272–376.
- [209] Dudley R.M., *The sizes of compact subsets of Hilbert space and continuity of Gaussian processes*, *J. Funct. Anal.* **1** (1967), no. 3, 290–330.
- [210] Dudley R.M., *Sample functions of the Gaussian processes*, *Ann. Probab.* **1** (1973), no. 1, 3–68.
- [211] Dudley R.M., Feldman J., Le Cam L., *On the seminorms and probabilities, and abstract Wiener spaces*, *Ann. Math.* **93** (1971), no. 2, 390–408.
- [212] Dudley R.M., Kanter M., *Zero-one laws for stable measures*, *Proc. Amer. Math. Soc.* **45** (1974), no. 2, 245–252; Correction: *ibid.* **88** (1983), no. 4, 689–690.
- [213] Duncan T.E., *Absolute continuity for abstract Wiener spaces*, *Pacif. J. Math.* **52** (1974), no. 2, 359–367.
- [214] Dunford N., Schwartz J.T., *Linear Operators, Part I*, Interscience Publ., 1960.
- [215] Dunn O.J., *Estimation of the means of dependent variables*, *Ann. Math. Statist.* **29** (1958), 1095–1111.
- [216] Dym H., McKean H.P., *Gaussian Processes, Function Theory, and the Inverse Spectral Problem*, Academic Press, New York, 1976.
- [217] Dynkin E.B., Yushkevich A.A., *Controlled Markov Processes*, Nauka, Moscow, 1975 (in Russian); English transl.: Springer, Berlin, 1979.
- [218] Eagleson G.K., *An extended dichotomy theorem for sequences of pairs of Gaussian measures*, *Ann. Probab.* **9** (1981), no. 3, 453–459.
- [219] Edgar G.A., *Measurability in a Banach space*, *Indiana Univ. Math. J.* **26** (1977), no. 4, 663–680.
- [220] Edwards R.E., *Functional Analysis. Theory and Applications*, Holt, Rinehart and Winston, New York — London, 1965.
- [221] Egorov A.D., Sobolevsky P.I., Yanovich L.A., *Functional Integrals: Approximate Evaluation and Applications*, Kluwer Academic Publ., Dordrecht, 1993.
- [222] Ehrhard A., *Symétrisation dans l'espace de Gauss*, *Math. Scand.* **53** (1983), 281–301.
- [223] Ehrhard A., *Un principe de symétrisation dans les espaces de Gauss*, *Lecture Notes in Math.* **990** (1983), 92–101.
- [224] Ehrhard A., *Inégalités isopérimétriques et intégrales de Dirichlet gaussiennes*, *Ann. Sci. Ec. Norm. Super.* **17** (1984), no. 2, 317–322.
- [225] Ehrhard A., *Eléments extrémaux pour les inégalités de Brunn–Minkowski gaussiennes*, *Ann. Inst. H. Poincaré* **22** (1986), no. 1, 149–168.
- [226] Eidlin V.L., *On certain classes of transformations preserving normality*, *Teor. Veroyatn. i Primenen.* **17** (1972), no. 3, 487–495 (in Russian); English transl.: *Theory Probab. Appl.* **17** (1972), no. 3, 463–471.
- [227] Ellis H.W., *Darboux properties and applications to non-convergent integrals*, *Canad. J. Math.* **3** (1951), 471–485.
- [228] Elworthy K.D., *Gaussian measures on Banach spaces and manifolds*, In: *Global Anal. and Appl.*, Vienna, 1974.

- [229] Enchev O., *Non linear transformation on the Wiener space*, Ann. Probab. **21** (1993), no. 4, 2169–2188.
- [230] Enchev O., Stroock D., *Rademacher's theorem for Wiener functionals*, Ann. Probab. **21** (1993), no. 1, 25–33.
- [231] Engelking R., *General Topology*, Polish Sci. Publ., Warszawa, 1977.
- [232] Evans S.N., *Equivalence and perpendicularity of local field Gaussian measures*, In: Seminar on Stochastic Processes (Vancouver, 1990), Progr. in Probab. **24**, pp. 173–181. Birkhauser, Boston, 1991.
- [233] Fang S., *Pseudo-théorème de Sard pour les applications réelles et connexité sur l'espace de Wiener*, Bull. Sci. Math. **113** (1989), no. 4, 483–492.
- [234] Fang S., *On the Ornstein-Uhlenbeck process*, Stochastics and Stochastics Reports **46** (1994), 141–159.
- [235] Fang S., *On derivatives of holomorphic functions on a complex Wiener space*, J. Math. Kyoto Univ. **34** (1994), no. 3, 637–640.
- [236] Fang S., Ren J., *Sur le squelette et les dérivées de Malliavin des fonctions holomorphes sur espace de Wiener complexe*, J. Math. Kyoto Univ. **33** (1993), no. 3, 749–764.
- [237] Federer H., *Geometric Measure Theory*, Springer, Berlin, 1969.
- [238] Feldman J., *Equivalence and perpendicularity of Gaussian processes*, Pacif. J. Math. **8** (1958), no. 4, 699–708; Correction: *ibid.* **9** (1959), 1295–1296.
- [239] Fernique X., *Continuité des processus gaussiens*, C. R. Acad. Sci. Paris **258** (1964), 6058–6060.
- [240] Fernique X., *Processus linéaires, processus généralisés*, Ann. Inst. Fourier (Grenoble) **17** (1967), 1–92.
- [241] Fernique X., *Intégrabilité des vecteurs gaussiens*, C. R. Acad. Sci. Paris **270** (1970), no. 25, 1698–1699.
- [242] Fernique X., *Une démonstration simple du théorème de R.M. Dudley et M. Kanter sur les lois zero-un pour les mesures stables*, Lecture Notes in Math. **381** (1974), 78–79.
- [243] Fernique X., *Régularité des trajectoires des fonctions aléatoires gaussiennes*, Lecture Notes in Math. **480** (1975), 2–187.
- [244] Fernique X., *Sur les théorèmes de Hajek-Feldman et de Cameron-Martin*, C. R. Acad. Sci. Paris **299** (1984), no. 8, 355–358.
- [245] Fernique X., *Comparaison de mesures gaussiennes et de mesures produit*, Ann. Inst. H. Poincaré, Probab. et Statist. **20** (1984), no. 2, 165–175.
- [246] Fernique X., *Comparaison de mesures gaussiennes et de mesures produit dans les espaces de Fréchet séparables*, Lecture Notes in Math. **1153** (1985), 179–197.
- [247] Fernique X., *Sur la convergence étroite des mesures gaussiennes*, Z. Wahrscheinlichkeitstheorie verw. Geb. **68** (1985), 331–336.
- [248] Fernique X., *Fonctions aléatoires dans les espaces lusiniens*, Expositiones Math. **8** (1990), 289–364.
- [249] Fernique X., *Régularité de fonctions aléatoires gaussiennes à valeurs vectorielles*, Ann. Probab. **18** (1990), 1739–1745.
- [250] Fernique X., *Sur la régularité de certaines fonctions aléatoires d'Ornstein-Uhlenbeck*, Ann. Inst. H. Poincaré **26** (1990), 399–417.
- [251] Feyel D., de La Pradelle A., *Espaces de Sobolev gaussiens*, Ann. Inst. Fourier **39** (1989), no. 4, 875–908.
- [252] Feyel D., de La Pradelle A., *Capacités gaussiens*, Ann. Inst. Fourier **41** (1991), no. 1, 49–76.
- [253] Feyel D., de La Pradelle A., *Hausdorff measures on the Wiener space*, Potential Anal. **1** (1992), 177–189.
- [254] Feyel D., de La Pradelle A., *Opérateurs linéaires gaussiens*, Potential Anal. **3** (1994), no. 1, 89–105.

- [255] Feyel D., de La Pradelle A., *Brownian processes in infinite dimension*, Potential Anal. **4** (1995), 173–183.
- [256] Feynman R.P., Hibbs A.R., *Quantum Mechanics and Path Integrals*, McGraw–Hill, New York, 1965.
- [257] Fitzsimmons P.J., *Brownian space-time functions of zero quadratic variation depend only on time*, Proc. Amer. Math. Soc. (1998).
- [258] Föllmer H., *Martin boundaries on Wiener space*, In: Diffusion Processes and Related Problems, Vol. 1, M. Pinsky ed., pp. 3–16. Birkhäuser, 1989.
- [259] Föllmer H., Wakolbinger A., *Time reversal of infinite dimensional diffusions*, Stoch. Process. and Appl. **22** (1986), no. 1, 59–77.
- [260] Fomin S.V., *Differentiable measures in linear spaces*, Uspehi Matem. Nauk **23** (1968), no. 1, 221–222 (in Russian).
- [261] Fortet R., Mourier E., *Les fonctions aléatoires comme éléments aléatoires dans les espaces de Banach*, Studia Math. **15** (1955), 62–73.
- [262] Fréchet M., *Généralization de la loi de probabilité de Laplace*, Ann. Inst. H. Poincaré **12** (1951), 1–29.
- [263] Freidlin M., *Functional Integration and Partial Differential Equations*, Princeton University Press, Princeton, 1985.
- [264] Freidlin M., Wentzell A., *Random Perturbations of Dynamical Systems*, Nauka, Moscow, 1979 (in Russian); English transl.: Springer–Verlag, Berlin, 1984.
- [265] Fremlin D.H., Talagrand M., *A Gaussian measure on l^∞* , Ann. Probab. **8** (1980), no. 6, 1192–1193.
- [266] Friedrichs K.O., *Mathematical Aspects of the Quantum Theory of Fields*, Interscience, New York, 1953.
- [267] Friedrichs K.O., Shapiro H.N., *Integration over Hilbert spaces and outer extensions*, Proc. Nat. Acad. Sci. USA **43** (1957), no. 4, 336–338.
- [268] Frolov N.N., *Embedding theorems for spaces of functions of countably many variables, I*, Proceedings Math. Inst. of Voronezh Univ., Voronezh University (1970), no. 1, 205–218 (in Russian).
- [269] Frolov N.N., *Embedding theorems for spaces of functions of countably many variables and their applications to the Dirichlet problem*, Dokl. Akad. Nauk SSSR **203** (1972), no. 1, 39–42 (in Russian); English transl.: Soviet Math. **13** (1972), no. 2, 346–349.
- [270] Frolov N.N., *On a coercitive inequality for an elliptic operator in infinitely many variables*, Matem. Sbornik **90** (1973), no. 3, 402–413 (in Russian); English transl.: Math. USSR Sbornik **19** (1973), 395–406.
- [271] Frolov N.N., *Imbedding theorems for spaces of functions of a countable number of variables and their applications*, Sibirsk. Matem. Zhurn. **22** (1981), no. 4, 199–217 (in Russian); English transl.: Siberian Math. J. **22** (1981), no. 4, 638–652.
- [272] Fukuda R., *Exponential integrability of sub-Gaussian vectors*, Probab. Theory Relat. Fields **85** (1990), 505–521.
- [273] Fukushima M., *Basic properties of Brownian motion and a capacity on the Wiener space*, J. Math. Soc. Jap. **36** (1984), no. 1, 161–176.
- [274] Fukushima M., *A note on capacities in infinite dimensions*, Lecture Notes in Math. **1299** (1988), 80–85.
- [275] Fukushima M., Kaneko H., *On (r, p) -capacities for general Markovian semigroups*, In: Infinite Dimensional Analysis and Stochastic Processes (Bielefeld, 1983), pp. 41–47. Boston, 1985.
- [276] Fuhrman M., *Hypercontractivité des semi-groupes de Ornstein–Uhlenbeck non symétriques*, C. R. Acad. Sci. Paris **321** (1995), no. 7, 929–932.
- [277] Gallamov M.M., *Wiener measures and some problems of approximation in Banach spaces*, Analysis Math. **18** (1992), no. 1, 25–36 (in Russian).
- [278] Garsia A.M., Posner E.C., Rodemich E.R., *Some properties of measures on function spaces induced by Gaussian processes*, J. Math. Anal. Appl. **21** (1968), 150–161.

- [279] Garsia A.M., Rodemich E., Rumsey H., *A real variable lemma and the continuity of paths of some Gaussian processes*, Indiana Math. J. **20** (1970), 565–578.
- [280] Gauss F., *Theoria Combinationis Observationum Erroribus Minimis Obnoxiae*, Göttingen, 1823.
- [281] Gaveau B., Moulinier J.-M., *Intégrales oscillantes stochastiques: estimation asymptotique de fonctionnelles caractéristiques*, J. Funct. Anal. **54** (1983), no. 2, 161–176.
- [282] Gaveau B., Moulinier J.-M., *Régularité des mesures et perturbation stochastiques de champs des vecteurs sur des espaces de dimension infinie*, Publ. Res. Inst. Sci. Kyoto Univ. **21** (1985), no. 3, 593–616.
- [283] Gaveau B., Trauber P., *L'intégral stochastique comme opérateur de divergence dans l'espace fonctionnel*, J. Funct. Anal. **46** (1982), no. 2, 230–238.
- [284] Gawarecki L., Mandrekar V., *On Girsanov type theorem for anticipative shifts*, In: Probability in Banach spaces, Vol. 9, J. Hoffmann-Jorgensen, J. Kuelbs, and M.B. Marcus eds., pp. 301–316. Birkhäuser, Boston — Basel — Berlin, 1994.
- [285] Gelfand I.M., Vilenkin N.Ya., *Generalized Functions*, Vol. 4, *Applications of Harmonic Analysis*, Nauka, Moscow, 1961 (in Russian); English transl.: Academic Press, New York — London, 1964.
- [286] Georgii H.-O., *Gibbs Measures and Phase Transitions*, de Gruyter, Berlin — New York, 1988.
- [287] Getzler E., *Degree theory for Wiener maps*, J. Funct. Anal. **68** (1988), no. 3, 388–403.
- [288] Gihman I.I., Skorohod A.V., *Densities of probability measures in function spaces*, Uspehi Matem. Nauk **21** (1966), no. 6, 83–152 (in Russian); English transl.: Russian Math. Surveys **21** (1966), no. 6, 83–156.
- [289] Gikhman I.I., Skorohod A.V., *The Theory of Stochastic Processes*, Vol. 1, Nauka, Moscow, 1971 (in Russian); English transl.: Springer-Verlag, Berlin, 1979.
- [290] Girsanov I.V., *On transforming a certain class of stochastic processes by absolutely continuous substitution of measures*, Teor. Veroyatn. i Primenen. **5** (1961), no. 3, 314–330 (in Russian); English transl.: Theory Probab. Appl. **5** (1960), 285–301.
- [291] Glimm J., Jaffe A., *Quantum Physics, a Functional Integral Point of View*, Springer, Berlin — New York, 1981.
- [292] Gnedenko B.V., *On a theorem of S.N. Bernstein*, Izvestia Akad. Nauk SSSR. Ser. Mat. **12** (1948), 97–100 (in Russian).
- [293] Gnedin A.V., *On mean-square epsilon dimension*, Math. Jap. **37** (1992), no. 4, 623–627.
- [294] Go F.-Z., Ma Z.-M., *Invariance of Malliavin fields on Ito's Wiener space and on abstract Wiener space*, J. Funct. Anal. **138** (1996), 449–476.
- [295] Götze F., Prohorov Yu.V., Ulyanov V., *Bounds for characteristic functions of polynomials in asymptotically normal variables*, Uspehi Matem. Nauk **51** (1996), no. 2, 3–26 (in Russian); English transl.: Russian Math. Surveys **51** (1996), no. 2, 181–204.
- [296] Gohberg I.G., Krein M.G., *Introduction to the Theory of Linear Nonselfadjoint Operators*, Nauka, Moscow, 1965 (in Russian); English transl.: Amer. Math. Soc., Providence, Rhode Island, 1969.
- [297] Gol'dshtein V.M., Reshetnyak Yu.G., *Quasiconformal Mappings and Sobolev Spaces*, Nauka, Novosibirsk, 1983 (in Russian); English transl.: Kluwer Academic Publ., Dordrecht, 1990.
- [298] Golosov Ju.I., *Gaussian measures equivalent to Gaussian Markov measures*, Doklady Akad. Nauk SSSR **166** (1966), 263–266 (in Russian); English transl.: Soviet Math. **7** (1966), no. 1, 48–52.
- [299] Golosov Ju.I., *A method for evaluating the Radon-Nikodym derivatives of two Gaussian measures*, Dokl. Akad. Nauk SSSR **170** (1966), no. 2, 242–245 (in Russian); English transl.: Soviet Math. **7** (1966), no. 5, 1162–1165.
- [300] Goodman V., *Quasi-differentiable functions on Banach spaces*, Proc. Amer. Math. Soc. **30** (1971), no. 2, 367–370.

