
Preface

The first time I cracked open a differential equations text, it was instant love. I do not remember the exact title of the book, or the author's name, but I do remember that the book was thinner than the ones recently selected for me to use in introductory courses. It was a book a student could read from cover to cover, while taking a course in the subject. I began to write course notes, with the aim of producing a text more to my liking. After a few years of this, the current book emerged.

This book has four chapters. I use Chapter 1 and parts of Chapters 2 and 3 for a first semester introduction to differential equations, and I use the rest of Chapters 2 and 3 together with Chapter 4 for the second semester.

Chapter 1 deals with single differential equations, first equations of order 1,

$$(1) \quad \frac{dx}{dt} = f(t, x),$$

then equations of order 2,

$$(2) \quad \frac{d^2x}{dt^2} = f(t, x, x').$$

We have a brief discussion of higher order equations. For second order equations, we concentrate on the case

$$(3) \quad \frac{d^2x}{dt^2} = f(x, x'),$$

which can be reduced to a first order equation for $v = x'$, as a function of x . Newton's law $F = ma$ for motion of a particle on a line gives such equations. We also specialize (2) to the linear case,

$$(4) \quad x'' + bx' + cx = f(t),$$

and discuss techniques for solving such equations.

While the study of single equations is the place to start, the subject of differential equations is and always has been mainly about *systems* of equations. This study requires a healthy dose of linear algebra. For a number of good reasons, it is not desirable to require a course in linear algebra as a prerequisite (or even a corequisite) for a course in differential equations, but rather the course includes some basic instruction in linear algebra. Chapter 2 provides the needed minicourse in linear algebra. We differ from most introductions to differential equations in providing complete proofs of the relevant results, including material on determinants, eigenvalues and eigenvectors of a linear transformation, and also generalized eigenvectors.

Chapter 3 deals with linear systems of differential equations. We start with the $n \times n$ system

$$(5) \quad \frac{dx}{dt} = Ax, \quad x(0) = x_0 \in \mathbb{C}^n,$$

where A is an $n \times n$ matrix, and define the matrix exponential e^{tA} , which produces the solution

$$(6) \quad x(t) = e^{tA}x_0.$$

Material from Chapter 2 plays a central role in analyzing this matrix exponential. We proceed from (5) to the inhomogeneous system

$$(7) \quad \frac{dx}{dt} = Ax + f(t), \quad x(0) = x_0.$$

We also study variable coefficient equations

$$(8) \quad \frac{dx}{dt} = A(t)x + f(t).$$

In particular, we study power series expansions for the solution, when $A(t)$ and $f(t)$ are given by convergent power series. We also consider expansions when (8) has a “regular singular point.” These power series topics are usually introduced in the context of a single second order equation, before the study of systems (and indeed, Chapter 1, §15 contains some exercises on this). We have moved the study here, both to speed the introduction to systems and because the presentation in the system context is simultaneously more compact and more general than in the context of a single, second order equation.

Chapter 4 crowns the text, with a study of nonlinear systems of differential equations. We begin with a general existence and uniqueness result, and proceed to some qualitative studies. This brings in the phase portrait,

which depicts the behavior of solution curves (also called orbits) for nonlinear $n \times n$ systems. From the point of view of visualization, the portraits work particularly well for $n = 2$, and are also quite useful for $n = 3$. We study a variety of problems from mathematical physics, and also some problems arising in mathematical biology. Numerical methods are also introduced in this chapter. We close with some results on systems with chaotic dynamics, which arise in dimension ≥ 3 .

We follow this introduction with a record of some standard notation that will be in use throughout the text.

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