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Advances in Mathematical Analysis of Partial Differential Equations

Darya Apushkinskaya
Alexander I. Nazarov
Editors
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Dedicated to the memory of
Olga Aleksandrovna Ladyzhenskaya
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Preface

This is a volume in memory of the outstanding Russian mathematician Olga Alexandrovna Ladyzhenskaya (1922–2004). She was one of the founders of the famous Leningrad (St. Petersburg) school of PDE’s and Mathematical Physics. Headed by Olga Alexandrovna, this school has made many fundamental contributions to the theory of PDE’s as we know it today.

As a genuine classic, Olga Alexandrovna had a great variety of topics in PDE’s and in Mathematical Physics to study on her own or together with her numerous students. Among the most important ones were problems from the spectral theory for differential operators, the convergence of finite difference methods, the calculus of variations, the theory of quasilinear equations, the mathematical fluid dynamics, and the theory of attractors for PDE’s. The results she achieved were spectacular and many of them still remain on the top of those areas. Olga Alexandrovna, together with her student Nina Nikolaevna Uraltseva, presented a complete solution to Hilbert’s 19th and 20th problems for a wide class of second order PDE’s.

It is not a secret that Olga Alexandrovna’s main mathematical “love” was a rigorous theory of fluid dynamics. Her very influential book *The Mathematical Theory of Viscous Incompressible Flow*, published in 1961, was translated into many languages and has become a classic in the field. Up to the present, it is an excellent introduction to the mathematical foundations of hydrodynamics.

Some of Olga Alexandrovna’s achievements related to the mathematical theory of the Navier–Stokes equations will be remembered the longest. The global well posedness for those equations in dimension 2 proved in 1950’ies is one of her great results of this kind. As it has been shown by her in collaboration with A. A. Kiselev, a similar result is valid in dimension 3 as well but holds only on a finite interval of time. It is worthy to notice that the global well posedness for the three-dimensional problem remains open up to this day. Moreover, the issue of existence and uniqueness of physically reasonable solutions to the NSEs in three dimensions has been chosen as one of the seven Millenium “million dollar” Prize Problems of the Clay Mathematical Institute.

Thinking on turbulence, she proved the existence of an invariant manifold attracting any bounded set of initial data for the 2D Navier-Stokes equations in the pioneering paper written in 1972. This result came to the attention not only of mathematicians, but of theoretical physicists as well. Moreover, this was a starting point for her to develop the theory of attractors of infinite-dimensional dynamical systems.

Olga Alexandrovna Ladyzhenskaya published more than 250 articles and authored or coauthored 7 monographs and a textbook. She has received many honors.
and awards, including the highest award of the Russian Academy of Sciences, the Lomonosov Gold Medal.

The present volume contains the proceedings of the international Workshop “Advances in Mathematical Analysis of Partial Differential Equations” held in the Institute Mittag-Leffler, Stockholm, Sweden, July 9–13, 2012. The workshop was partially supported by the European Mathematical Society and dedicated to the 90th anniversary of the O. A. Ladyzhenskaya birthday. The workshop has been attended by well-known experts in the field of partial differential equations from 10 countries. They came to honor Olga Alexandrovna Ladyzhenskaya’s memory and to pay tribute to her scientific legacy.

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