- [301] Goodman V., *A divergence theorem for Hilbert space*, Trans. Amer. Math. Soc. **164** (1972), 411–426.
- [302] Goodman V., *Some probability and entropy estimates for Gaussian measures*, In: Probab. in Banach spaces, Vol. 6, pp. 150–156. Birkhäuser, Boston, 1990.
- [303] Goodman V., Kuelbs J., *Rates of clustering for weakly convergent Gaussian random vectors and some applications*, In: Probab. in Banach Spaces, Vol. 8, pp. 304–324. Birkhäuser, Boston, 1992.
- [304] Goodman V., Kuelbs J., *Cramer functional estimates for Gaussian measures*, In: Diffusion Processes and Related Problems in Analysis. Progress in Probab., Vol. 22, pp. 473–495. Birkhäuser, Boston, 1990.
- [305] Goodman V., Kuelbs J., *Gaussian chaos and functional laws of the iterated logarithm for Ito-Wiener integrals*, Ann. Inst. H. Poincaré **29** (1993), 485–512.
- [306] Gordon Y., *Some inequalities for Gaussian processes and applications*, Israel J. Math. **50** (1985), 265–289.
- [307] Gordon Y., *Gaussian processes and almost spherical sections of convex bodies*, Ann. Probab. **16** (1988), 180–188.
- [308] Gordon Y., *Majorization of Gaussian processes and geometric applications*, Probab. Theory Relat. Fields **91** (1992), 251–266.
- [309] Gowers W.T., Maurey B., *The unconditional basic sequence problem*, J. Amer. Math. Soc. **6** (1993), no. 4, 857–874.
- [310] Graves R.E., *Additive functionals over a space of continuous functions*, Ann. Math. **54** (1951), no. 2, 275–285.
- [311] Grenander U., *Stochastic processes and statistical inference*, Ark. Math. **1** (1950), no. 3, 195–277.
- [312] Gross L., *Integration and nonlinear transformations in Hilbert space*, Trans. Amer. Math. Soc. **94** (1960), no. 3, 404–440.
- [313] Gross L., *Harmonic analysis on Hilbert space*, Mem. Amer. Math. Soc. **46** (1963), 1–62.
- [314] Gross L., *Abstract Wiener spaces*, In: Proc. 5th Berkeley Symp. Math. Stat. Probab., Part 1, pp. 31–41. University of California Press, Berkeley, 1965.
- [315] Gross L., *Potential theory on Hilbert space*, J. Funct. Anal. **1** (1967), no. 2, 123–181.
- [316] Gross L., *Abstract Wiener measure and infinite dimensional potential theory*, Lecture Notes in Math. **140** (1970), 84–119.
- [317] Gross L., *Logarithmic Sobolev inequalities*, Amer. J. Math. **97** (1975), no. 4, 1061–1083.
- [318] Gross L., *Logarithmic Sobolev inequalities and contractive properties of semigroups*, Lecture Notes in Math. **1563** (1993), 54–82.
- [319] Guerin M., *Non-hilbertian structure of the Wiener measure*, Colloq. Math. **28** (1973), 145–146.
- [320] Guichardet A., *Symmetric Hilbert Spaces and Related Problems*, Lect. Notes in Math. **261**, Springer-Verlag, 1972.
- [321] Gupta S.S., *Bibliography on the multivariate normal integrals and related topics*, Ann. Math. Statist. **34** (1963), 829–838.
- [322] Gutiérrez C., *On the Riesz transforms for Gaussian measures*, J. Funct. Anal. **120** (1994), 107–134.
- [323] de Guzmán M., *Differentiation of Integrals in \mathbb{R}^n* , Lecture Notes in Math. **481**, Springer-Verlag, Berlin — Heidelberg — New York, 1975.
- [324] Hajek J., *On a property of normal distributions of any stochastic process*, Czech. Math. J. **8** (1958), 610–618 (in Russian); English transl.: Selecta Transl. Math. Statist. and Probab., Vol. 1, pp. 245–252. Inst. Math. Statist. and Amer. Math. Soc., Providence, Rhode Island, 1961.
- [325] Hajek J., *A property of J -divergences of marginal probability distributions*, Czech. Math. J. **8** (1958), 460–463.

- [326] Halmos P., Sunder V.S., *Bounded Integral Operators on L^2 Spaces*, Springer-Verlag, Berlin — New York, 1978.
- [327] Hardy G.H., *A theorem concerning Fourier transforms*, J. London Math. Soc. (2) **8** (1933), 227–231.
- [328] Hazod W., *Stable probability measures on groups and on vector spaces*, Lecture Notes in Math. **1210** (1986), 304–352.
- [329] Heinrich P., *Zero-one laws for polynomials in Gaussian random variables: a simple proof*, J. Theor. Probab. **9** (1996), no. 4, 1019–1027.
- [330] Herer W., *A characterization of Gaussian measures on Hilbert space*, Bull. Acad. Polon. Sci. Math. Astronom. Phys. **17** (1969), 443–446.
- [331] Hertle A., *Gaussian surface measures and the Radon transform on separable Banach spaces*, Lecture Notes in Math. **794** (1980), 513–531.
- [332] Hertle A., *Gaussian plane and spherical means in separable Hilbert spaces*, Lecture Notes in Math. **945** (1982), 314–335.
- [333] Hertle A., *On the asymptotic behaviour of Gaussian spherical integrals*, Lecture Notes in Math. **990** (1983), 221–234.
- [334] Heyer H., *Probability Measures on Locally Compact Groups*, Springer-Verlag, Berlin, 1977.
- [335] Hida T., *Canonical representations of Gaussian processes and their applications*, Mem. Coll. Sci. Univ. Kyoto, A (Math.) **33** (1960), 109–155.
- [336] Hida T., *Quadratic functionals of Brownian motion*, J. Multivar. Anal. **1** (1971), 58–69.
- [337] Hida T., *Brownian Motion*, Springer, Berlin, 1980.
- [338] Hida T., Hitsuda M., *Gaussian Processes*, Amer. Math. Soc., Providence, Rhode Island, 1993.
- [339] Hida T., Kuo H., Pothoff J., Streit L., *White Noise Calculus*, Kluwer Academic Publ., Dordrecht, 1993.
- [340] Hida T., Nomoto H., *Gaussian measure on the projective limit space of measures*, Proc. Japan Acad. **40** (1964), 301–304.
- [341] Hirsch F., *Theory of capacity on the Wiener space*, In: Stochastic Analysis and Related Topics, V (The Silivri Workshop, 1994), H. Kőrezlioglu, B. Øksendal, and A.S. Üstünel eds., pp. 69–98. Birkhäuser, Boston — Basel — Berlin, 1996.
- [342] Hitsuda M., *Representation of Gaussian processes equivalent to Wiener measure*, Osaka J. Math. **5** (1968), no. 2, 299–312.
- [343] Hoeffding W., *On a theorem of V.M. Zolotarev*, Teor. Veroyatn. i Primenen. **9** (1964), no. 1, 96–99 (in Russian); English transl.: Theory Probab. Appl. **9** (1964), 89–91.
- [344] Hoffmann-Jorgensen J., *The Theory of Analytic Spaces*, Aarhus Various Publ. Series, Vol. 10, Aarhus, 1970.
- [345] Hoffmann-Jorgensen J., Shepp L.A., Dudley R., *On the lower tail of Gaussian seminorms*, Ann. Probab. **7** (1979), 319–342.
- [346] Holley R., Stroock D., *The D.L.R. conditions for translation invariant Gaussian measures on $\mathcal{S}'(\mathbf{R}^d)$* , Z. Wahrscheinlichkeitstheorie verw. Geb. **53** (1980), no. 3, 293–304.
- [347] Hörmander L., *The Analysis of Linear Partial Differential Operators*, Vol. 2, Springer-Verlag, Berlin — New York, 1983.
- [348] Houdré C., Kagan A., *Variance inequalities for functions of Gaussian variables*, J. Theor. Probab. **8** (1995), 23–30.
- [349] Houdré C., Pérez-Abreu V., *Covariance identities and inequalities for functionals on Wiener and Poisson spaces*, Ann. Probab. **23** (1995), no. 1, 400–419.
- [350] Hu Y., *Itô-Wiener chaos expansion with exact residual and correlation, variance inequalities*, J. Theor. Probab. **10** (1997), no. 4, 835–848.
- [351] Hu Y.Z., Meyer P.A., *Sur les intégrales multiples de Stratonovich*, Lecture Notes in Math. **1321** (1988), 72–81.

- [352] Hu Y.Z., Meyer P.A., *Chaos de Wiener et intégrale de Feynman*, Lecture Notes in Math. **1321** (1988), 51–71.
- [353] Huang S.T., Cambanis S., *Stochastic and multiple Wiener integrals for Gaussian processes*, Ann. Probab. **6** (1978), 585–614.
- [354] Hunt G.A., *Random Fourier transforms*, Trans. Amer. Math. Soc. **71** (1951), 38–69.
- [355] Hwang C.R., *Gaussian measure of large balls in a Hilbert space*, Proc. Amer. Math. Soc. **78** (1980), no. 1, 107–110; Erratum: *ibid.* **94** (1985), no. 1, 188.
- [356] Ibragimov I.A., *On the probability that a Gaussian vector with values in a Hilbert space hits a sphere of small radius*, J. Soviet Math. **20** (1982), 2164–2174.
- [357] Ibragimov I.A., *On conditions for the smoothness of trajectories of random functions*, Teor. Veroyatn. i Primenen. **28** (1983), no. 2, 229–250 (in Russian); English transl.: Theory Probab. Appl. **28** (1983), no. 2, 240–262.
- [358] Ibragimov I.A., *Conditions for Gaussian homogeneous fields to belong to classes H_p^r* , Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov **184** (1990), 126–143 (in Russian); English transl.: J. Math. Sci. **68** (1994), no. 4, 484–497.
- [359] Ibragimov I.A., Rozanov Yu.A., *Gaussian Random Processes*, Nauka, Moscow, 1970 (in Russian); English transl.: Springer-Verlag, New York — Berlin, 1978.
- [360] Ibramhalilov I.Sh., Skorohod A.V., *Consistent Estimates of Parameters of Random Processes*, Naukova Dumka, Kiev, 1980 (in Russian).
- [361] Ikeda N., Watanabe S., *Stochastic Differential Equations and Diffusion Processes*, North-Holland, 1981.
- [362] Imkeller P., Nualart D., *Integration by parts on Wiener space and the existence of occupation densities*, Ann. Probab. **22** (1994), 469–493.
- [363] Inglot T., *An elementary approach to the zero-one laws for Gaussian measures*, Colloq. Math. **40** (1979), no. 2, 319–325.
- [364] Inglot T., Weron A., *On Gaussian random elements in some non-Banach spaces*, Bull. Polon. Sci. Ser. Math., Astronom., Phys. **22** (1974), 1039–1043.
- [365] Iscoe I., Marcus M.B., McDonald D., Talagrand M., Zinn J., *Continuity of l^2 -valued Ornstein-Uhlenbeck processes*, Ann. Probab. **18** (1990), 68–84.
- [366] Itô K., *Multiple Wiener integral*, J. Math. Soc. Japan. **3** (1951), 157–169.
- [367] Itô K., *The topological support of Gaussian measures on Hilbert space*, Nagoya Math. J. **38** (1970), 181–183.
- [368] Itô K., *Infinite dimensional Ornstein-Uhlenbeck processes*, In: Taniguchi Symp. SA, Katata, 1982, pp. 197–224. North-Holland, Amsterdam, 1984.
- [369] Itô K., McKean H.P., *Diffusion Processes and their Sample Paths*, Springer-Verlag, Berlin, 1974.
- [370] Itô K., Nisio M., *On the oscillation functions of Gaussian processes*, Math. Scand. **22** (1968), no. 1, 209–232.
- [371] Itô K., Nisio M., *On the convergence of sums of independent Banach space valued random variables*, Osaka J. Math. **5** (1968), no. 1, 35–48.
- [372] Jain N., *A zero-one law for Gaussian processes*, Proc. Amer. Math. Soc. **29** (1971), no. 3, 585–587.
- [373] Jain N., Kallianpur G., *A note on uniform convergence of stochastic processes*, Ann. Math. Statist. **41** (1970), 1360–1362.
- [374] Jain N., Kallianpur G., *Norm convergent expansions for Gaussian processes in Banach spaces*, Proc. Amer. Math. Soc. **25** (1970), no. 4, 890–895.
- [375] Jain N., Kallianpur G., *Oscillation function of a multiparameter Gaussian process*, Nagoya Math. J. **47** (1972), 15–28.
- [376] Jain N.C., Marcus M.B., *Sufficient conditions for continuity for stationary Gaussian processes and applications to random series of functions*, Ann. Inst. Fourier **24** (1974), no. 2, 117–141.
- [377] Jain N.C., Marcus M.B., *Continuity of subgaussian processes*, In: Probability on Banach spaces, J. Kuelbs ed., pp. 81–196. Marcel Dekker, New York, 1978.

