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Part IV: Long-Time Solutions and Related Topics

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Contents

Preface

Keys to ignition, use at your discretion.

– From "Starin' Through My Rear View" by Tupac Shakur

This is Part IV (a.k.a. $R^{\#}_{ijk\ell}$), the sequel to Volume One ([**75**]; a.k.a. g_{ij}) and Parts I, II, III ([**69**], [**70**], [**71**]; a.k.a. $R_{ijk\ell}$, $\frac{\partial}{\partial t}R_{ijk\ell}$, $\Delta R_{ijk\ell}$, respectively) of Volume Two on techniques and applications of the Ricci flow. For the reader's convenience, we have included the titles of each chapter on the pages that follow.

In this part we mainly discuss aspects of the *long-time behavior* of solutions to the Ricci flow, including the geometry of noncompact gradient Ricci solitons, ancient solutions, Hamilton's classification of 3-dimensional nonsingular solutions, and the stability of the Ricci flow. Any theory about singularities of the Ricci flow requires an understanding of ancient solutions and, in particular, gradient Ricci solitons. Building on the success in dimensions at most 3, the study of higher-dimensional Ricci solitons is currently an active field; we discuss some of the progress in this direction. We also present recent progress on (1) the classification of ancient 2-dimensional solutions without the κ-noncollapsing hypothesis and (2) Type I ancient solutions and singularities. In a direction complementary to the study of singularities, we discuss 3-dimensional nonsingular solutions. These solutions underlie the Ricci flow approach to the geometrization conjecture; Hamilton's work on this is a precursor to Perelman's more general theory of immortal solutions to the Ricci flow with surgery. Finally, a largely unexplored direction in the Ricci flow concerns the sensitivity of solutions to their initial data; the study of stability of solutions represents an aspect of this.

The choice of topics is based on our familiarity and taste. Due to the diversity of the field of Ricci flow, we have inevitably omitted many important works. We have also omitted some topics originally slated for this part, such as the linearized Ricci flow and the space-time formulation of the Ricci flow. We now give detailed descriptions of the chapter contents.

Chapter 27. This chapter is a continuation of Chapter 1 of Part I. Here we discuss some recent progress on the geometry of noncompact gradient Ricci solitons (**GRS**), including some qualitatively sharp estimates for the volume growth, potential functions, and scalar curvatures of GRS. We also discuss the logarithmic Sobolev inequality for shrinking GRS as well as shrinking GRS with nonnegative Ricci curvature.

Chapter 28. This chapter complements the discussion in Part III on Perelman's theory of 3-dimensional ancient κ-solutions. The topics discussed are a local lower bound for the scalar curvature under Ricci flow, some geometric properties of 3-dimensional singularity models, noncompact 2-dimensional ancient solutions without the κ-noncollapsed condition, and classifying certain ancient solutions with positive curvature.

Chapter 29. In this chapter we present the results of Daskalopoulos, Hamilton, and Sesum that any simply-connected ancient solution to the Ricci flow on a closed surface must be either a round shrinking 2-sphere or the rotationally symmetric King–Rosenau solution. The proof involves an eclectic collection of geometric and analytic methods. Monotonicity formulas that rely on being in dimension 2 are used.

Chapter 30. This chapter is focused on the general study of Type I singularities and Type I ancient solutions. We study properties and applications of Perelman's reduced distance and reduced volume based at the singular time for Type I singular solutions. We also discuss the result that Type I singular solutions have unbounded scalar curvature.

Chapter 31. In the study of nonsingular solutions to the Ricci flow on closed 3-manifolds in the subsequent chapters, of vital importance are finite-volume hyperbolic limits. In this chapter we present some prerequisite knowledge on the geometry and topology of hyperbolic 3-manifolds. Key topics are the Margulis lemma (including the ends of finite-volume hyperbolic manifolds) and the Mostow rigidity theorem.

Chapter 32. Hamilton's celebrated result says that for solutions to the normalized Ricci flow on closed 3-manifolds which exist for all forward time and have uniformly bounded curvature, the underlying differentiable 3-manifold admits a geometric decomposition in the sense of Thurston. The proof of the main result requires an understanding of the asymptotic behavior of the solution as time tends to infinity. If collapse occurs in the sense of Cheeger and Gromov, then the underlying differentiable 3-manifold admits an F-structure and in particular admits a geometric decomposition. Otherwise, one may extract limits of noncollapsing sequences by the uniformly bounded curvature assumption. In the cases where these limits have nonnegative sectional curvature, we can topologically classify the original 3-manifolds.

Chapter 33. In the cases where the limits do not have nonnegative sectional curvature, they must be hyperbolic 3-manifolds with finite volume, which may be either compact or noncompact. If these hyperbolic limits are compact, then they are diffeomorphic to the original 3-manifold. On the other hand, if these hyperbolic limits are noncompact, then the difficult result is that their truncated embeddings in the original 3-manifold are such that the boundary tori are incompressible in the complements. To establish this, one proves the stability of hyperbolic limits by the use of harmonic maps and Mostow rigidity. Then, assuming the compressibility of any boundary tori, one applies a minimal surface argument to obtain a contradiction.

