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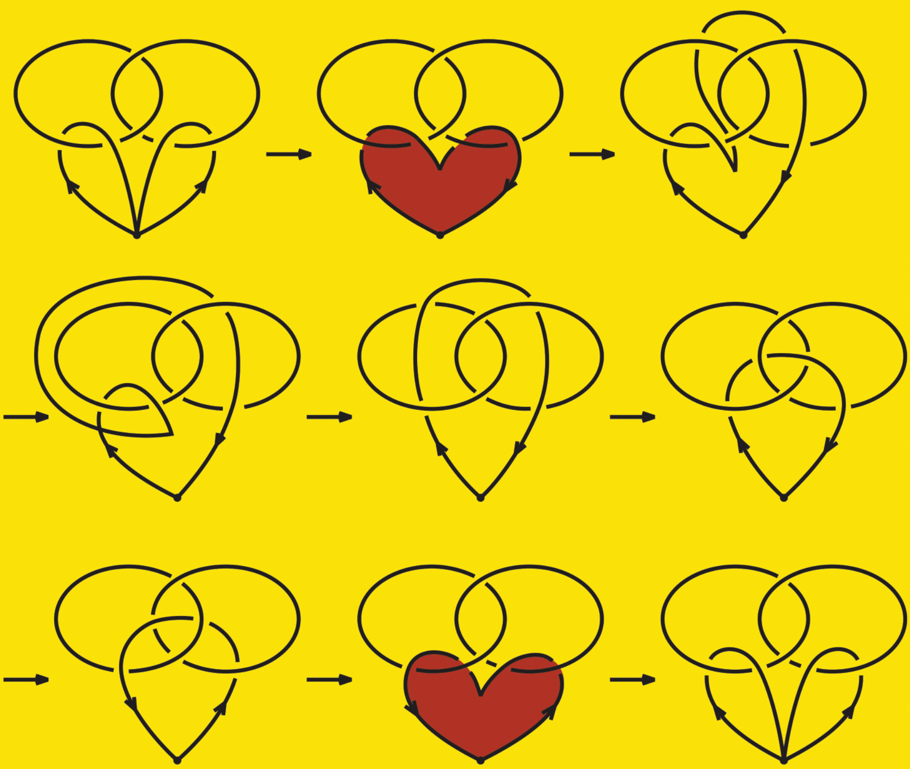
TOPOLOGY NOW!

Robert Messer & Philip Straffin



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Topology Now!

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Preface

Topology is a branch of mathematics packed with intriguing concepts, fascinating geometric objects, and ingenious methods for studying them. The authors have written this textbook to make this material accessible to undergraduate students who may be at the beginning of their study of upper-level mathematics and who may not have covered the extensive prerequisites required for a traditional course in topology. Our preference is to cultivate the intuitive ideas of continuity, convergence, and connectedness so that students can quickly delve into knot theory, surfaces, fixed-points, and even obtain a taste of algebraic topology. We believe that students should see the exciting geometric ideas of topology now (!) rather than later.

We acknowledge the danger in building on less than a solid foundation, and one of our goals is to provide adequate reinforcement. Principles we introduce without a rigorous development are supported with numerous examples and explicit statements. We have also provided a selection of careful proofs. For example, we include a complete proof that the Alexander polynomial is a well-defined invariant for knots—a rare feat for a text at this level. After working with the material in this text, students will be well equipped to study the intricacies and abstractions of more advanced courses in point-set and algebraic topology. We hope that many of them will be motivated to take these courses.

The geometric approach to topology in this text also exposes students to the interrelation among the various branches of mathematics. Continuity and geometry are of course at the heart of the matter. This is a natural preparation for courses in real analysis, geometry, and further work in topology. Students will also see strong ties with linear algebra in the dimension of various objects, with abstract algebra in the fundamental group of a space, and with discrete mathematics in a variety of combinatorial and counting arguments.

