Preface

Introduction to Analysis is designed to bridge the gap between the intuitive calculus normally offered at the undergraduate level and the sophisticated analysis courses the student encounters at the senior or first-year-graduate level. Through a rigorous approach to the usual topics handled in one-dimensional calculus—limits, continuity, differentiation, integration, and infinite series—the book offers a deeper understanding of the ideas encountered in the calculus. Although the text assumes that the reader has completed several semesters of calculus, this assumption is necessary only for some of the motivation (of theorems) and examples.

The book has been written with two important goals in mind for its readers: the development of a rigorous foundation for the basic topics of analysis, and the less tangible acquisition of an accurate intuitive feeling for analysis. In the interest of these goals, considerable time is devoted to motivating and developing new concepts. Economy of space is often sacrificed so that ideas can be introduced in a natural fashion.

This 5th edition contains a number of changes recommended by the reviewers and users of earlier editions of the book. Chapter 0 contains introductory material on sets, functions, relations, mathematical induction, recursion, equivalent and countable sets, and the set of real numbers. As in the 4th edition, the set of real numbers is postulated as an ordered field with the least upper bound property. Chapters 1 through 4 contain the material on sequences, limits of functions, continuity, and differentiation. Chapter 5 is devoted to the Riemann integral, rather than the Riemann–Stieltjes integral treated in the first edition. Chapter 6 treats infinite series, and Chapter 7 contains material on sequences and series of functions.

The exercise sets offer a selection of exercises with level of difficulty ranging from very routine to quite challenging. The starred exercises are of particular importance, because they contain facts vital to the development of later sections. The star is *not* used to indicate the more difficult exercises.

At the end of each chapter, you will find several PROJECTS. The purpose of a PROJECT is to give the reader a substantial mathematical problem and the necessary guidance to solve that problem. A PROJECT is distinguished from an exercise in that the PROJECT is a multi-step problem that can be very difficult for the beginner without significant assistance. PROJECTS are sometimes used to cover material not included in the chapter discussions or to offer alternative methods for proving important theorems. For example, PROJECT 2.2 includes left- and right-hand limits. PROJECT 3.1 offers an approach to uniform continuity without the use of compactness. Other PROJECTS are used to generalize theorems and proofs in the text.

In the course of this exposition, a number of famous names are mentioned: Cauchy, Bolzano, Weierstrass, Riemann, Caratheodory, and others. A serious student should seek to know something about the persons who have made important contributions to analysis. The reader is urged to indulge in a little historical research when encountering the names of these people.

TO THE INSTRUCTOR

Suppose you have just been assigned to teach the junior-senior level course in analysis this coming semester. The designated text will be *Introduction to Analysis*, 5th Edition. What should you know before and during the planning of the course you will offer to your students?

First of all, you should be aware of some of the special features of the book. Each chapter is divided into sections and the exercises are grouped in a similar fashion. So, for example, if you are discussing Section 2.3 in class, the exercises related to that material will be found in Section 2.3 of the exercise set. As mentioned previously, some exercises are starred to indicate that they contain facts vital to later material. The starred exercises are not necessarily the most difficult ones.

An INSTRUCTOR'S MANUAL is available to assist you in planning your course. It contains suggestions for solutions and comments concerning exercises and PROJECTS. It allows you to see at a glance what is needed for the solution of a particular exercise and assist you in deciding when to assign a problem and what hints you might need to offer your students. PROJECTS are multi-stage problems and probably should be assigned over a period of several class meetings. These problems are broken down into small steps that eventually lead to the solution. In the early part of the semester, you may need to lead your class step-by-step, but as students gain confidence, they will find success comes more easily.

Some of the PROJECTS are designed to offer you alternatives in approaching certain important theorems. In Chapter 1, the Bolzano-Weierstrass Theorem is proven and used to show that every Cauchy sequence is convergent. PROJECT 1.5 and PROJECT 1.7 offer two ways of obtaining the convergence of Cauchy sequences without the need for the Bolzano-Weierstrass Theorem. In Chapter 3, PROJECT 3.1 gives a proof without the use of compact sets that a function continuous on a closed and bounded set is uniformly continuous. In addition, PROJECT 3.1 yields the result that the continuous image of a closed and bounded set is also closed and bounded.

As you plan for the coming semester, you will need to make some choices. Unless your class is exceptional, it will be impossible to cover the entire book in one semester. So, what do you cover, and in what order? Most students will have some familiarity with the contents of Sections 0.1, 0.2, and 0.3. How much time you devote to those sections will depend on the extent of that familiarity. Section 0.5 is especially critical as many students have little appreciation of the structure of the system of real numbers as a complete ordered field. For that reason, a comprehensive coverage of Section 0.5 is crucial. Chapter 1, *Sequences*, is vital to the study of analysis and deserves careful attention. At this stage, you have some choices. Chapter 6, *Infinite Series*, is almost independent of Chapters 2–5, enough so that you can go directly from Chapter 1 to Chapter 6 if you so desire. Otherwise, Chapters 2, 3, 4, and 5, should be covered in that order.

This text is designed to offer you flexibility in designing your course and assistance in that effort. I hope you enjoy teaching this course as much as I enjoyed writing the book. If you do, I'm sure your students will share the pleasure with you.

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Edward D. Gaughan