

American Mathematical Society

# TRANSLATIONS

Series 2 • Volume 196

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Advances in the Mathematical Sciences

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## Northern California Symplectic Geometry Seminar

Ya. Eliashberg

D. Fuchs

T. Ratiu

A. Weinstein

Editors



American Mathematical Society

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Northern California  
Symplectic Geometry  
Seminar

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Advances in the Mathematical Sciences — 45

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*(Formerly Advances in Soviet Mathematics)*

## Northern California Symplectic Geometry Seminar

Ya. Eliashberg  
D. Fuchs  
T. Ratiu  
A. Weinstein  
Editors



**American Mathematical Society**  
Providence, Rhode Island

ADVANCES IN THE MATHEMATICAL SCIENCES  
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ABSTRACT. The Northern California Symplectic Geometry Seminar was established in the Fall of 1989 to encourage ongoing interaction between geometers at Stanford University and the University of California campuses at Berkeley, Davis, and Santa Cruz. The lectures over the years, by visitors, local faculty, and advanced graduate students, have provided a panorama of developments in symplectic and contact geometry and topology, Poisson geometry, quantization theory, and applications.

The present volume contains papers by some of the seminar participants. They reflect the diversity of topics of the seminar, ranging from symplectic topology to Hamiltonian dynamics, from quantum cohomology and mirror symmetry to infinite-dimensional symplectic geometry, the theory of Hamiltonian group actions, and quantization.

The book can be used by researchers and graduate students working in symplectic geometry and topology, and applications.

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## Introduction

The Northern California Symplectic Geometry Seminar was established in the Fall of 1989 to encourage ongoing interaction between geometers at Stanford University and the University of California campuses at Berkeley, Davis, and Santa Cruz. Since then, the seminar has met on the first Monday of each month from October to May (except January), alternately at Stanford and Berkeley, with two lectures at each meeting. Beginning in the Fall of 1992, after the tragic death of Andreas Floer, who was among the original participants in the seminar, one of the lectures at the October meeting has been designated as the Andreas Floer Memorial Lecture. The Floer Memorial Lectures have been given by Helmut Hofer (1992), Dusa McDuff (1993), Yasha Eliashberg (1994), Jon Wolfson (1995), Vladimir Arnold (1996), Simon Donaldson (1997), and Maxim Kontsevich (1998).

The lectures over the years, by visitors, local faculty, and advanced graduate students, have provided a panorama of developments in symplectic and contact geometry and topology, Poisson geometry, quantization theory, and applications.

All the authors of the current volume, except Marcin Poźniak, were participants and/or speakers at the seminar.

The paper by Ana Cannas da Silva and Victor Guillemin concerns symplectic orbifolds with a Hamiltonian action by a group  $G$ . The authors define a quantization space  $V$  which is a representation of  $G$ , and describe a decomposition of  $V$  into irreducible representations of  $G$ .

Simon Donaldson in his paper describes and studies the infinite dimensional symmetric space structure which arises naturally on the space of Kähler metrics with a fixed cohomology class.

The paper by Viktor Ginzburg is a survey of counter-examples to the “Hamiltonian Seifert conjecture” and more generally of Hamiltonian systems without periodic orbit on a compact energy level.

Alexander Givental presents a “shortcut” to his celebrated original proof of the Mirror Theorem for the case of quintic threefolds, thus giving a proof of the formula of Candelas, de la Ossa, Green and Parkes for the the number of rational curves of degree  $d$  on a quintic threefold in  $\mathbb{C}^4$ .

François Lalonde and Charles Pestieau prove that the basic symplectic inequalities – the non-squeezing theorem and the energy-capacity inequality – are in a certain sense stable under product with any closed symplectic manifold.

Dusa McDuff’s paper is devoted to a construction of the virtual moduli cycle. Virtual moduli cycles are essential for the theory of Gromov-Witten invariants, Floer homology theory, etc. McDuff’s paper provides further clarification of this important theory.

The paper by Richard Montgomery may serve as a good introduction to the emerging theory of the geometry and topology of Engel structures on 4-manifolds. In it Montgomery establishes some foundational facts in this theory.

Yongbin Ruan contributed to the volume an expository article about some newly discovered relations between quantum cohomology and birational geometry, and the surgery techniques which play a fundamental role in these discoveries.

Dietmar Salamon's paper complements his earlier joint work with S. Dostoglou, which contained a proof of the Atiyah-Floer conjecture based on the construction of a natural isomorphism between instanton and symplectic Floer homology theories for the mapping tori of symplectomorphisms of surfaces. In the current paper, Salamon proves that this isomorphism intertwines the product structures in the two theories.

The work of Paul Seidel concerns the topology of the group of symplectomorphisms of the product of complex projective spaces; it also partially extends earlier results of Gromov, Abreu and McDuff about the group of symplectic automorphisms of  $S^2 \times S^2$ . In particular, Seidel shows that the inclusion of the group of symplectic diffeomorphisms into the group of all diffeomorphisms induces a non-injective map on some homotopy groups.

Susan Tolman and Jonathan Weitsman consider a symplectic manifold with a Hamiltonian torus action, and give a simple proof of the Goresky-Kottwitz-MacPherson formula for the image of the homomorphism induced between equivariant cohomology of the manifold and the fixed point set of the torus action.

Marcin Poźniak was a PhD student of Dietmar Salamon. He died on 29 November 1996 as a result of a tragic accident. As Salamon remembers, "Poźniak began his graduate studies at Warwick in October 1989. For his MSc degree he worked on triangulations and Morse theory, and his PhD thesis, completed in 1994, was devoted to the theory of so-called clean Lagrangian intersections. After his doctoral studies Marcin Poźniak returned to his position at the Jagellonian University of Krakow. Marcin Poźniak was a very thoughtful person and a talented young mathematician. He had many friends among the students and staff at Warwick. We shall miss him very much." Because of his tragic death, Poźniak's PhD thesis remained unpublished, although its results have been used and quoted by many authors working in the area. Thus we consider it appropriate to publish Poźniak's thesis in this volume. The manuscript was prepared for publication by Davide Castelvecchi.

The Editors

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