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Horst R. Thieme

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To Ruth and Reese
and in memory of Adelheid

Contents

Preface

A fundamental question in the theory of discrete and continuous time population models concerns the conditions for the extinction or persistence of populations. This question is addressed by persistence theory ([**33**, **68**, **187**, **226**] and the references therein). Since some time, it has been recognized that, if the dynamics of the population are mathematically captured by continuous or discrete semiflows and if these semiflows have first order approximations, the spectral radii of certain bounded linear operators (better known as basic reproduction numbers) act as thresholds between population extinction and persistence [**12**, **44**, **50**, **62**, **63**, **118**–**121**, **150**, **196**, **214**].

Work on continuous (positively) homogeneous operators (of degree one) by John Mallet-Paret and Roger Nussbaum [**156**, **157**] suggested to me that the spectral radii of these more general operators can play a similar role in population models that include the mating between two sexes. I thank Roger Nussbaum [**8**, **145**, **146**, **167**–**170**] for guiding me through some of the literature on homogeneous operators. Indeed, as we will see, in discrete-time models of structured populations with two sexes, the spectral radius of an order-preserving homogeneous operator that is the first order approximation of the population turnover operator acts as threshold between population extinction and population persistence. Discrete-time models (difference equations) are chosen because the theory of homogeneous operators applies more readily than in continuous-time models ((partial, delay, integro) differential equations). For the role of homogeneous operators in differential equations models, see [**107**, **108**, **120**, **153**, **221**, **222**].

Work on the material in this monograph began as a series of papers [**199**–**202**], some of them coauthored by Wen Jin and/or Hal Smith [**127**–**130**, **187**, **188**], whom I gratefully acknowledge, and the dissertation by Wen Jin [**126**]. The mathematics play out in the framework of normed vector spaces, usually ordered by a closed cone. I thank Ulrich Krause for walking me through his book [**138**] which got me interested in some geometric aspects (Sections 2.1.3 and 2.6). I always wondered whether there are interesting ordered normed vector spaces that are not complete themselves or are even ordered by a cone that is not complete. I got my answer from a talk by Piotr Gwiazda at the Fields Institute in Toronto and in subsequent discussions with him, Karolina Kropielnicka and Anna Marciniak-Czochra ([**34**, **72**, **97**–**102**], Chapter 3): The cone of nonnegative Borel measures on a separable metric space is complete under the flat (aka dual bounded Lipschitz) norm if and only if the underlying metric space is complete ([**113**, Thm.3.8], Section 3.2.4). This has largely influenced how the theory of ordered normed vector spaces is presented in this monograph and has led to the concept of a serially complete additive subset and a monotonically complete cone of a normed vector space (Sections 2.1.2 and 2.1.5). Chapter 2 can be used as a text for a course on ordered normed vector spaces.

I thank Odo Diekmann and Eugenia Franco for carefully reading [**203**] and motivating me to bring it into its present form (the original one is partially covered in Chapter 19). I thank Gaël Raoul for carefully reading [**205**] and convincing me that Feller kernels have no good backward interpretation (see Section 18.1.1). Fei Cao read a part of the monograph during a reading course at Arizona State University and did a couple of the exercises. My late colleagues John McDonald and Neil Weiss at ASU gave me a complimentary copy of their Real Analysis book [**159**], which turned out to be a wonderful resource I refer to frequently. I am grateful to the anonymous reviewers for their constructive comments and to my editor Dr. Ina Mette for encouragement that extended over more than a decade.

I thank my wife Adelheid for her love and support over the many years I worked on this monograph [**191**]; sadly, the love of my life did not live to see the book's completion. I dedicate it to her memory and to my daughters Ruth and Reese (Clara) in gratitude for their tolerance.

Finally, praise to the Holy Spirit, source of knowledge and persistence, for the completion of this book.

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