- [378] Jain N.C., Monrad D., *Gaussian submartingales*, Z. Wahrscheinlichkeitstheorie verw. Geb. **59** (1982), 139–159.
- [379] Jain N.C., Monrad D., *Gaussian measures in B_p* , Ann. Probab. **11** (1983), no. 1, 46–57.
- [380] Jamison B., Orey S., *Subgroups of sequences and paths*, Proc. Amer. Math. Soc. **24** (1970), no. 4, 739–744.
- [381] Janson S., *Gaussian Hilbert Spaces*, Cambridge University Press, Cambridge, 1997.
- [382] Johnson G.W., Kallianpur G., *Multiple Wiener integrals on abstract Wiener spaces and liftings of p -linear forms*, In: White noise analysis (Bielefeld, 1989), pp. 208–219. World Sci. Publishing, River Edge, 1990.
- [383] Johnson G.W., Skoug D.L., *Scale-invariant measurability in Wiener space*, Pacific J. Math. **83** (1979), no. 1, 157–176.
- [384] Johnson N.L., Kotz S., *Distributions in Statistics: Continuous Multivariate Distributions*, Wiley, New York, 1972.
- [385] Kac M., *On a characterization of the normal distribution*, Amer. J. Math. **61** (1939), 726–728.
- [386] Kac M., *On distributions of certain Wiener functionals*, Trans. Amer. Math. Soc. **65** (1949), 1–13.
- [387] Kac M., *Integration in Function Spaces and Some of its Applications*, Scuola Normale Superiore, Pisa, 1980.
- [388] Kadota T., Shepp L.A., *Conditions for the absolute continuity between a certain pair of probability measures*, Z. Wahrscheinlichkeitstheorie verw. Geb. **16** (1970), 250–260.
- [389] Kagan A.M., Linnik Yu.V., Rao C.R., *Characterization Problems in Mathematical Statistics*, Nauka, Moscow, 1972 (in Russian); English transl.: Wiley, New York, 1973.
- [390] Kahane J.-P., *Some Random Series of Functions*, 2nd edn., Cambridge University Press, Cambridge, 1985.
- [391] Kahane J.-P., *Une inégalité du type de Slepian et Gordon sur les processus gaussiens*, Israel J. Math. **55** (1986), 109–110.
- [392] Kahane J.-P., *A century of interplay between Taylor series, Fourier series and Brownian motion*, Bull. London Math. Soc. **29** (1997), no. 3, 257–279.
- [393] Kailath T., *The structure of Radon–Nikodym derivatives with respect to Wiener and related measures*, Ann. Math. Statist. **42** (1971), no. 3, 1054–1067.
- [394] Kailath T., Zakai M., *Absolute continuity and Radon–Nikodym derivatives for certain measures relative to Wiener measure*, Ann. Math. Statist. **42** (1971), no. 1, 130–140.
- [395] Kakutani S., *On equivalence of infinite product measures*, Ann. Math. **49** (1948), 214–224.
- [396] Kallianpur G., *The role of reproducing kernel Hilbert spaces in the study of Gaussian processes*, In: Advances in Probab., Vol. 2, pp. 49–83. Marcel Dekker, New York, 1970.
- [397] Kallianpur G., *Zero-one laws for Gaussian processes*, Trans. Amer. Math. Soc. **149** (1970), no. 1, 199–211.
- [398] Kallianpur G., *Abstract Wiener spaces and their reproducing kernel Hilbert spaces*, Z. Wahrscheinlichkeitstheorie verw. Geb. **17** (1971), 113–123.
- [399] Kallianpur G., *Stochastic Filtering Theory*, Springer–Verlag, New York — Berlin, 1980.
- [400] Kallianpur G., Oodaira H., *The equivalence and singularity of Gaussian processes*, In: Proc. Symp. on Time Series Analysis, pp. 279–291. Wiley, New York, 1963.
- [401] Kallianpur G., Oodaira H., *Non-anticipative representations of equivalent Gaussian processes*, Ann. Probab. **1** (1973), 104–122.
- [402] Kallianpur G., Oodaira H., *Freidlin–Wentzell type estimates for abstract Wiener spaces*, Sankhya, ser. A, **40** (1978), 116–137.

- [403] Kallianpur G., Kannan D., Karandikar R.L., *Analytic and sequential Feynman integrals on abstract Wiener and Hilbert spaces, and a Cameron–Martin formula*, Ann. Inst. H. Poincaré. Probab. et Statist. **21** (1985), no. 4, 323–361.
- [404] Kaneko H., *On (τ, p) -capacities for Markov processes*, Osaka J. Math. **23** (1984), 325–336.
- [405] Kannan D., Kannappan Pl., *On a characterization of Gaussians measures in a Hilbert space*, Ann. Inst. H. Poincaré **11** (1975), no. 4, 397–404.
- [406] Karatzas I., Shreve S.E., *Brownian Motion and Stochastic Calculus*, Springer–Verlag, New York — Berlin — Heidelberg, 1988.
- [407] Karhunen K., *Ueber lineare Methoden in der Wahrscheinlichkeitsrechnung*, Ann. Acad. Sci. Fennicae, Ser. A, Math. Phys. **37** (1947), 3–79.
- [408] Kats M.P., *Continuity of universally measurable linear maps*, Sibirsk. Matem. Zhurn. **23** (1982), no. 3, 83–90 (in Russian); Correction: *ibid.* **24** (1983), no. 3, 217; English transl.: Siberian Math. J. **23** (1982), no. 3, 358–364.
- [409] Katznelson Y., Malliavin P., *Image des points critiques d’une application régulière*, Lecture Notes in Math. **1322** (1987), 85–92.
- [410] Katznelson Y., Malliavin P., *Un contre-exemple au théorème de Sard en dimension infinie*, C. R. Acad. Sci. Paris **306** (1988), 37–41.
- [411] Kazumi T., *Refinements in terms of capacities of certain limit theorems on an abstract Wiener space*, J. Math. Kyoto Univ. **32** (1992), 1–33.
- [412] Kazumi T., Shigekawa I., *Measures of finite (r, p) -energy and potentials on a separable metric space*, Lecture Notes in Math. **1526** (1992), 415–444.
- [413] Kendall M.G., Stewart A., *The Advanced Theory of Statistics*, vol. 1: *Distribution Theory*, 4th edn., Goffin and Co., London, 1977.
- [414] Khafisov M.U., *Some new results on differentiable measures*, Vestnik Moskovsk. Univ. Ser. I Mat. Mekh. (1990), no. 4, 63–66 (in Russian); English transl.: Moscow State Univ. Math. Bull. **45** (1990), no. 4, 34–36.
- [415] Khafisov M.U., *A quasi-invariant smooth measure on the diffeomorphisms group of a domain*, Matem. Zamet. **48** (1990), no. 3, 134–142 (in Russian); English transl.: Math. Notes **48** (1990), no. 3–4, 968–974.
- [416] Khatri C., *On certain inequalities for normal distributions and their applications to simultaneous confidence bounds*, Ann. Math. Statist. **38** (1967), 1853–1867.
- [417] Khrennikov A.Yu., *The Dirichlet problem in a Banach space*, Matem. Zamet. **34** (1983), no. 4, 629–636; English transl.: Math. Notes **34** (1983), 804–808.
- [418] Khrennikov A.Yu., *Functional super-analysis*, Uspehi Matem. Nauk **43** (1988), no. 2, 87–114; English transl.: Russian Math. Surveys **43** (1988), no. 2, 103–137.
- [419] Khrennikov A.Yu., *p -Adic Valued Distributions in Mathematical Physics*, Kluwer Academic Publ., Dordrecht, 1994.
- [420] Kobanenko K.N., *On extensions of generalized Lipschitzian mappings*, Matem. Zamet. **63** (1998), no. 5, 789–791; English transl.: Math. Notes. **63** (1998).
- [421] Kolmogoroff A., *Grundbegriffe der Wahrscheinlichkeitsrechnung*, Berlin, 1933; English transl.: Kolmogorov A.N., *Foundations of the Theory of Probability*, Chelsea Publ. Co., New York, 1950.
- [422] Kolmogoroff A., *La transformation de Laplace dans les espaces linéaires*, C. R. Acad. Sci. Paris **200** (1935), 1717–1718.
- [423] Kolmogoroff A., Leontowitsch M., *Zur Berechnung der mittleren Brownschen Fläche*, Phys. Z. Sowjetunion **4** (1933), 1–13; English transl.: Kolmogorov A.N., Leontovich M.A. On evaluation of the average Brownian area, In: Selected works of A.N. Kolmogorov, Vol. 2 (A.N. Shyrayev ed.), pp. 128–138. Kluwer Academic Publ., Dordrecht, 1992.
- [424] Kôno N., *On the modulus of continuity of sample functions of Gaussian processes*, J. Math. Kyoto Univ. **10** (1970), 493–536.

- [425] Koval'chik I.M., *Wiener integral*, Uspekhi Matem. Nauk **18** (1963), no. 1, 97–134 (in Russian); English transl.: Russian Math. Surveys **18** (1963).
- [426] Krée M., *Propriété de trace en dimension infinie, d'espaces du type Sobolev*, C. R. Acad. Sci. Paris **279** (1974), no. 5, 157–164.
- [427] Krée M., *Propriété de trace en dimension infinie, d'espaces du type Sobolev*, Bull. Soc. Math. France **105** (1977), no. 2, 141–163.
- [428] Krée M., Krée P., *Continuité de la divergence dans les espace de Sobolev relatifs à l'espace de Wiener*, C. R. Acad. Sci. Paris **296** (1983), no. 20, 833–836.
- [429] Kruglov V.M., *Topics in Probability Theory*, Visshaya Shkola, Moscow, 1984 (in Russian).
- [430] Krylov N.V., *Introduction to the Theory of Diffusion Processes*, Amer. Math. Soc., Providence, Rhode Island, 1995.
- [431] Krylov N.V., *On SPDEs and superdiffusions*, Ann. Probab. **25** (1997), 1789–1809.
- [432] Kuelbs J., *Abstract Wiener spaces and applications to analysis*, Pacif. J. Math. **31** (1969), no. 2, 433–450.
- [433] Kuelbs J., *Gaussian measures on a Banach space*, J. Funct. Anal. **5** (1970), no. 3, 354–367.
- [434] Kuelbs J., *Expansions of vectors in a Banach space related to Gaussian measures*, Proc. Amer. Math. Soc. **27** (1971), no. 2, 364–370.
- [435] Kuelbs J., *Some results for probability measures on linear topological vector spaces with an application to Strassen's LogLog law*, J. Funct. Anal. **14** (1973), no. 1, 28–43.
- [436] Kuelbs J., Li W.V., *Metric entropy and the small ball problem for Gaussian measures*, J. Funct. Anal. **116** (1993), no. 1, 133–157.
- [437] Kuelbs J., Li W.V., *Small ball problems for Brownian motion and the Brownian sheet*, J. Theor. Probab. **5** (1993), 547–577.
- [438] Kuelbs J., Li W.V., *Gaussian samples approach "smooth points" slowest*, J. Funct. Anal. **124** (1994), 333–348.
- [439] Kuelbs J., Li W.V., *Some large deviation results for Gaussian measures*, In: Probability in Banach Spaces, Vol. 9 (J. Hoffmann-Jorgensen, J. Kuelbs, and M.B. Marcus eds.), pp. 251–270. Birkhäuser, Boston — Basel — Berlin, 1994.
- [440] Kuelbs J., Li W.V., Linde W., *The Gaussian measure of shifted balls*, Probab. Theory Relat. Fields **98** (1994), no. 2, 146–162.
- [441] Kuelbs J., Li W.V., Shao Q.-M., *Small ball probabilities for Gaussian processes with stationary increments under Hölder norms*, J. Theor. Probab. **8** (1995), 361–386.
- [442] Kuelbs J., Li W.V., Talagrand M., *Lim inf results for Gaussian samples and Chung's functional LIL*, Ann. Probab. **22** (1994), 1879–1903.
- [443] Kullback S., *Information Theory and Statistics*, Wiley, New York, 1958.
- [444] Kuo H.H., *Integration theory in infinite dimensional manifolds*, Trans. Amer. Math. Soc. **159** (1971), 57–78.
- [445] Kuo H., *Gaussian Measures in Banach spaces*, Lecture Notes in Math. **463**, Springer, Berlin — Heidelberg — New York, 1975.
- [446] Kuo H., *White Noise Distribution Theory*, CRC Press, Boca Raton, New York, 1996.
- [447] Kusuoka S., *The nonlinear transformation of Gaussian measure on Banach space and its absolute continuity I, II*, J. Fac. Sci. Univ. Tokyo. Sec. 1A. **29** (1982), no. 3, 567–597; **30** (1983), no. 1, 199–220.
- [448] Kusuoka S., *Dirichlet forms and diffusion processes on Banach spaces*, J. Fac. Sci. Univ. Tokyo. Sec. 1A **29** (1982), no. 1, 79–95.
- [449] Kusuoka S., *Analytic functionals of Wiener processes and absolute continuity*, Lecture Notes in Math. **923** (1982), 1–46.
- [450] Kusuoka S., *On the absolute continuity of the law of a system of multiple Wiener integrals*, J. Fac. Sci. Univ. Tokyo. Sec. 1A **30** (1983), no. 1, 191–197.