Chapter 34. The purpose of this chapter is to prove, by the implicit function theorem, two results used in the previous chapter. We first show that almost hyperbolic cusps are swept out by constant mean curvature tori. Second, for any metric g on a compact manifold with negative Ricci curvature and concave boundary and for any metric $\tilde{g}$ sufficiently close to g, we prove the existence of a harmonic diffeomorphism from g to $\tilde{g}$ near the identity map.

Chapter 35. A potentially useful direction in Ricci flow is to study the perturbational aspects of the flow, in particular, stability of solutions, dependence on initial data, and properties of generic solutions and 1-parameter families of solutions. In this chapter we discuss the stability of solutions. The analysis of stability is partly dependent on understanding the Ricci flow coupled to the Lichnerowicz Laplacian heat equation for symmetric 2-tensors.

Chapter 36. In this chapter we survey a numerical approach, due to Garfinkle and one of the authors, to modeling rotationally symmetric degenerate neckpinches including the reflectionally symmetric case of two Bryant solitons simultaneously forming as limits. We also survey the matched asymptotic analysis of rotationally symmetric degenerate neckpinches and the related Ważewski retraction method.

Appendix K. In this appendix we recall some concepts and results about the analysis on manifolds that are used in various places in the book. In particular, we discuss the implicit function theorem, Hölder and Sobolev spaces of sections of bundles, formulas for harmonic maps, and the eigenvalues of the Hodge–de Rham Laplacian acting on differential forms on the round sphere.

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I didn't think I never dreamed

That I would be around to see it all come true.

– From "Nineteen Hundred and Eighty-Five" by Paul McCartney and Wings

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Contents of Volume One and Parts I, II, and III of Volume Two

Volume One: An Introduction

Part I: Geometric Aspects

Notation and Symbols

Doesn't mean that much to me

To mean that much to you.

– From "Old Man" by Neil Young

The following is a list of some of the notation and symbols which we use in this book.

∇	covariant derivative
$\square^*$	adjoint heat operator
$\square_L$	Lichnerowicz Laplacian heat operator
$\square_L^*$	adjoint Lichnerowicz Laplacian heat operator
$\doteqdot$	defined to be equal to
$\cdot$	dot product or multiplication
$\tilde{\measuredangle}$	Euclidean comparison angle
$\nabla^2 f$	Hessian of f
$\overline{\wedge}$	Kulkarni–Nomizu product
#	sharp operator
$\otimes_S$	symmetric tensor product
$\alpha^\natural$	dual vector field to the 1-form α
$W_\top$	tangential component of the vector W
$W_\perp$	normal component of the vector W
Area	area of a surface or volume of a hypersurface
ASCR	asymptotic scalar curvature ratio
AVR	asymptotic volume ratio
$B_p\left(r\right)$	ball of radius r centered at p
b	Bianchi map
B_{abcd}	the quadratic 4-tensor $-R_{apbq}R_{cpdq}$
bounded curvature	bounded *sectional* curvature (for time-dependent metrics, where the bound may depend on time)
$C_V\mathcal{J}$	tangent cone at V of a convex set $\mathcal{J} \subset \mathbb{R}^k$
const	constant

CK	vector space of conformal Killing vector fields
Conf	group of conformal diffeomorphisms
$\frac{d^+}{dt}$, $\frac{d^-}{dt}$, $\frac{d_+}{dt}$, $\frac{d_-}{dt}$	a Dini time derivative
d	distance
d_{GH}	Gromov–Hausdorff distance
$d\mu$	volume form
$d\mu_{\mathbb{E}}$	Euclidean volume form
$d\sigma$ *or* dA	volume form on boundary or hypersurface
Δ, Δ_L, Δ_d	Laplacian, Lichnerowicz Laplacian, Hodge–de Rham Laplacian
diam	diameter
div	divergence
$\mathbb{E}^n$	$\mathbb{R}^n$ with the flat Euclidean metric
$\mathbb{E}^{n,1}$	Minkowski $(n+1)$-space
$E_r(x,t)$	heat ball of radius r based at (x,t)
exp	exponential map
$\mathcal{F}$	Perelman's energy functional
Γ_{ij}^k	Christoffel symbols
$g(X,Y) = \langle X,Y \rangle$	metric or inner product
$g(t)$	time-dependent metric, e.g., solution of the Ricci flow
g_∞ *or* $g_\infty(t)$	limit Riemannian metric or solution of Ricci flow
GRS	gradient Ricci soliton
h *or* II	second fundamental form
H	mean curvature
$H_V \mathcal{J}$ for $V \in \partial \mathcal{J}$	set of closed half-spaces H containing $\mathcal{J} \subset \mathbb{R}^k$ with $V \in \partial H$
Hess f	Hessian of f (same as $\nabla^2 f$)
id	identity
Im	imaginary part
Inn (G)	inner automorphism group
int	interior
inj	injectivity radius
Isom	group of isometries of a Riemannian manifold
IVP	initial-value problem
J	Jacobian of the exponential map
$J^k(\mathcal{M},\mathcal{N})$	bundle of k-jets of maps
KV	vector space of Killing vector fields
L	length

