American Mathematical Society
TRANSLATIONS
Series 2 • Volume 187
Advances in the Mathematical Sciences

Coherent Transform, Quantization, and Poisson Geometry

M. V. Karasev
Editor

American Mathematical Society

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Advances in the Mathematical Sciences — 40

(*Formerly Advances in Soviet Mathematics*)

Coherent Transform, Quantization, and Poisson Geometry

M. V. Karasev
Editor

American Mathematical Society
Providence, Rhode Island

1991 *Mathematics Subject Classification.* Primary 58F05, 81R30, 78H05, 81Sxx;
Secondary 33Cxx, 51H15, 53C05.

ABSTRACT. This book consists of three long articles devoted to advanced problems in quantization theory, as well as in symplectic and Poisson geometry. The main topics are: coherent states and irreducible representations for algebras with non-Lie permutation relations, Hamilton dynamics and quantization over stable isotropic submanifolds, and infinitesimal tensor complexes over degenerate symplectic leaves in Poisson manifolds. The articles include many examples (in particular, examples from physics) and complete proofs. The book will be of interest to researchers and graduate students working in these areas of mathematics and mathematical physics.

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Preface

The present collection conrains three paper on the quantization theory and symplectic and Poisson geometry.

In the paper "*Non-Lie permutation relations, coherent states, and quantum embedding*" by Mikhail Karasev and Elena Novikova, irreducible representations for some interesting classes of algebras with non-Lie permutation relations are constructed. The approach used in construction these represenations is related to the theory of coherent states. The solution of the well-known problem of quantum embedding of surfaces (symplectic leaves) is presented. The relationship between quantum surfaces (of revolution, of birevolution, etc.), irreducible representations of non-Lie algebras and the theory of (generalized) hypergeometric functions is investigated. Explicit formulas for $\hbar$-expansions of the Weyl and Wick quantum products are found in geometric terms. Meromorphic coherent states are used to establish a duality between Wick and anti-Wick symbols. Numerous examples of non-Lie permutation relations, in particular, some examples of quadratic algebras that are important in physics, are considered in details.

The paper "*Adapted connections, Hamilton dynamics, geometric phases, and quantization over isotropic submanifolds*" by Mikhail Karasev and Yurii Vorobjev is devoted to symplectic geometry and Hamilton dynamics in a neighborhood of a stable isotropic submanifold (for example, a torus). The approach in the paper is based on the notion of adapted symplectic connection. The results generalize and develop a variety of classical theorems related to the case of a stable trajectory. Flat coisotropic extensions, Darboux atlases, Lagrangian inflations, Floquet multipliers in the symplectic setting, and other structures on isotropic submanifolds are introduced and studied. The method of adapted connections is also applied to the quantization problem and to the spectral geometry. Analogs and generalizations of the Gelfand–Lidskii index, the Johnson–Moser rotation number, and the Maslov class are defined for isotropic submanifolds of dimension more than 1 and less than $\frac{1}{2}$(phase space dimension). These invariants are combined together in a quantization condition.

In the paper "*Infinitesimal Poisson cohomology*" by Vladimir Itskov, Mikhail Karasev, and Yurii Vorobjev infinitesimal geometry of a degenerate symplectic leaf of a Poisson manifold is studied. Using linear connection and the Lie algebroid structure in the corresponding normal bundle, the infinitesimal Poisson cohomology is studied in detail, and applications to the problem of deformations of Poisson brackets are considered.

The articles in the book continue and substantially develop the authors' previous results about nonlinear Poisson brackets, Hamilton dynamics, and quantization.

The collection can be viewed as a summary of some new ideas and approaches suggested by participants of our research seminar in the last five years. This seminar takes place in the Quantum Dynamical Systems Laboratory at the Applied Mathematics Department of the Moscow Institute of Electronics and Mathematics (MIEM). The participants of the seminar, who are my co-authors in this collection, graduated from MIEM in different years. For example, Yurii Vorobjev graduated from a special group initiated by V. P. Maslov in 1972. Students of this group attended lectures by V. Arnold, Yu. Manin, and other outstanding mathematicians. Our collaboration with Yurii started back then. Elena Novikova recently completed graduate studies (very successfully) and received the PhD degree. Vladimir Itskov is now a graduate student; he is certainly highly talented.

Discussions with my students Mikhail Kozlov and Alexander Pereskokov, who also are participants of the seminar, contributed to the common work. Unfortunately, for various reasons, this collection does not contain their papers.

The friendly scientific atmosphere at the Chair of Applied Mathematics greatly facilitates the collection. The help offered by M. A. Shishkova in translating and typesetting the articles in this collection was invaluable.

Mikhail Karasev

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