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Complex Differential Geometry

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Preface

Complex manifold has been sitting in the overlap of quite a few branches of mathematics, such as differential geometry, algebraic geometry, several complex variables, global analysis, topology, algebraic number theory, mathematical physics, etc..

On one hand, complex manifolds provide a rich class of geometric objects. For instance, the (common) zero locus of any (generic set of) complex polynomials is always a complex manifold. In fact, until very recently, the connected sum of simply connected algebraic surfaces has been the only known examples of simply connected smooth 4-manifolds. On the other hand, complex manifolds behave rather differently than generic smooth manifolds. They are much more coherent. The existence of a complex structure often imposes topological consequences on the underlying smooth manifold. For instance, only a rather special kind of compact smooth manifold can admit a Kählerian complex structure. As another example, the group of biholomorphisms of a compact complex manifold is finite dimensional, while the diffeomorphism group of a smooth manifold is always infinite dimensional.

This rich yet restrictive character makes complex manifold a rather special and interesting object of study. This book is about the differential geometric aspects of complex manifolds. My main goal is to provide a self-contained, comprehensive approach to the subject, and saving the readers from traveling back and forth between two sets of books in learning the subject. Also, in the last chapter, we include some more recent results in the field, most of which are not yet collected in textbooks.

The book is divided into three parts, each contains three chapters. The first part contains standard materials from general topology, differentiable manifold, and basic Riemannian geometry. The second part discusses complex manifolds and analytic varieties, sheaves and holomorphic vector bundles, as well as a brief account of the surface classification theory, which provide the readers with some concrete examples of complex manifolds. The last part is the main purpose of the book, in which we discuss metric, connection, curvature and the various roles they played in the study of complex manifolds. I also collected some exercises at the end of each chapter. Most of them are ranging from straight forward to medium challenging, and serve the purpose of enhancing the comprehension of the topics. A few of them are more challenging, and could be regarded as questions leading to supplementary readings.

Part of the materials in this book have been used by the author in the lectures of a graduate course given at Ohio State University in 1998/99, as well as in a mini course taught in the 1998 summer school at Nanjing University. I would like to thank the students who participated for their feedback and suggestions, which made the book relatively more focused and readable. I would also like to thank G. Tian, H. Wu and S-T Yau for several valuable corrections and suggestions towards the manuscript. Finally, I would like to thank NSF, NSA, Sloan Foundation and the Ohio State University for the financial support, which made the writing of this book possible.

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