LHS	left-hand side
log	natural logarithm
$\mathcal{I}$	a time interval for the Ricci flow
$\mathcal{J}$	a time interval for the backward Ricci flow
λ	λ-invariant
L	Perelman's L-distance
ℓ	reduced distance or ℓ-function
$\mathcal{L}$	Lie derivative *or* $\mathcal{L}$-length
$\mathcal{L}\,\mathrm{Cut}$	$\mathcal{L}$-cut locus
$\mathcal{L}\exp$	$\mathcal{L}$-exponential map
$\mathcal{L}\,\mathrm{I}$	$\mathcal{L}$-index form
$\mathcal{L}\,\mathrm{J}_V$	$\mathcal{L}$-Jacobian
$L(v, X)$	linear trace Harnack quadratic
$(\mathcal{M}, \hat{g})$	static Riemannian manifold
μ	μ-invariant
MCF	mean curvature flow
$\mathfrak{Met}$	space of Riemannian metrics on a manifold
MVP	mean value property
Möb	group of Möbius transformations
$\times$	multiplication, when a formula does not fit on one line
ν	ν-invariant *or* unit outward normal
$\mathfrak{M}_{n,\kappa}$	collection of n-dimensional κ-solutions
$\mathfrak{M}_{n,\kappa}^{\mathrm{Harn}}$	n-dimensional κ-solutions with Harnack
$n\omega_n$	volume of the unit Euclidean $(n-1)$-sphere
NRF	normalized Ricci flow
ω_n	volume of the unit Euclidean n-ball
ODE	ordinary differential equation
Out	outer automorphism group
$\mathrm{PSL}(n, \mathbb{C})$	projective complex special linear group
P_{ijk}	the symmetric 3-tensor $\nabla_i R_{jk} - \nabla_j R_{ik}$
PDE	partial differential equation
PIC	positive isotropic curvature
$R_{ijk\ell}$	$\sum_m R_{ijk}^m g_{m\ell}$ (opposite of Hamilton's convention)
R_{jk}	$\sum_i R_{ijk}^i = \sum_{i,\ell} g^{i\ell} R_{ijk\ell}$ (components of Ricci)
$\mathcal{R}_{jk}$	a symmetric 2-tensor ($\mathcal{R}_{jk} = R_{jk}$ is a special case)
$\mathrm{Ray}_{\mathcal{M}}(O)$	space of rays emanating from O in $\mathcal{M}$
$\mathbb{R}_{>0}$	set of positive real numbers
RF	Ricci flow

RHS	right-hand side
R, Rc, Rm	scalar, Ricci, and Riemann curvature tensors
$\mathrm{Rm}^{\#}$	the quadratic $\mathrm{Rm} \# \mathrm{Rm}$
$\mathbf{R}$	algebraic curvature operator
$\mathrm{Rc}(\mathbf{R})$	a trace of $\mathbf{R}$ (of two indices)
$\mathbb{R}^n$	n-dimensional Euclidean space
$\mathrm{SL}(n, \mathbb{C})$	complex special linear group
$\mathrm{SO}(n, \mathbb{R})$	real orthogonal group
$S_B^2(\mathfrak{so}(n))$	space of algebraic curvature operators
$S\Omega_T$	side boundary $\partial\Omega \times (0, T]$
$S_V \mathcal{J}$ for $V \in \partial\mathcal{J}$	set of support functions of $\mathcal{J} \subset \mathbb{R}^k$ at V
sect	sectional curvature
$\mathcal{S}^n$	unit radius n-dimensional sphere
supp	support of a function
$T_x\mathcal{M}$	tangent space of $\mathcal{M}$ at x
$T_x^*\mathcal{M}$	cotangent space of $\mathcal{M}$ at x
$\tau(t)$	function satisfying $\frac{d\tau}{dt} = -1$
tr *or* trace	trace
$\tilde{V}$	reduced volume
$\hat{V}_\infty$	mock reduced volume
$\mathcal{V}$	vector bundle
Vol	volume of a manifold
$\mathcal{W}$	Perelman's entropy functional
$W^{k,p}$	Sobolev space of functions with $\leq k$ weak derivatives in L^p
$W^{k,p}_{\mathrm{loc}}$	space of functions locally in $W^{k,p}$
WMP	weak maximum principle

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Ricci flow is a powerful technique using a heat-type equation to deform Riemannian metrics on manifolds to better metrics in the search for geometric decompositions. With the fourth part of their volume on techniques and applications of the theory, the authors discuss long-time solutions of the Ricci flow and related topics.

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