- [451] Kusuoka S., *A diffusion process on a fractal*, In: Probabilistic methods in mathematical physics, Proceedings of Taniguchi International Symp. (1985), pp. 251–274. Kinokuniga, Tokyo, 1987.
- [452] Kusuoka S., *Some remarks on Getzler's degree theorem*, Lecture Notes in Math. **1299** (1988), 239–249.
- [453] Kusuoka S., *Analysis on Wiener spaces I, nonlinear maps*, J. Funct. Anal. **98** (1991), 122–168; *II, Differential forms*, *ibid.* **103** (1992), 229–274.
- [454] Kusuoka S., Stroock D., *Precised asymptotics of certain Wiener functionals*, J. Funct. Anal. **99** (1991), 1–74.
- [455] Kwapien S., *Decoupling inequalities for polynomial chaos*, Ann. Probab. **15** (1987), 1062–1071.
- [456] Kwapien S., *A remark on the median and the expectation of convex functions of Gaussian vectors*, In: Probability in Banach spaces, Vol. 9 (J. Hoffmann-Jorgensen, J. Kuelbs, and M.B. Marcus eds.), pp. 271–272. Birkhäuser, Boston — Basel — Berlin, 1994.
- [457] Kwapien S., Szymanski B., *Some remarks on Gaussian measures on Banach spaces*, Probab. and Math. Statist. **1** (1980), 59–65.
- [458] Kwapien S., Sawa J., *On some conjecture concerning Gaussian measures of dilations of convex symmetric sets*, Studia Math. **105** (1993), no. 2, 173–187.
- [459] Kwapien S., Pycia M., Schachermayer W., *A proof of a conjecture of Bobkov and Houdré*, Electronics Communications in Probability **1** (1996), no. 2, 7–10.
- [460] Kwapien S., Woyczyński W.A., *Random Series and Stochastic Integrals: Single and Multiple*, Birkhäuser, Boston, 1992.
- [461] Landau H.J., Shepp L.A., *On the supremum of a Gaussian process*, Sankhya A **32** (1970), 369–378.
- [462] Langevin P., *Sur la théorie du mouvement brownien*, C. R. Acad. Sci. Paris **146** (1908), 530–533.
- [463] Laplace P.S., *Mémoire sur les intégrales définies et leur application aux probabilités, et spécialement à la recherche du milieu qu'il faut choisir entre les résultats des observations*, Mémoires de l'Institut Impérial de France (1810), 279–347.
- [464] Lascar B., *Propriétés locales d'espaces de type Sobolev en dimension infinie*, Commun. Partial Diff. Equations **1** (1976), 561–584.
- [465] Latala R., *A note on the Ehrhard inequality*, Studia Math. **118** (1996), no. 3, 169–174.
- [466] Ledoux M., *A note on large deviations for Wiener chaos*, Lecture Notes in Math. **1426** (1990), 1–14.
- [467] Ledoux M., *On an integral criterion for hypercontractivity of diffusion semigroups and extremal functions*, J. Funct. Anal. **105** (1992), 444–465.
- [468] Ledoux M., *L'algèbre de Lie des gradients itérés d'un générateur Markovien*, C. R. Acad. Sci. Paris **317** (1993), no. 2, 1049–1052.
- [469] Ledoux M., *Semigroup proofs of the isoperimetric inequality in Euclidean and Gauss space*, Bull. Sci. Math. **118** (1994), 485–510.
- [470] Ledoux M., *L'algèbre de Lie des gradients itérés d'un générateur Markovien — développements de moyennes et entropies*, Ann. Sci. Ecole Norm. Sup. **28** (1995), 435–460.
- [471] Ledoux M., *Isoperimetry and Gaussian analysis*, Lecture Notes in Math. **1648** (1996), 165–294.
- [472] Ledoux M., Talagrand M., *Probability in Banach Spaces. Isoperimetry and Processes*, Springer-Verlag, Berlin — New York, 1991.
- [473] Lee D., Wasilkowski G.W., *Approximation of linear functionals on a Banach space with a Gaussian measure*, J. Complexity **2** (1986), 12–43.
- [474] Lee Y.J., *Sharp inequalities and regularity of heat semigroup on infinite-dimensional spaces*, J. Funct. Anal. **71** (1987), no. 1, 69–87.

- [475] Leindler L., *On a certain converse of Hölder's inequality II*, Acta Sci. Math. (Szeged) **33** (1972), 217–223.
- [476] LePage R.D., *Log Log law for Gaussian processes*, Z. Wahrscheinlichkeitstheorie verw. Geb. **25** (1973), no. 2, 103–108.
- [477] LePage R.D., *Subgroups of paths and reproducing kernels*, Ann. Probab. **1** (1973), no. 2, 345–347.
- [478] LePage R.D., Mandrekar V., *Equivalence-singularity dichotomies from zero-one laws*, Proc. Amer. Math. Soc. **31** (1972), 251–254.
- [479] Lescot P., *Sard's theorem for hyper-Gevrey functionals on the Wiener space*, J. Funct. Anal. **129** (1995), no. 1, 191–220.
- [480] Lévy P., *A special problem of Brownian motion, and a general theory of Gaussian random functions*, In: Proc. Third Berkeley Symp. Math. Statist. and Probability, Vol. 2, pp. 133–175. University of California Press, Berkeley — Los Angeles, 1956.
- [481] Lévy P., *Processus Stochastiques et Mouvement Brownien*, 2nd edn., Paris, 1965.
- [482] Lewandowski M., *A note on functions which separate Gaussian measures*, Math. Z. **201** (1989), no. 1, 145–150.
- [483] Lewandowski M., Ryznar M., Zak T., *Anderson inequality is strict for Gaussian and stable measures*, Proc. Amer. Math. Soc. **123** (1995), no. 12, 3875–3880.
- [484] Li W.V., *Comparison results for the lower tail of Gaussian seminorm*, J. Theoret. Probab. **5** (1992), 1–32.
- [485] Lieb E.H., *Gaussian kernels have only Gaussian maximizers*, Invent. Math. **102** (1990), 179–208.
- [486] Lifshits M.A., *The absolute continuity of the supremum-type functionals of Gaussian processes*, Zap. Nauch. Sem. Leningrad. Otdel. Mat. Inst. Steklov (LOMI) **119** (1982), 154–166 (in Russian); English transl.: J. Soviet Math.
- [487] Lifshits M.A., *Distribution of the maximum of a Gaussian process*, Teor. Veroyatn. i Primenen. **31** (1986), no. 1, 134–142; English transl.: Theory Probab. Appl. **31** (1986), no. 1, 125–132.
- [488] Lifshits M.A., *The oscillation and the lower distribution boundary of the maximum of a Gaussian process*, Zap. Nauch. Sem. Leningrad. Otdel. Mat. Inst. Steklov (LOMI) **177** (1989), 78–82 (in Russian); English transl.: J. Soviet Math.
- [489] Lifshits M.A., *Gaussian large deviations of a smooth seminorm*, Zap. Nauch. Sem. Leningrad. Otdel. Mat. Inst. Steklov (LOMI) **194** (1992), 106–113 (in Russian); English transl.: J. Math. Sci. **75** (1995), no. 5, 1940–1943.
- [490] Lifshits M., *Tail probabilities of Gaussian suprema and Laplace transform*, Ann. Inst. H. Poincaré, Probab. et Stat., **30** (1994), no. 2, 163–180.
- [491] Lifshits M.A., *Gaussian Random Functions*, TViMS, Kiev, 1995 (in Russian); English transl.: Kluwer Academic Publ., Dordrecht, 1995.
- [492] Lifshits M.A., *On the lower tail probabilities of some random series*, Ann. Probab. **25** (1997), no. 1, 424–442.
- [493] Lifshits M.A., Tsirel'son B.S., *Small deviations of Gaussian fields*, Teor. Veroyatn. i Primenen. **31** (1987), no. 3, 632–633 (in Russian); English transl.: Theory Probab. Appl. **31** (1987), 557–558.
- [494] Linde W., *Probability in Banach Spaces — Stable and Infinitely Divisible Distributions*, Wiley, New York, 1986.
- [495] Linde W., *Uniqueness theorems for Gaussian measures in l_q , $1 \leq q < \infty$* , Math. Z. **197** (1988), no. 3, 319–341.
- [496] Linde W., *Gaussian measure of translated balls in a Banach space*, Teor. Veroyatn. i Primenen. **34** (1989), no. 2, 349–359 (in Russian); English transl.: Theory Probab. and Appl. **34** (1989), no. 2, 307–317.
- [497] Linde W., *Gaussian measures of large balls in l^p* , Ann. Probab. **19** (1991), 1264–1279.

- [498] Linde W., *Comparison results for the small ball behavior of Gaussian random variables*, In: Probability in Banach Spaces, Vol. 9 (J. Hoffmann-Jorgensen, J. Kuelbs, and M.B. Marcus eds.), pp. 273–297. Birkhäuser, Boston — Basel — Berlin, 1994.
- [499] Linde W., Pietsch A., *Mappings of Gaussian cylindrical measures in Banach spaces*, Teor. Veroyatn. i Primenen. **19** (1974), 472–487 (in Russian); English transl.: Theory Probab. Appl. **19** (1974), 445–460.
- [500] Linde W., Rosinski J., *Exact behavior of Gaussian measures of translated balls in Hilbert spaces*, J. Multivar. Anal. **50** (1994), 1–16.
- [501] Linde W., Tarieladze V.I., Chobanyan S.A., *Characterization of certain classes of Banach spaces by properties of Gaussian measures*, Teor. Veroyatn. i Primenen. **25** (1980), no. 1, 162–167 (in Russian); English transl.: Theory Probab. Appl. **25** (1980), no. 1.
- [502] Linnik Yu.V., *Decompositions of Random Variables and Vectors*, Nauka, Moscow, 1972 (in Russian); English transl.: Amer. Math. Soc., Providence, Rhode Island, 1977.
- [503] Linnik Yu.V., Eidlin V.L., *Remark on analytic transformations of normal vectors*, Teor. Veroyatn. i Primenen. **13** (1968), no. 4, 752–754; English transl.: Theory Probab. Appl. **13** (1968), no. 4, 707–710.
- [504] Liptser R.S., Shirayev A.N., *Statistics of Random Processes*, Vol. 1, Springer-Verlag, Berlin, 1977.
- [505] Lobuzov A.A., *The first boundary value problem for a parabolic equation in an abstract Wiener space*, Matem. Zamet. **30** (1981), no. 2, 221–233 (in Russian); English transl.: Math. Notes. **30** (1981), 592–599.
- [506] Loève M., *Quelques propriétés des fonctions aléatoires de second ordre*, C. R. Acad. Sci. Paris **222** (1946), 469–470.
- [507] Lusin N., *Leçons sur les Ensembles Analytiques et leurs Applications*, Gauthiers-Villars, Paris, 1930; 2nd edn.: Chelsea, New York, 1972.
- [508] Lions P.L., Toscani G., *A strengthened central limit theorem for smooth densities*, J. Funct. Anal. **129** (1995), 148–167.
- [509] Lyons T., Zeitouni O., *Conditional exponential moments for iterated Wiener integrals, with application to Onsager-Machlup functionals*, Preprint (1997).
- [510] Maiorov V.E., *About widths of Wiener space in the L_q -norm*, J. Complexity **12** (1996), 47–57.
- [511] Major P., *Multiple Ito Integral*, Lecture Notes in Math. **849**, Springer, Berlin, 1980.
- [512] Malliavin P., *Stochastic calculus of variation and hypoelliptic operators*, In: Proc. Intern. Symp. SDE Kyoto (1976), pp. 195–263. Wiley, Tokyo, 1978.
- [513] Malliavin P., *Implicit functions in finite corank on the Wiener space*, In: Proc. Taniguchi Intern. Symp. on Stochast. Anal., pp. 369–386. Kinokuniya, Katata and Kyoto, 1982.
- [514] Malliavin P., *Analyticité réelle des lois conditionnelles de fonctionnelles additives*, C. R. Acad. Sci. Paris **302** (1986), no. 2, 73–78.
- [515] Malliavin P., *Infinite dimensional analysis*, Bull. Sci. Math. **117** (1993), 63–90.
- [516] Malliavin P., *Integration and Probability*, Springer-Verlag, Berlin — New York, 1995.
- [517] Malliavin P., *Stochastic Analysis*, Springer, Berlin — New York, 1997.
- [518] Malliavin P., Nualart D., *Quasi sure analysis of stochastic flows and Banach space valued smooth functionals on the Wiener space*, J. Funct. Anal. **112** (1993), no. 2, 287–317.
- [519] Malliavin P., Taniguchi S., *Analytic functions, Cauchy formula, and stationary phase on a real abstract Wiener space*, J. Funct. Anal. **143** (1997), 470–528.
- [520] Malyshev V.A., Minlos R.A., *Gibbs Random Fields. Cluster Expansions*, Nauka, Moscow, 1985 (in Russian); English transl.: Kluwer Academic Publ., 1991.
- [521] Mandelbaum A., *Linear estimators and measurable linear transformations on a Hilbert space*, Z. Wahrscheinlichkeitstheorie verw. Geb. **65** (1984), 385–397.