The prerequisite for this approach to topology is some exposure to the geometry of objects in higher-dimensional Euclidean spaces, together with appreciation of precise mathematical definitions and proofs. We recommend courses in multivariable calculus and linear algebra, and one further proof-oriented course.

Organization. The first chapter introduces various ways objects can be considered to be the same. We begin with set-theoretical concepts and introduce the concept of continuity for preserving the geometrical properties of a space. For an object that is a subset of a larger space, we also consider a continuous family of deformations of the embedded object.

In the second chapter we apply these concepts to knots and links. Although we rely on intuitive ideas about polyhedral structure of sets and subsets, we point out the difficulties and how to address them before sweeping them under the rug. We develop some useful ways of distinguishing among knots and sample some polynomials of recent discovery that are quite powerful invariants of knots.

Chapters 3 and 4 present some basic examples of surfaces and their three-dimensional analogs. These classical results are among the most beautiful ideas of geometric topology. The proofs are honest, lacking only some of the technical details.

Chapter 5 takes a side tour into the theory of fixed-points. This provides an interesting application of topology that in turn has useful applications in disciplines beyond mathematics.

Chapter 6 introduces an algebraic system of dealing with loops in a space. Although we only scratch the surface of algebraic topology, students have an opportunity to work with the basic concepts and develop a sense of the power of algebraic techniques.

Chapter 7 presents material that is often the starting point in traditional texts. Our idea is that the abstractions of point-set topology will make more sense after the students have seen the geometric examples that motivate these abstractions. For example, the general concept of a quotient topology is nicely motivated by the geometric technique of gluing together edges of a polygonal disk to form a closed surface.

A course covering Chapters 1 through 4 would provide a geometric introduction to topology. The later chapters are somewhat independent. Chapter 5 connects topology with analysis, Chapter 6 introduces the algebraic aspects of topology, and Chapter 7 introduces abstract topological spaces.

We have included some harder material that will challenge students but which is not necessary to the flow of ideas. For example, the proofs that the Alexander polynomial is a well-defined knot invariant can easily be omitted from the end of Section 2.5 without loss of continuity. Likewise, the discussion of Heegaard splittings in Section 4.4, the Contraction Mapping Theorem in Section 5.2, and the material on words and relations in Section 6.5 can be regarded as supplemental topics.

For over twenty years, the second author has taught a junior-level course at Beloit College based on the material in Chapters 1 through 4 (omitting the Alexander polynomial proofs at the end of Section 2.5 and the material in Section 4.4) and most of Chapter 6, and tying everything together with Chapter 7.

Features. Each chapter begins with an informal introduction. This gives students a historical or mathematical context for topics contained in the chapter. The individual sections and topics within the sections are also linked with transitional comments that place the topics into a larger context.

An abundance of examples illustrate the new concepts. These are stated as problems that the students might encounter as homework or on exams. The solutions provide models for dealing with the concepts as well as illustrations of the level of rigor expected in deriving results.

Each section contains a rich variety of exercises. Many exercises give students practice with the definitions and theorems of the section. Other exercises relate the current material to previous topics or provide motivation for future developments. Exercises frequently ask students to fill in gaps in arguments given in the section. The most challenging exercises extend topics in new directions, offering possibilities for independent study or undergraduate research.

A Web site is maintained at http://www.albion.edu/math/ram/TopologyNow! to provide additional support material for this text. Students and instructors are invited to visit this site and to submit comments, suggestions, questions, exercises, sample syllabi, and supplementary course material that might be of value to others.

Acknowledgments. The authors hope both students and teachers will enjoy this text. We have worked hard to make this material clear and comprehensible while maintaining a standard of honest mathematics. We encourage readers to contact us with suggestions and comments. The following students and professors have made comments and suggestions for improvements in preliminary material for this book. We gratefully acknowledge their contributions.