- [522] Mandelbrot B.B., Van Ness J., *Fractional Brownian motions, fractional noises and applications*, SIAM Review **10** (1968), 422–437.
- [523] Marcus M.B., *Continuity of Gaussian processes and random Fourier series*, Ann. Probab. **1** (1973), 968–981.
- [524] Marcus M.B., *A comparison of continuity conditions for Gaussian processes*, Ann. Probab. **1** (1973), 123–130.
- [525] Marcus M.B., Shepp L., *Continuity of Gaussian processes*, Trans. Amer. Math. Soc. **151** (1970), 377–392.
- [526] Marcus M.B., Shepp L., *Sample behavior of Gaussian processes*, In: Proc. 6th Berkeley Symp. Math. Statist. Probab. Vol. 2, pp. 423–442. University of California Press, Berkeley, 1971.
- [527] Martynov G.V., *Omega-square Tests*, Nauka, Moscow, 1978 (in Russian).
- [528] Martynov G.V., *Calculation of the function of normal distribution*, Teor. Veroyatn., Matem. Statist. i Teor. Kibern., T. 19, pp. 57–84, Itogi Nauki i Tehn. VINITI, Moscow, 1982 (in Russian); English transl.: J. Soviet Math.
- [529] Maruyama G., *Notes on Wiener integrals*, Kodai Math. Semin. Rep. (1950), no. 2, 41–44.
- [530] Maruyama G., *On the transition probability functionals of the Markov process*, Natural Sci. Rep. Ochanomizu Univ. **5** (1954), no. 1, 10–20.
- [531] Masani P., *The homogeneous chaos from the standpoint of vector measures*, Philos. Trans. Roy. Soc. London Ser. A **355** (1997), no. 1727, 1099–1258.
- [532] Mathai A.M., Pederzoli G., *Characterizations of the Normal Probability Law*, Wiley, New York, 1977.
- [533] *Mathematics of the 19th century*, A.N. Kolmogorov and A.L. Yushkevich eds., Nauka, Moscow, 1978 (in Russian); English transl.: Birkhäuser Verlag, Basel -- Boston -- Berlin, 1992.
- [534] Mayer-Wolf E., Nualart D., Pérez-Abreu V., *Large deviations for multiple Wiener–Ito integral processes*, Lecture Notes in Math. **1526** (1992), 11–31.
- [535] Mayer-Wolf E., Zeitouni O., *The probability of small Gaussian ellipsoids and associated conditional moments*, Ann. Probab. **21** (1993), no. 1, 14–24.
- [536] Maz'ja V., *Sobolev Spaces*, Springer, Berlin, 1985.
- [537] Mazziotto G., Millet A., *Absolute continuity of the law of an infinite-dimensional Wiener functional with respect to the Wiener probability*, Probab. Theory Relat. Fields **85** (1990), no. 3, 403–411.
- [538] McKean H.P., *Geometry of differential space*, Ann. Probab. **1** (1973), 197–206.
- [539] McShane E.J., *Extension of range of functions*, Bull. Amer. Math. Soc. **40** (1934), 837–842.
- [540] Mehler F.G., *Ueber die Entwicklung einer Funktion von beliebig vielen Variablen nach Laplaceschen Funktionen höherer Ordnung*, J. Reine Angewandte Math. **66** (1866). 161–176.
- [541] Meyer P.-A., *Probability and Potentials*, Blaisdell Publ. Co., 1965.
- [542] Meyer P.-A., *Note sur les processus d'Ornstein–Uhlenbeck*, Lecture Notes in Math. **920** (1982), 95–133.
- [543] Meyer P.-A., *Quelques resultats analytiques sur semigroupe d'Ornstein–Uhlenbeck en dimension infinie*, Theory and Appl. of Random Fields, Lecture Notes in Control and Information Sci. **49** (1983), 201–214.
- [544] Meyer P.-A., *Transformation de Riesz pour les lois gaussiennes*, Lecture Notes in Math. **1059** (1984), 179–193.
- [545] Miller K.S., *Multidimensional Gaussian Distributions*, John Wiley and Sons, New York, 1964.
- [546] Millet A., Smolenski W., *On the continuity of Ornstein–Uhlenbeck processes in infinite dimensions*, Probab. Theory Relat. Fields **92** (1992), 529–547.

- [547] Milman V., Pisier G., *Gaussian processes and mixed volumes*, Ann. Probab. **15** (1987), 292–304.
- [548] Minkova L.D., Hadzhiev D.I., *Representation of Gaussian processes equivalent to a Gaussian martingale*, Stochastics **3** (1980), 251–266.
- [549] Minlos R.A., *Generalized random processes and their extension to a measure*, Trudy Moskovsk. Matem. Obsc. **8** (1959), 497–518 (in Russian); English transl.: Selecta Transl. Math. Statist. and Probab., Vol. 3, pp. 291–314. Inst. Math. Statist. and Amer. Math. Soc., Providence, Rhode Island, 1961.
- [550] Mogul'skii A.A., *Fourier method for determining the asymptotic behavior of small deviations of a Wiener process*, Sibirsk. Matem. Zhurn. **23** (1982), no. 3, 161–174 (in Russian); English transl.: Siberian Math. J. **23** (1982), no. 3, 420–431.
- [551] de Moivre A., *The Doctrine of Chances*, 2d edn., 1738.
- [552] Molčan G.M., *Characterization of Gaussian fields with Markovian property*, Dokl. Akad. Nauk. SSSR **197** (1971), no. 4, 784–787 (in Russian); English transl.: Soviet Math. Dokl. **12** (1971), no. 2, 563–567.
- [553] Monrad D., Rootzeń H., *Small values of Gaussian processes and functional laws of the iterated logarithm*, Probab. Theory Relat. Fields **101** (1995), 173–192.
- [554] Moulinier J.-M., *Absolute continuité de probabilités de transition par rapport à une mesure gaussienne dans une espace de Hilbert*, J. Funct. Anal. **64** (1985), no. 2, 257–295.
- [555] Moulinier J.-M., *Fonctionnelles oscillantes stochastiques et hypoellipticité*, Bull. Sci. Math. **109** (1985), 37–60.
- [556] Mourier E., *Éléments aléatoires dans un espace de Banach*, Ann. Inst. H. Poincaré **19** (1953), 161–244.
- [557] Mushtari D.Kh., *Probabilities and Topologies in Banach Spaces*, Kazanskii Univ., Kazan, 1989 (in Russian).
- [558] Nagaev S.V., *On probabilities of large deviations of a Gaussian distribution in a Banach space*, Izv. Akad. Nauk UzSSR Ser. Fiz.-Mat. Nauk (1981), no. 5, 18–21 (in Russian).
- [559] Nagaev S.V., *On the asymptotics of a Wiener measure for a narrow band*, Teor. Veroyatn. i Primenen. **26** (1981), 630 (in Russian); English transl.: Theory Probab. Appl. **26** (1981), 625–626.
- [560] Nelson E., *Dynamical Theories of Brownian Motion*, Princeton University Press, 1967.
- [561] Nelson E., *The free Markoff field*, J. Funct. Anal. **12** (1973), no. 2, 211–227.
- [562] Neretin Ju.A., *Some remarks on quasi-invariant actions of loop groups and the group of diffeomorphisms of the circle*, Commun. Math. Phys. **164** (1994), 599–626.
- [563] Neveu J., *Processus Aléatoires Gaussiens*, Les presses de l'université de Montréal, Montréal, 1968.
- [564] Neveu J., *Sur l'espérance conditionnelles par rapport à un mouvement brownien*, Ann. Inst. H. Poincaré **B12** (1976), no. 2, 105–112.
- [565] Nguyen T.T., Rempala G., Wesolowski J., *Non-Gaussian measures with Gaussian structure*, Probab. Math. Stat. **16** (1996), no. 2, 287–298.
- [566] Nomoto H., *On a class of metrical automorphisms on Gaussian measure space*, Nagoya Math. J. **38** (1970), 21–25.
- [567] Norin N.V., *Stochastic integrals and differentiable measures*, Teor. Veroyatn. i Primenen. **32** (1987), no. 1, 114–124 (in Russian); English transl.: Theory Probab. Appl. **32** (1987), no. 1, 107–116.
- [568] Norin N.V., Smolyanov O.G., *Some results on logarithmic derivatives of measures on locally convex spaces*, Matem. Zamet. **54** (1993), no. 6, 135–138 (in Russian); English transl.: Math. Notes **54** (1993), no. 6, 1277–1279.
- [569] Novikov A.A., *Small deviations of Gaussian processes*, Matem. Zamet. **29** (1981), no. 2, 291–301 (in Russian); English transl.: Math. Notes **29** (1981), no. 2, 150–155.

- [570] Nualart D., *The Malliavin Calculus and Related Topics*, Springer-Verlag, Berlin — New York, 1995.
- [571] Nualart D., Üstünel A.S., *Geometric analysis of conditional independence on Wiener space*, Probab. Theory Relat. Fields **89** (1991), no. 4, 407–422.
- [572] Nualart D., Üstünel A.S., Zakai M., *On the moments of a multiple Wiener–Ito integral and the space induced by the polynomials of the integral*, Stochastics **25** (1988), 233–240.
- [573] Nualart D., Üstünel A.S., Zakai M., *Some relations among classes of σ -fields on the Wiener spaces*, Probab. Theory Relat. Fields **85** (1990), 119–129.
- [574] Nualart D., Zakai M., *Generalized stochastic integrals and the Malliavin calculus*, Probab. Theory Relat. Fields **73** (1986), no. 2, 255–280.
- [575] Nualart D., Zakai M., *Multiple Wiener–Ito integrals possessing a continuous extension*, Probab. Theory Relat. Fields **85** (1990), no. 1, 131–145.
- [576] Obata N., *White noise calculus and Fock Spaces*, Lecture Notes in Math. **1577**, Springer, Berlin — New York, 1994.
- [577] Okazaki Y., *Gaussian measure on topological vector space*, Mem. Fac. Sci. Kyushu Univ. Ser. A **34** (1980), no. 1, 1–21.
- [578] Okazaki Y., *Bochner’s theorem on measurable linear functionals of a Gaussian measure*, Ann. Probab. **9** (1981), no. 4, 663–664.
- [579] Onsager L., Machlup S., *Fluctuations and irreversible processes*, Phys. Rev. **91** (1953), 1505–1515.
- [580] Oleszkiewicz K., *On certain characterization of normal distribution*, Statistics and Probab. Letters **33** (1997), 237–240.
- [581] Ostrovskii E.I., *On the local structure of normal fields*, Dokl. Akad. Nauk SSSR **195** (1970), no. 1, 40–42 (in Russian); English transl.: Soviet Math. (1970).
- [582] Ostrovskii E.I., *Convergence of the canonical expansion for normal fields*, Matem. Zamet. **28** (1973), no. 3, 565–572 (in Russian); English transl.: Math. Notes **28** (1973).
- [583] Ostrovskii E.I., *Covariance operators and some estimates of Gaussian random vectors*, Dokl. Akad. Nauk SSSR **236** (1977), no. 3, 541–543 (in Russian); English transl.: Soviet Math. Dokl. **18** (1977), no. 5, 1234–1236.
- [584] Ostrovskii E.I., *Exact asymptotics of the density of the distribution of multiple stochastic integrals*, Problemi Peredachi Inform. **28** (1992), 60–67 (in Russian); English transl.: Problems Inform. Transmission **28** (1992), no. 3, 250–257.
- [585] Paley R.E.A.C., Wiener N., *Fourier Transforms in the Complex Domain*, Amer. Math. Soc., Providence, Rhode Island, 1934.
- [586] Paley R.E.A.C., Wiener N., Zygmund A., *Notes on random functions*, Math. Z. **37** (1933), 647–668.
- [587] Pap G., *Dependence of Gaussian measure on covariance in Hilbert space*, Lecture Notes in Math. **1080** (1984), 188–194.
- [588] Pap G., *Analog of heat equation for Gaussian measure of a ball in Hilbert space*, J. Theor. Probab. **3** (1990), no. 4, 563–577.
- [589] Park Ch., Skoug D., *Linear transformations of Wiener integrals*, Proc. Amer. Math. Soc. **116** (1992), no. 2, 445–456.
- [590] Parzen E., *Probability density functionals and reproducing kernel Hilbert spaces*, In: Proc. Symp. Time Series Analysis, pp. 155–169. Wiley, New York, 1963.
- [591] Patel J.K., Read C.B., *Handbook of the Normal Distribution*, 2d edn, Marcel Dekker, New York, 1996.
- [592] Paulauskas V.I., Rachkauskas A.Yu., *The Accuracy of Approximation in the Central Limit Theorem in Banach Spaces*, Mokslas, Vilnius, 1987 (in Russian); English transl.: Kluwer Academic Publ., 1989.
- [593] Pearson K., *Notes on the history of correlation*, Biometrika **13** (1920), 25–45; reprinted in [730], pp. 185–205.

- [594] Peirce C.S., *On the theory of errors of observations*, Appendix no. 21 of Reports of the Superintendent of the U.S. Coast Survey for the year ending June 1870, pp. 200–224. G.P.O. Washington, 1873; reprinted in *Writings of Charles S. Peirce*, Vol. 3, pp. 114–160. Indiana University Press, Bloomington, 1986.
- [595] Peters G., *Flows on the Wiener space generated by vector fields with low regularity*, C. R. Acad. Sci. Paris **320** (1995), 1003–1008.
- [596] Peters G., *Anticipating flows on the Wiener space generated by vector fields of low regularity*, J. Funct. Anal. **142** (1996), no. 1, 129–192.
- [597] Petrovskii I.G., *Über das Irrfahrtproblem*, Math. Ann. **109** (1934), 425–444.
- [598] Pettis B.-J., *On the Radon–Nikodym theorem*, Lecture Notes in Math. **644** (1978), 340–355.
- [599] Phelps R.R., *Gaussian null sets and differentiability of Lipschitz map on Banach spaces*, Pacif. J. Math. **77** (1978), no. 2, 523–531.
- [600] Piech M.A., *The Ornstein–Uhlenbeck semigroup in an infinite dimensional L^2 setting*, J. Funct. Anal. **18** (1975), no. 3, 271–285.
- [601] Piech M.A., *Smooth functions on Banach spaces*, J. Math. Anal. Appl. **57** (1977), no. 1, 56–67.
- [602] Piech M.A., *Differentiability of measures associated with parabolic equation on infinite dimensional spaces*, Trans. Amer. Math. Soc. **253** (1979), 191–209.
- [603] Pietsch A., *Operator Ideals*, North-Holland, Amsterdam, 1980.
- [604] Pinsker M.S., *On asymptotic properties of distributions for quadratic functionals of Gaussian stochastic processes*, Teor. Veroyatn. i Primenen. **6** (1961), 365–366 (in Russian); English transl.: Theory Probab. Appl. **6** (1961), 334–335.
- [605] Pisier J., *Probability methods in the geometry of Banach spaces*, Lecture Notes in Math. **1206** (1985), 167–241.
- [606] Pisier G., *Riesz transforms: a simpler analytic proof of P.-A. Meyer’s inequality*, Lecture Notes in Math. **1321** (1988), 485–501.
- [607] Pitcher T.S., *Likelihood ratios of Gaussian processes*, Ark. Mat. **4** (1960), 35–44.
- [608] Pitcher T.S., *Likelihood ratios for diffusion processes with shifted mean value*, Trans. Amer. Math. Soc. **101** (1961), no. 1, 168–176.
- [609] Pitcher T.S., *On the sample functions of processes which can be added to a Gaussian process*, Ann. Math. Statist. **34** (1963), no. 1, 329–333.
- [610] Piterbarg V.I., *Gaussian random processes*, Itogi Nauki i Tehn. Teor. Veroyatn., Matem. Statist. i Teor. Kibern., Vol. 19, pp. 155–199. Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow, 1982 (in Russian); English transl.: J. Soviet Math. **23** (1983), 2599–2626.
- [611] Piterbarg V.I., *Asymptotic Methods in the Theory of Gaussian Processes and Fields*, Izdat. Moskovsk. Univ., Moscow, 1988 (in Russian); English transl.: Amer. Math. Soc., Providence, Rhode Island, 1996.
- [612] Piterbarg V.I., Fatalov V.R., *Laplace’s method for probability measures in Banach spaces*, Uspehi Matem. Nauk **50** (1995), no. 6, 57–150 (in Russian); English transl.: Russian Math. Surveys **50** (1995).
- [613] Pitt L., *A Markov property for Gaussian processes with a multidimensional parameter*, Arch. Ration. Mech. Anal. **43** (1971), 367–391.
- [614] Pitt L., *A Gaussian correlation inequality for symmetric convex sets*, Ann. Probab. **5** (1977), 470–474.
- [615] Pitt L.D., *Positively correlated normal variables are associated*, Ann. Probab. **10** (1982), 496–499.
- [616] Plackett R.L., *A reduction formula for normal multivariate integrals*, Biometrika **41** (1954), 351–360.
- [617] Plana G.A.A., *Mémoire sur divers problèmes de probabilité*, Mémoires Acad. Impériale de Turin pour les Années 1811–1812 **20** (1813), 355–498.

- [618] Polya G., *Herleitung des Gauss'schen Fehlergesetzes aus einer Funktionalgleichung*, Math. Z. **18** (1923), 96–108.
- [619] Ponomarenko L.S., *Inequalities for the distributions of normalized quadratic forms in random variables*, Teor. Veroyatn. i Primenen. **23** (1978), no. 3, 676–680 (in Russian); English transl.: Theory Probab. Appl. **23** (1978), no. 3, 652–656.
- [620] Ponomarenko L.S., *On estimation of the distributions of normalized quadratic forms in normally distributed random variables*, Teor. Veroyatn. i Primenen. **30** (1985), no. 3, 545–549 (in Russian); English transl.: Theory Probab. Appl. **30** (1985), no. 3, 580–584.
- [621] Prat J.-J., *Équation de Schrödinger: analyticité transverse de la densité de la loi d'une fonctionnelle additive*, Bull. Sci. Math. **115** (1991), 133–176.
- [622] Preiss D., *Gaussian measures and covering theorems*, Comment. Math. Univ. Carol. **20** (1979), no. 1, 95–99.
- [623] Preiss D., *Gaussian measures and the density theorems*, Comment. Math. Univ. Carol. **22** (1981), no. 1, 181–193.
- [624] Preiss D., *Differentiation of measures in infinitely dimensional spaces*, In: Proc. Conf. Topology and Measure III (Vitte/Hiddensee, Oct. 19–25, 1980), Part 2, pp. 201–207. Wissen. Beitrage Greifswald Univ., Greifswald, 1982.
- [625] Preiss D., *Differentiability of Lipschitz functions on Banach spaces*, J. Funct. Anal. **91** (1990), 312–345.
- [626] Preiss D., Tišer J., *Differentiation of Gaussian measures on Hilbert space*, Lecture Notes in Math. **945** (1981), 194–207.
- [627] Preiss D., Zajiček L., *Fréchet differentiation of convex functions in a Banach space with a separable dual*, Proc. Amer. Math. Soc. **91** (1984), 202–204.
- [628] Prékopa A., *Logarithmic concave measures with applications*, Acta Sci. Math. **32** (1971), 301–316.
- [629] Preston C., *Continuity properties of some Gaussian processes*, Ann. Math. Statist. **43** (1972), 285–292.
- [630] Prokhorov Yu.V., *Convergence of random processes and limit theorems in probability theory*, Teor. Veroyatn. i Primenen. **1** (1956), no. 2, 177–238 (in Russian); English transl.: Theory Probab. Appl. **1** (1956), 157–214.
- [631] Prohorov Yu.V., Fish M., *A characterization of normal distributions in Hilbert space*, Teor. Veroyatn. i Primenen. **2** (1957), 475–477 (in Russian); English transl.: Theory Probab. Appl. **2** (1957), 468–469.
- [632] Rademacher H., *Eineindeutige Abbildungen und Meßbarkeit*, Monatsh. für Mathematik und Physik **27** (1916), 183–290.
- [633] Rajput B.S., *On Gaussian measures in certain locally convex spaces*, J. Multivar. Anal. **2** (1972), no. 3, 282–306.
- [634] Rajput B.S., *Gaussian measures on L_p spaces, $1 \leq p < \infty$* , J. Multivar. Anal. **2** (1972), 382–403.
- [635] Rajput B.S., Cambanis S., *Gaussian processes and Gaussian measures*, Ann. Math. Statist. **43** (1972), 1944–1952.
- [636] Ramer R., *On nonlinear transformations of Gaussian measures*, J. Funct. Anal. **15** (1974), no. 3, 166–187.
- [637] Rao C.R., *Linear Statistical Inference and its Applications*, Wiley, New York, 1965.
- [638] Rao C.R., Varadarajan V.S., *Discrimination of Gaussian processes*, Sankhya A **25** (1963), 303–330.
- [639] Revuz D., Yor M., *Continuous Martingales and Brownian Motion*, Springer-Verlag, Berlin — New York, 1991.
- [640] Rhee Wan Soo, *On the distribution of the norm for a Gaussian measure*, Ann. Inst. H. Poincaré, Probab. et Statist. **20** (1984), no. 3, 277–286.
- [641] Rhee Wan Soo, Talagrand M., *Bad rates of convergence for the central limit theorem in Hilbert space*, Ann. Probab. **12** (1984), no. 3, 843–850.

- [642] Rhee Wan Soo, Talagrand M., *Uniform convexity and the distribution of the norm for a Gaussian measure*, Probab. Theory Relat. Fields. **71** (1986), no. 1, 59–68.
- [643] Richter W.-D., *Laplace–Gauss integrals, Gaussian measure asymptotic behaviour and probabilities of moderate deviations*, Z. Anal. Anwend. **4** (1985), no. 3, 257–267.
- [644] Rieffel M.A., *The Radon–Nikodym theorem for the Bochner integral*, Trans. Amer. Math. Soc. **131** (1968), no. 2, 466–487.
- [645] Rihaoui I., *Continuité et différentiabilité des translation dans un espace gaussien*, Math. Scand. **51** (1982), no. 1, 179–192.
- [646] Röckner M., *On the parabolic Martin boundary of the Ornstein–Uhlenbeck process on Wiener space*, Ann. Probab. **20** (1992), no. 2, 1063–1085.
- [647] Roelly S., Zessin H., *Une caractérisation des diffusions par le calcul des variations stochastiques*, C. R. Acad. Sci. Paris **313** (1991), 309–312.
- [648] Roelly S., Zessin H., *Une caractérisation des mesures de Gibbs sur $C(0, 1)^{\mathbb{Z}^d}$ par le calcul des variations stochastiques*, Ann. Inst. H. Poincaré **29** (1993), 327–338.
- [649] Rosenblatt M., *Independence and dependence*, In: Proc. Fourth Berkeley Symp. on Math. Statist. and Probability, pp. 411–443. University of California Press, Berkeley — Los Angeles, 1961.
- [650] Rosinski J., Samorodnitsky G., Taqqu M.S., *Zero-one laws for multilinear forms in Gaussian and other infinitely divisible random variables*, J. Multivar. Anal. **46** (1993), 61–82.
- [651] Rotar’ V.I., Shervashidze T.L., *Some estimates of the distributions of quadratic forms*, Teor. Veroyatn. i Primenen. **30** (1985), no. 3, 549–554 (in Russian); English transl.: Theory Probab. Appl. **30** (1985), no. 3, 585–590.
- [652] Royer G., *Comparaison des mesures de Cauchy en dimension infinie*, Z. Wahrscheinlichkeitstheorie verw. Geb. **64** (1983), no. 1, 7–14.
- [653] Rozanov Yu.A., *On the density of one Gaussian measure with respect to another*, Teor. Veroyatn. i Primenen. **7** (1962), 84–89 (in Russian); English transl.: Theory Probab. Appl. **7** (1962), 82–87.
- [654] Rozanov Yu.A., *On the problem of the equivalence of probability measures corresponding to stationary Gaussian processes*, Teor. Veroyatn. i Primenen. **8** (1963), no. 3, 241–250 (in Russian); English transl.: Theory Probab. Appl. **8** (1963), no. 3, 223–231.
- [655] Rozanov Yu.A., *On probability measures in function spaces corresponding to Gaussian random processes*, Teor. Veroyatn. i Primenen. **9** (1964), no. 3, 448–465 (in Russian); English transl.: Theory Probab. Appl. **9** (1964), no. 3, 404–420.
- [656] Rozanov Yu.A., *On the densities of Gaussian distributions and Wiener–Hopf integral equations*, Teor. Veroyatn. i Primenen. **11** (1966), no. 2, 170–179 (in Russian); English transl.: Theory Probab. Appl. **11** (1966), no. 2, 152–160.
- [657] Rozanov Yu.A., *On Gaussian fields with given conditional distributions*, Teor. Veroyatn. i Primenen. **12** (1967), 433–443 (in Russian); English transl.: Theory Probab. Appl. **12** (1967), 381–391.
- [658] Rozanov Yu.A., *Infinite-dimensional Gaussian distributions*, Trudy Matem. Steklov Inst. **108** (1968), 1–161 (in Russian); English transl.: Proc. Steklov Inst. Math. **108**, American Math. Soc., Providence, Rhode Island, 1971.
- [659] Rozanov Yu.A., *Markov Random Fields*, Nauka, Moscow, 1981 (in Russian); English transl.: Springer–Verlag, New York — Berlin, 1982.
- [660] Šatašvili A.D., *On a certain class of absolutely continuous non-linear transformations of Gaussian measures*, Trudy Vychisl. Centra Akad. Nauk Gruzin. SSR **5** (1965), 69–105 (in Russian).
- [661] Sato H., *On the equivalence of Gaussian measures*, J. Math. Soc. Japan **19** (1967), 159–172.
- [662] Sato H., *Gaussian measure on a Banach space and abstract Wiener measure*, Nagoya Math. J. **36** (1969), 65–81.

- [663] Sato H., *Souslin support and Fourier expansion of a Gaussian Radon measure*, Lecture Notes in Math. **860** (1981), 299–313.
- [664] Sato H., *Characteristic functional of a probability measure absolutely continuous with respect to a Gaussian Radon measure*, J. Funct. Anal. **61** (1985), no. 2, 222–245.
- [665] Sato H., *Gaussian measurable dual and Bochner's theorem*, Ann. Probab. **9** (1981), no. 4, 656–662.
- [666] Sato H., *Gaussian measures on locally convex spaces and related topics*, Soochow J. Math. **18** (1992), no. 4, 461–496.
- [667] Sato H., Okazaki Y., *Separabilities of a Gaussian Radon measure*, Ann. Inst. H. Poincaré **B 11** (1975), no. 3, 287–298.
- [668] Sazonov V.V., *A remark on characteristic functionals*, Teor. Veroyatn. i Primenen. **3** (1958), no. 2, 201–205 (in Russian); English transl.: Theory Probab. Appl. **3** (1958), no. 2, 188–192.
- [669] Sazonov V.V., *Normal Approximations. Some Recent Advances*, Lecture Notes in Math. **879**, Springer, Berlin — New York, 1981.
- [670] Schaefer H.H., *Topological Vector Spaces*, Springer-Verlag, Berlin — New York, 1971.
- [671] Schechtman G., Schlumprecht Th., Zinn J., *On the Gaussian measure of the intersection of symmetric, convex sets*, Preprint (1995).
- [672] Schilder M., *Some asymptotic formulas for Wiener integrals*, Trans. Amer. Math. Soc. **125** (1966), 63–85.
- [673] Schreiber M., *Fermeture en probabilité des chaos de Wiener*, C. R. Acad. Sci. Paris **265** (1967), 859–862.
- [674] Schwartz L., *Radon Measures on Arbitrary Topological Spaces and Cylindrical Measures*, Oxford University Press, London, 1973.
- [675] Scott A., *A note on conservative confidence regions for the mean of a multivariate normal distribution*, Ann. Math. Statist. **38** (1967), 278–280.
- [676] Segal I., *Tensor algebras over Hilbert spaces. I*, Trans. Amer. Math. Soc. **81** (1956), no. 2, 106–134.
- [677] Segal I., *Distributions in Hilbert space and canonical system of operators*, Trans. Amer. Math. Soc. **88** (1958), no. 1, 12–41.
- [678] Segal I., *Mathematical Problems of Relativistic Physics*, American Math. Soc., Providence, Rhode Island, 1963.
- [679] Seidman T.I., *Linear transformation of a functional integral, I*, Commun. Pure Appl. Math. **12** (1959), 611–621.
- [680] Seidman T.I., *Linear transformation of a functional integral, II*, Commun. Pure Appl. Math. **17** (1964), 493–508.
- [681] Sevast'yanov B.A., *A class of limit distributions for quadratic forms from normal stochastic quantities*, Teor. Veroyatn. i Primenen. **6** (1961), no. 3, 386–372 (in Russian); English transl.: Theory Probab. Appl. **6** (1961), no. 3, 337–340.
- [682] Ševčík V.V., *On subspaces of a Banach space that coincide with the ranges of continuous linear operators*, Dokl. Akad. Nauk SSSR **263** (1982), no. 4, 817–819 (in Russian); English transl.: Soviet Math. Dokl. **25** (1982), no. 2, 454–456.
- [683] Shale D., *Linear symmetries of the free boson fields*, Trans. Amer. Math. Soc. **103** (1962), no. 1, 149–167.
- [684] Shao Q.-M., Wang D., *Small ball probabilities of Gaussian fields*, Probab. Theory Relat. Fields **102** (1995), 511–517.
- [685] Shavgulidze E.T., *A measure that is quasi-invariant with respect to the action of a group of diffeomorphisms of a finite-dimensional manifold*, Dokl. Akad. Nauk SSSR **303** (1988), no. 4, 811–814 (in Russian); English transl.: Soviet Math. Dokl. **38** (1989), 622–625.
- [686] Shepp L.A., *Distinguishing a sequence of random variables from a translate of itself*, Ann. Math. Statist. **36** (1965), 1107–1112.