Albion College: Elizabeth Chen, Brad Emmons, Robert Gray, Miles Horak, Frederick Horein, Jamie Kucab, Martha O'Kennon, David Reimann, Timothy Schafer, Aaron St. John, David Tollefson, Matthew Woods

Beloit College: students in Phil Straffin's topology courses for over twenty years

Cranbrook Kingswood School: Yuyin Chen

University of Akron: Lisa Lackney, Benjamin Marko, Ian Deters, Lori McDonnell, Tom Price, Joel Rabe, Charles Williams

University of Detroit Mercy: Gillian Carney

University of Wyoming: Sylvia Hobart

Valparaiso University: Elizabeth Brondos, Ross Corliss, Steven Klee, Kimberly Pearson, Paul Schmid, Dan Tesch, Philip Whaley

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About the Authors

Robert Messer studied mathematics as an undergraduate at the University of Chicago. He wrote his thesis in geometric topology at the University of Wisconsin under D. Russell McMillan, receiving his PhD in 1975. He was a John Wesley Young Research Instructor at Dartmouth College and has taught at Western Michigan University and Vanderbilt University. He has been at Albion College since 1981 where he has served as chair of the Department of Mathematics and Computer Science from 1997 to 2002.

In addition to research in topology, he is the author of the textbook *Linear Algebra: Gateway to Mathematics* (1994) and one of the co-authors of *Learning by Discovery: A Lab Manual for Calculus* (MAA, 1993). He helped to organize and coach the Michigan All-Star Math Team for the American Regions Mathematics League Competitions and has served as director of the Michigan Mathematics Prize Competition for the Michigan Section of the Mathematical Association of America. He enjoys the combinatorics and symmetry of English change ringing as well as traditional American and English country dance.

Philip Straffin earned his undergraduate degree in mathematics from Harvard University. He learned knot theory from Ray Lickorish at Cambridge University on a Marshall Scholarship, and received his PhD from the University of California at Berkeley, with a thesis in algebraic topology under Emery Thomas. He has taught at Beloit College since 1970, and served as Chair of Mathematics and Computer Science from 1980 to 1990. He has twice been chosen as Beloit College's Teacher of the Year, and received the MAA's Haimo Award for Distinguished College Teaching of Mathematics in 1993.

Professor Straffin has published over 30 research and expository papers, and has won the Allendoerfer Award and the Trevor Evans Award for mathematical exposition from the MAA. His books include *Topics in the Theory of Voting* (1980) and *Game Theory and Strategy* (MAA, 1993), and edited collections *Political and Related Models* (with Steven

Brams and William Lucas, 1983) and *Applications of Calculus* (MAA, 1993). He is a member of the American Mathematical Society, the Mathematical Association of America and the Association for Women in Mathematics. For the MAA, he has been Chair of the Wisconsin Section, Editor of the Anneli Lax New Mathematical Library, and served on the MAA Notes Editorial Board, the Haimo Teaching Award Committee, the Beckenbach Book Prize Committee, the Council on Publications, and the Coordinating Council on Awards. He enjoys the challenge of scaling peaks in the mountains of Colorado.

MATHEMATICS/TOPOLOGY

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Topology is a branch of mathematics packed with intriguing concepts, fascinating geometrical objects and ingenious methods for studying them. The authors have written this textbook to make this material accessible to undergraduate students without requiring extensive prerequisites in upper-level mathematics. The approach is to cultivate the intuitive ideas of continuity, convergence, and connectedness so students can quickly delve into knot theory, the topology of surfaces and three-dimensional manifolds, fixed points, and elementary homotopy theory. The fundamental concepts of point-set topology appear at the end of the book when students can see how this level of abstraction provides a sound logical basis for the geometrical ideas that have come before. This organization exposes students to the exciting geometrical ideas of topology now(!) rather than later.

Students using this textbook should have some exposure to the geometry of objects in higher-dimensional Euclidean spaces together with an appreciation of precise mathematical definitions and proofs. Multivariable calculus, linear algebra, and one further proof-oriented mathematics course are suitable preparation.

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