- [687] Shepp L.A., *Radon–Nikodym derivatives of Gaussian measures*, Ann. Math. Statist. **37** (1966), no. 2, 321–354.
- [688] Shepp L.A., *Gaussian measures in function space*, Pacif. J. Math. **17** (1966), no. 1, 167–173.
- [689] Shepp L.A., Zeitouni O., *A note on conditional exponential moments and the Onsager–Machlup functional*, Ann. Probab. **20** (1992), no. 1, 652–654.
- [690] Shevlyakov A.Yu., *Distributions of square-integrable functionals of Gaussian measures*, Theory of random processes, no. 13, pp. 104–110. Naukova Dumka, Kiev, 1985 (in Russian).
- [691] Shi Z., *Small ball probabilities for a Wiener process under weighted sup-norms, with an application to the supremum of Bessel local times*, J. Theor. Probab. **9** (1996), no. 4, 915–929.
- [692] Shigekawa I., *Absolute continuity of probability laws of Wiener functionals*, Proc. Jap. Acad. Ser. A **54** (1978), no. 8, 230–233.
- [693] Shigekawa I., *Derivatives of Wiener functionals and absolute continuity of induced measures*, J. Math. Kyoto Univ. **20** (1980), no. 2, 263–289.
- [694] Shigekawa I., *Existence of invariant measures of diffusions on an abstract Wiener space*, Osaka J. Math. **24** (1987), no. 1, 37–59.
- [695] Shigekawa I., *Sobolev spaces over the Wiener space based on an Ornstein–Uhlenbeck operator*, J. Math. Kyoto Univ. **32** (1992), no. 4, 731–748.
- [696] Shilov G.E., Fan Dyk Tin', *Integral, Measure and Derivative on Linear Spaces*, Nauka, Moscow, 1967 (in Russian).
- [697] Shiryaev A.N., *Probability Theory*, Springer, Berlin, 1984.
- [698] Shiryaev A.N., Kabanov Yu.M., Kramkov D.O., Melnikov A.V., *Towards the theory of pricing of options of both European and American types*, Teor. Veroyatn. i Primenen. **39** (1994), no. 1, 23–129 (in Russian); English transl.: Theory Probab. Appl. **39** (1994), no. 1, 14–102.
- [699] Šidák Z., *Rectangular confidence regions for the means of multivariate normal distributions*, J. Amer. Statist. Assoc. **62** (1967), 626–633.
- [700] Šidák Z., *On multivariate normal probabilities of rectangles: their dependence on correlations*, Ann. Math. Statist. **39** (1968), no. 5, 1425–1434.
- [701] Šidák Z., *A note on C.G. Khatri's and A. Scott's papers on multidimensional normal distributions*, Ann. Inst. Statist. Math. **27** (1975), 181–184.
- [702] Siebert E., *Operator-decomposability of Gaussian measures on separable Banach spaces*, J. Theor. Probab. **5** (1992), no. 2, 333–347.
- [703] Simon B., *The $P(\varphi)_2$ Euclidean (Quantum) Field Theory*, Princeton University Press, 1974.
- [704] Simon B., *Functional Integration and Quantum Physics*, Academic Press, New York, 1979.
- [705] Skitovich V.P., *Linear forms of independent random variables and the normal distribution*, Izv. Akad. Nauk SSSR **18** (1954), 185–200 (in Russian).
- [706] Skorohod A.V., *On the differentiability of measures corresponding to Markov processes*, Teor. Veroyatn. i Primenen. **5** (1960), no. 1, 45–53 (in Russian); English transl.: Theory Probab. and Appl. **5** (1960), 40–49.
- [707] Skorohod A.V., *Studies in the Theory of Random Processes*, Kiev, 1961 (in Russian); English transl.: Addison–Wesley, 1965.
- [708] Skorohod A.V., *Nonlinear transformations of stochastic measures in functional spaces*, Dokl. Akad. Nauk SSSR **168** (1966), 1269–1271 (in Russian); English transl.: Soviet Math. **7** (1966), 838–840.
- [709] Skorohod A.V., *A remark on Gaussian measures in a Banach space*, Teor. Veroyatn. i Primenen. **15** (1970), no. 3, 519–520 (in Russian); English transl.: Theory Probab. Appl. **15** (1970), no. 3, 508–509.

- [710] Skorohod A.V., *Integration in Hilbert Space*, Springer-Verlag, Berlin — New York, 1974.
- [711] Skorohod A.V., *On a generalization of a stochastic integral*, Teor. Veroyatn. i Primenen. **20** (1975), no. 2, 223–237 (in Russian); English transl.: Theory Probab. Appl. **20** (1975), 219–233.
- [712] Skorohod A.V., Šatašvili A.D., *On the absolute continuity of a Gaussian measure under a nonlinear transformation*, Teor. Veroyatn. i Matem. Statist. **15** (1976), 139–151 (in Russian); English transl.: Theory Probab. Math. Statist. **15** (1978), 144–155.
- [713] Slepian D., *The one-sided barrier problem for Gaussian noise*, Bell. Syst. Tech. J. **41** (1962), no. 2, 463–501.
- [714] Slutsky E., *Alcune proposizioni sulla teoria delle funzioni aleatorie*, Giorn. Ist. Italiano degli Attuari **8** (1937), 193–199.
- [715] Smolyanov O.G., *Measurable polylinear and power functionals in certain linear spaces with a measure*, Dokl. Akad. Nauk SSSR **170** (1966), no. 3, 526–529 (in Russian); English transl.: Soviet Math. **7** (1966), no. 5, 1242–1246.
- [716] Smolyanov O.G., Uglanov A.V., *Every Hilbert subspace of a Wiener space has measure zero*, Matem. Zamet. **14** (1973), no. 3, 369–374 (in Russian); English transl.: Math. Notes **14** (1973), 772–774.
- [717] Smolyanov O.G., Shavgulidze E.T., *Continual Integrals*, Izdat. Moskovsk. Univ., Moscow, 1989 (in Russian).
- [718] Sodnomov B.S., *On the arithmetic sums of sets*, Dokl. Akad. Nauk SSSR **80** (1951), no. 2, 173–175 (in Russian).
- [719] Sokolova S.D., *Equivalence of Gaussian measures corresponding to solutions of stochastic differential equations*, Teor. Veroyatn. i Primenen. **28** (1983), no. 2, 429–433 (in Russian); English transl.: Theory Probab. Appl. **28** (1983), no. 2, 451–454.
- [720] Sonis M.G., *Generalized large numbers laws for Gaussian measures*, Vestnik Mosk. Univ. (1967), no. 4, 31–37 (in Russian).
- [721] Stengle G., *A divergence theorem for Gaussian stochastic process expectations*, J. Math. Anal. Appl. **21** (1968), no. 3, 537–546.
- [722] Stigler S.M., *Mathematical statistics in the early States*, Ann. Statist. **2** (1978), no. 2, 239–265.
- [723] Stigler S.M., *The History of Statistics: the Measurement of Uncertainty before 1900*, Belknap Press, Cambridge, Mass., 1995.
- [724] Stolz W., *Une méthode élémentaire pour l'évaluation de petites boules browniennes*, C. R. Acad. Sci. Paris **316** (1993), 1217–1220.
- [725] Stolz W., *Some small ball probabilities for Gaussian processes under nonuniform norms*, J. Theor. Probab. **9** (1996), no. 3, 613–630.
- [726] Strassen V., *An invariance principle for the law of the iterated logarithm*, Z. Wahrscheinlichkeitstheorie verw. Geb. **3** (1964), no. 3, 211–226.
- [727] Stroock D., *The Malliavin calculus, a functional analytic approach*, J. Funct. Anal. **44** (1981), no. 2, 212–257.
- [728] Stroock D., *Homogeneous chaos revisited*, Lect. Notes in Math. **1247** (1987), 1–7.
- [729] Stroock D., *Gaussian measures in traditional and not so traditional settings*, Bull. Amer. Math. Soc. **33** (1996), no. 3, 135–155.
- [730] *Studies in the history of statistics and probability*, Kendall M., Plackett R.L., eds. C. Griffin and Co., London, 1977.
- [731] Sudakov V.N., *Gaussian measures, Cauchy measures and ε -entropy*, Dokl. Akad. Nauk SSSR **185** (1969), no. 1, 51–53 (in Russian); English transl.: Soviet Math. **10** (1969), 310–313.
- [732] Sudakov V.N., *Gaussian random processes and measures of solid angles in Hilbert spaces*, Dokl. Akad. Nauk SSSR **197** (1971), no. 1, 43–45 (in Russian); English transl.: Soviet Math. Dokl. **12** (1971), 412–415.

- [733] Sudakov V.N., *Geometric problems of the theory of infinite-dimensional probability distributions*, Trudy Matem. Inst. Steklov **141** (1976), 1–190 (in Russian); English transl.: Proc. Steklov Inst. Math. (1979), no. 2, 1–178.
- [734] Sudakov V.N., *Conditional distributions of the maximum of a Gaussian random field*, Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov **184** (1990), 260–263 (in Russian); English transl.: J. Math. Sci. **68** (1994), no. 4, 585–587.
- [735] Sudakov V.N., *Remarks on modifications of random processes*, Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov **194** (1992), 150–169 (in Russian); English transl.: J. Math. Sci. **75** (1995), no. 5, 1969–1981.
- [736] Sudakov V.N., Tsirel'son B.S., *Extremal properties of half-spaces for spherically invariant measures*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov (LOMI) **41** (1974), 14–24 (in Russian); English transl.: J. Soviet Math. **9** (1978), 9–17.
- [737] Suetin P.K., *Classical Orthogonal Polynomials*, Nauka, Moscow, 1976 (in Russian).
- [738] Sugita H., *Sobolev spaces of Wiener functionals and Malliavin calculus*, J. Math. Kyoto Univ. **25** (1985), no. 1, 31–48.
- [739] Sugita H., *On a characterization of the Sobolev spaces over an abstract Wiener space*, J. Math. Kyoto Univ. **25** (1985), no. 4, 717–725.
- [740] Sugita H., *Positive Wiener functionals and potential theory over abstract Wiener spaces*, Osaka J. Math. **25** (1988), no. 3, 665–698.
- [741] Sugita H., *Hu–Meyer multiple Stratonovich integrals and essential continuity of multiple Wiener integrals*, Bull. Sci. Math. **113** (1989), 463–474.
- [742] Sugita H., *Various topologies on the Wiener space and Lévy's stochastic area*, Probab. Theory Relat. Fields **91** (1992), 286–296.
- [743] Sugita H., *Properties of holomorphic Wiener functions – skeleton, contraction, and local Taylor expansion*, Probab. Theory Relat. Fields **100** (1994), no. 1, 117–130.
- [744] Sugita H., *Regular version of holomorphic Wiener function*, J. Math. Kyoto Univ. **34** (1994), 849–857.
- [745] Sunouchi G., *Harmonic analysis and Wiener integrals*, Tohoku Math. J. **3** (1951), 187–196.
- [746] Sytaya G.N., *On certain asymptotic representations for a Gaussian measure in Hilbert space*, Theory of Stochastic Processes, no. 2, pp. 93–104, Kiev, 1974 (in Russian).
- [747] Sytaya G.N., *On asymptotics of the Wiener measure of small spheres*, Teor. Veroyatn. i Matem. Statist. **16** (1977), 121–135 (in Russian); English transl.: Theory Probab. Math. Statist. **16** (1977), 131–144.
- [748] Sytaya G.N., *On asymptotics of measure of small spheres for Gaussian processes equivalent to the Wiener process*, Teor. Veroyatn. i Matem. Statist. **19** (1978), 128–133 (in Russian); English transl.: Theory Probab. Math. Statist. **19** (1980), 149–154.
- [749] Sztencel R., *On the lower tail of stable seminorm*, Bull. Acad. Polon. Sci. Ser. Sci. Math. **32** (1984), no. 11–12, 715–719.
- [750] Takahashi Y., Okazaki Y., *On some properties of Gaussian covariance operators in Banach spaces*, Math. J. Okayama Univ. **29** (1987), 221–232.
- [751] Takeda M., *(r, p) -capacity on the Wiener space and properties of Brownian motion*, Z. Wahrscheinlichkeitstheorie verw. Geb. **68** (1984), no. 2, 149–162.
- [752] Talagrand M., *La τ -régularité des mesures gaussiennes*, Z. Wahrscheinlichkeitstheorie verw. Geb. **57** (1981), no. 2, 213–221.
- [753] Talagrand M., *Mesures gaussiennes sur un espace localement convexe*, Z. Wahrscheinlichkeitstheorie verw. Geb. **64** (1983), 181–209.
- [754] Talagrand M., *Sur l'intégrabilité des vecteurs gaussiens*, Z. Wahrscheinlichkeitstheorie verw. Geb. **68** (1984), no. 1, 1–8.
- [755] Talagrand M., *Regularity of Gaussian processes*, Acta Math. **159** (1987), no. 1–2, 99–149.

- [756] Talagrand M., *Small tails for the supremum of a Gaussian process*, Ann. Inst. H. Poincaré, Probab. et Statist., **24** (1988), no. 2, 307–315.
- [757] Talagrand M., *A note on Gaussian measure of translates of balls*, In: Geometry of Banach spaces (Strobl, 1989), pp. 253–256. London Math. Soc. Lect. Note Ser. **158**. Cambridge University Press, Cambridge, 1990.
- [758] Talagrand M., *Sudakov-type minoration for Gaussian processes*, Israel J. Math. **79** (1992), 207–224.
- [759] Talagrand M., *A simple proof of the majorizing measure theorem*, Geom. Funct. Anal. **2** (1992), 118–125.
- [760] Talagrand M., *New Gaussian estimates for enlarged balls*, Geom. Funct. Anal. **3** (1993), 502–526.
- [761] Talagrand M., *Sharper bounds for Gaussian and empirical processes*, Ann. Probab. **22** (1994), 28–76.
- [762] Talagrand M., *The small ball problem for the Brownian sheet*, Ann. Probab. **22** (1994), 1331–1354.
- [763] Taqqu M.S., *Weak convergence to fractional Brownian motion and the Rosenblatt process*, Z. Wahrscheinlichkeitstheorie verw. Geb. **31** (1975), 287–302.
- [764] Taqqu M.S., *Convergence of integrated processes of arbitrary Hermite rank*, Z. Wahrscheinlichkeitstheorie verw. Geb. **50** (1979), no. 1, 53–83.
- [765] Timan A.F., *Theory of Approximation of Functions of a Real Variable*, Nauka, Moscow, 1959 (in Russian); English transl.: Pergamon Press, New York, 1963.
- [766] Tišer J., *Differentiation theorem for Gaussian measures on Hilbert space*, Trans. Amer. Math. Soc. **308** (1988), no. 2, 655–666.
- [767] Titchmarsh E.C., *The Theory of Functions*, 2nd edn., Oxford University Press, London, 1968.
- [768] Tolmachev A.N., *A property of distributions of diffusion processes*, Matem. Zamet. **54** (1993), no. 3, 106–113 (in Russian); English transl.: Math. Notes **54** (1993), 946–950.
- [769] Tong Y.L., *Probability Inequalities in Multivariate Distributions*, Academic Press, New York, 1980.
- [770] Tong Y.L., *The Multivariate Normal Distribution*, Springer-Verlag, Berlin — New York, 1990.
- [771] Tortrat A., *Lois $e(\lambda)$ dans les espaces vectoriels et lois stables*, Z. Wahrscheinlichkeitstheorie verw. Geb. **37** (1976), no. 2, 175–182.
- [772] Tortrat A., *Prolongements τ -réguliers et applications aux probabilités gaussiennes*, In: Symp. Math. Ist. Naz. Alta Mat., Vol. 21, pp. 117–138. London – New York, 1977.
- [773] Traub J.F., Wasilkowski G.W., Wozniakowski H., *Information-Based Complexity*, Academic Press, New York, 1988.
- [774] Tsirelson B.S., *A natural modification of a random process, and its application to series of random functions and to Gaussian measures*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov (LOMI) **55** (1976), 35–63 (in Russian); English transl.: J. Soviet Math. **16** (1981), 940–956.
- [775] Tsirelson B.S., *Addendum to the article on natural modification*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **72** (1977), 202–211 (in Russian); English transl.: J. Soviet Math. **23** (1983), 2363–2369.
- [776] Tsirelson B.S., *The density of the maximum of a Gaussian process*, Teor. Veroyatn. i Primenen. **20** (1975), no. 4, 865–873 (in Russian); English transl.: Theory Probab. Appl. **20** (1975), 847–856.
- [777] Tsirelson B.S., *A geometrical approach to maximum likelihood estimates for infinite-dimensional Gaussian location, I, II, III*, Teor. Veroyatn. i Primenen. **27** (1982), no. 2, 388–395; **30** (1985), no. 4, 772–773; **31** (1986), no. 3, 537–549 (in Russian);

- English transl.: Theory Probab. Appl. **27** (1982), 411–418; **30** (1985), 820–828; **31** (1986), 470–483.
- [778] Uglanov A.V., *Surface integrals in a Banach space*, Matem. Sbornik **110** (1979), 189–217 (in Russian); English transl.: Math. USSR Sbornik **38** (1981), 175–199.
- [779] Uglanov A.V., *On the division of generalized functions of an infinite number of variables by polynomials*, Dokl. Akad. Nauk SSSR **264** (1982), no. 5, 1096–1099 (in Russian); English transl.: Soviet Math. Dokl. **25** (1982), no. 3, 843–846.
- [780] Uglanov A.V., *Smoothness of distributions of functionals of random processes*, Teor. Veroyatn. i Primenen. **33** (1988), no. 3, 535–544 (in Russian); English transl.: Theory Probab. Appl. **33** (1988), 500–508.
- [781] Uglanov A.V., *Hilbert carriers of Wiener measure*, Matem. Zamet. **51** (1992), no. 6, 91–96 (in Russian); English transl.: Math. Notes **51** (1992), no. 5–6, 589–592.
- [782] Uhlenbeck G.E., Ornstein L.S., *On the theory of Brownian motion. I*, Phys. Rep. **36** (1930), 823–841.
- [783] Ulyanov V., *On Gaussian measure of balls in H* , In: Frontiers in Pure and Applied Probability II, Proc. of the Fourth Russian–Finish Symposium on Probab. Theory and Mathem. Statistics, TVP Science Publ., Moscow, 1995.
- [784] Umemura Y., *On the infinite dimensional Laplacian operator*, J. Math. Kyoto Univ. **4** (1965), 477–492.
- [785] Üstünel A.S., *An Introduction to Analysis on Wiener Space*, Lecture Notes in Math. **1610**, Springer, Berlin — New York, 1995.
- [786] Üstünel A.S., Zakai M., *On the structure of independence on Wiener space*, J. Funct. Anal. **90** (1990), no. 1, 113–137.
- [787] Üstünel A.S., Zakai M., *Transformations of Wiener measure under anticipative flows*, Probab. Theory Relat. Fields **93** (1992), 91–136.
- [788] Üstünel A.S., Zakai M., *Applications of the degree theorem to absolute continuity on Wiener space*, Probab. Theory Relat. Fields **95** (1993), 509–520.
- [789] Üstünel A.S., Zakai M., *Transformation of the Wiener measure under non-invertible shifts*, Probab. Theory Relat. Fields **99** (1994), 485–500.
- [790] Üstünel A.S., Zakai M., *The composition of Wiener functionals with non absolutely continuous shifts*, Probab. Theory Relat. Fields **98** (1994), 163–184.
- [791] Üstünel A.S., Zakai M., *Random rotations of the Wiener path*, Probab. Theory Relat. Fields **103** (1995), no. 3, 409–430.
- [792] Üstünel A.S., Zakai M., *Extension of Lipschitz functions on Wiener space*, In: New trends in stochastic analysis (Proc. of the Taniguchi Internat. Symp.), D. Elworthy et al., eds., pp. 465–470. World Scientific, New York, 1996.
- [793] Üstünel A.S., Zakai M., *The construction of filtration on abstract Wiener space*, J. Funct. Anal. **143** (1997), no. 1, 10–32.
- [794] Üstünel A.S., Zakai M., *Degree theory on Wiener space*, Probab. Theory Relat. Fields **108** (1997), 259–279.
- [795] Üstünel A.S., Zakai M., *The Sard inequality on Wiener space*, J. Funct. Anal. **149** (1997), 226–244.
- [796] Vakhania N.N., *Probability Distributions on Linear Spaces*, Tbilisi, 1971 (in Russian); English transl.: North-Holland, Amsterdam, 1981.
- [797] Vakhania N.N., *Correspondence between Gaussian measures and the Gaussian processes*, Matem. Zamet. **26** (1979), no. 2, 293–297 (in Russian); English transl.: Math. Notes **26** (1979), no. 2, 638–640.
- [798] Vakhania N.N., *Canonical factorization of Gaussian covariance operators and some of its applications*, Teor. Veroyatn. i Primenen. **38** (1993), no. 3, 481–490 (in Russian); English transl.: Theory Probab. Appl. **38** (1993), no. 3, 498–505.

- [799] Vakhania N.N., Tarieladze V.I., *Covariance operators of probability measures in locally convex spaces*, Teor. Veroyatn. i Primenen. **23** (1978), no. 1, 3–26 (in Russian); English transl.: Theory Probab. Appl. **23** (1978), no. 1, 1–21.
- [800] Vakhania N.N., Tarieladze V.I., Chobanyan S.A., *Probability Distributions in Banach Spaces*, Nauka, Moscow, 1984 (in Russian); English transl.: Kluwer Academic Publ., Dordrecht, 1987.
- [801] Varadhan S.R.S., *Limit theorems for sums of independent random variables with values in a Hilbert space*, Sankhya A **24** (1962), 213–238.
- [802] Varberg D.E., *On equivalence of Gaussian measures*, Pacif. J. Math. **11** (1961), 751–762.
- [803] Varberg D.E., *Gaussian measures and a theorem of T.S. Pitcher*, Proc. Amer. Math. Soc. **13** (1962), 799–807.
- [804] Varberg D.E., *On Gaussian measures equivalent to Wiener measure*, Trans. Amer. Math. Soc. **113** (1964), no. 2, 262–273.
- [805] Varberg D.E., *On Gaussian measures equivalent to Wiener measure II*, Math. Scand. **18** (1966), no. 3, 143–160.
- [806] Varberg D.E., *Linear transformations of Gaussian measures*, Trans. Amer. Math. Soc. **122** (1966), no. 1, 98–111.
- [807] Varberg D.E., *Convergence of quadratic forms in independent random variables*, Ann. Math. Statist. **37** (1966), 567–576.
- [808] Varberg D.E., *Equivalent Gaussian measures with a particularly simple Radon–Nikodym derivative*, Ann. Math. Statist. **38** (1967), no. 4, 1027–1030.
- [809] Varberg D.E., *Almost sure convergence of quadratic forms in independent random variables*, Ann. Math. Statist. **39** (1968), 1502–1506.
- [810] Vershik A.M., *General theory of Gaussian measures in linear spaces*, Uspehi Matem. Nauk **19** (1964), no. 1, 210–212 (in Russian).
- [811] Vershik A.M., *Some characteristic properties of Gaussian stochastic processes*, Teor. Veroyatn. i Primenen. **9** (1964), no. 2, 390–394 (in Russian); English transl.: Theory Probab. Appl. **9** (1964), no. 2, 353–356.
- [812] Vershik A.M., *Duality in the theory of measure in linear spaces*, Dokl. Akad. Nauk SSSR **170** (1966), no. 3, 497–500 (in Russian); English transl.: Soviet Math. **7** (1966), no. 5, 1210–1214.
- [813] Vershik A.M., Sudakov V.N., *Probability measures in infinite-dimensional spaces*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov (LOMI) **12** (1969), 7–67 (in Russian); English transl.: Seminars in Math. Steklov Math. Inst. **12** (1971), 1–28.
- [814] Voiculescu D.V., Dykema K.J., Nica A., *Free Random Variables*, CRM Monograph Series, Amer. Math. Soc., Providence, Rhode Island, 1992.
- [815] Walsh J., *A note on uniform convergence of stochastic processes*, Proc. Amer. Math. Soc. **18** (1967), no. 1, 129–132.
- [816] Wang M.C., Uhlenbeck G.E., *On the theory of Brownian motion II*, Rev. Mod. Phys. **17** (1945), no. 2–3, 323–342.
- [817] Wang Y., *Small ball problem via wavelets for Gaussian processes*, Statistics and Probab. Letters **32** (1997), 133–139.
- [818] Wasilkowski G.W., *Optimal algorithms for linear problems with Gaussian measures*, Rocky Mountain J. Math. **16** (1986), no. 4, 727–749.
- [819] Watanabe S., *Analysis of Wiener functionals (Malliavin calculus) and its applications to heat kernels*, Ann. Probab. **15** (1987), no. 1, 1–39.
- [820] Watanabe S., *Short time asymptotic problems in Wiener functional integration theory. Applications to heat kernels and index theorems*, Lecture Notes in Math. **1444** (1990), 1–62.
- [821] Watanabe S., *Fractional order Sobolev spaces on Wiener space*, Probab. Theory Relat. Fields **95** (1993), 175–198.

- [822] Wentzell A.D., *A Course in the Theory of Stochastic Processes*, Nauka, Moscow, 1975 (in Russian); English transl.: McGraw-Hill, New York, 1981.
- [823] Wiener N., *The average of an analytic functional*, Proc. Nat. Acad. Sci. **7** (1921), no. 9, 253–260.
- [824] Wiener N., *The average value of an analytic functional and the Brownian movement*, Proc. Nat. Acad. Sci. **7** (1921), no. 10, 294–298.
- [825] Wiener N., *Differential space*, J. Math. and Phys. **2** (1923), 131–174.
- [826] Wiener N., *The average value of a functional*, Proc. London Math. Soc. **22** (1924), 454–467.
- [827] Wiener N., *The homogeneous chaos*, Amer. J. Math. **60** (1938), 879–936.
- [828] Woodward D.A., *A general class of linear transformations of Wiener integrals*, Trans. Amer. Math. Soc. **100** (1961), 459–480.
- [829] Yadrenko M.I., *Spectral Theory of Random Fields*, Vischa Shkola, Kiev, 1980 (in Russian); English transl.: Optimization Software, New York, 1983.
- [830] Yaglom A.M., *On the equivalence and perpendicularity of two Gaussian probability measures in function space*, In: Proc. Sympos. Time Series Analysis, pp. 327–346. Wiley, New York, 1963.
- [831] Yeh J., *Singularity of Gaussian measures in function spaces with factorable covariance functions*, Pacif. J. Math. **31** (1969), 547–554.
- [832] Yeh J., *Stochastic Processes and the Wiener Integral*, Marcel Dekker, New York, 1973.
- [833] Yoshida N., *A large deviation principle for (r, p) -capacities on the Wiener space*, Probab. Theory Relat. Fields **94** (1993), 473–488.
- [834] Yost D., *If every donut is a teacup, then every Banach space is a Hilbert space*, In: Seminar on Functional Analysis, Vol. 1, pp. 127–148. Univ. de Murcia, 1987.
- [835] Yurinsky V., *Sums and Gaussian Vectors*, Lecture Notes in Math. **1617**, Springer, Berlin, 1995.
- [836] Yurinsky V., *Some asymptotic formulae for Gaussian distributions*, J. Multivar. Anal. **56** (1996), 302–332.
- [837] Zak T., *On the difference of Gaussian measure of two balls in Hilbert spaces*, Lecture Notes in Math. **1391** (1989), 401–405.
- [838] Zakai M., *Stochastic integration, trace and the skeleton of Wiener functionals*, Stochastics and Stoch. Rep. **32** (1985), 93–108.
- [839] Zakai M., Zeitouni O., *When does Ramer formula look like Girsanov formula?*, Ann. Probab. **20** (1992), no. 3, 1436–1440.
- [840] Zalgaller V.A., *Mixed volumes and the probability of hitting in convex domains for a multidimensional normal distribution*, Matem. Zamet. **2** (1967), no. 1, 105–114 (in Russian); English transl.: Math. Notes **2** (1967), no. 1, 542–545.
- [841] Zhao Z.-X., *Quasilinear transformation in abstract Wiener spaces*, Sci. Sinica **24** (1981), no. 1, 1–12.
- [842] Zolotarev V.M., *Concerning a certain probability problem*, Teor. Veroyatn. i Primenen. **6** (1961), no. 2, 219–222 (in Russian); English transl.: Theory Probab. Appl. **6** (1961), no. 2, 201–204.
- [843] Zolotarev V.M., *Asymptotic behavior of the Gaussian measure in l_2* , J. Soviet Math. **24** (1986), 2330–2334.
- [844] Zygmund A., *Trigonometric Series*, Cambridge University Press, Cambridge, 1968.

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