
Contents

Preface	ix
Chapter 1. Single Differential Equations	1
§1. The exponential and trigonometric functions	3
§2. First order linear equations	15
§3. Separable equations	19
§4. Second order equations – reducible cases	25
§5. Newton’s equations for motion in 1D	27
§6. The pendulum	31
§7. Motion with resistance	38
§8. Linearization	40
§9. Second order constant coefficient linear equations – homo- geneous	42
§10. Nonhomogeneous equations I – undetermined coefficients	48
§11. Forced pendulum – resonance	54
§12. Spring motion	58
§13. RLC circuits	60
§14. Nonhomogeneous equations II – variation of parameters	63
§15. Variable coefficient second order equations	66
§16. Higher order linear equations	72
§A. Where Bessel functions come from	75

Chapter 2. Linear Algebra	79
§1. Vector spaces	80
§2. Linear transformations and matrices	83
§3. Basis and dimension	88
§4. Matrix representation of a linear transformation	94
§5. Determinants and invertibility	97
§6. Eigenvalues and eigenvectors	107
§7. Generalized eigenvectors and the minimal polynomial	109
§8. Triangular matrices	116
§9. Inner products and norms	120
§10. Norm, trace, and adjoint of a linear transformation	126
§11. Self-adjoint and skew-adjoint transformations	130
§12. Unitary and orthogonal transformations	134
§A. The Jordan canonical form	140
§B. Schur's upper triangular representation	142
§C. The fundamental theorem of algebra	142
Chapter 3. Linear Systems of Differential Equations	145
§1. The matrix exponential	146
§2. Exponentials and trigonometric functions	157
§3. First order systems derived from higher order equations	160
§4. Nonhomogeneous equations and Duhamel's formula	163
§5. Simple electrical circuits	167
§6. Second order systems	171
§7. Curves in \mathbb{R}^3 and the Frenet-Serret equations	179
§8. Variable coefficient systems	186
§9. Variation of parameters and Duhamel's formula	192
§10. Power series expansions	195
§11. Regular singular points	205
§A. Logarithms of matrices	219
Chapter 4. Nonlinear Systems of Differential Equations	223
§1. Existence and uniqueness of solutions	225
§2. Dependence of solutions on initial data and other parameters	236

§3. Vector fields, orbits, and flows	240
§4. Gradient vector fields	259
§5. Newtonian equations	267
§6. Central force problems and two-body planetary motion	272
§7. Variational problems and the stationary action principle	283
§8. The brachistochrone problem	294
§9. The double pendulum	299
§10. Momentum-quadratic Hamiltonian systems	304
§11. Numerical study – difference schemes	310
§12. Limit sets and periodic orbits	318
§13. Predator-prey equations	330
§14. Competing species equations	345
§15. Chaos in multidimensional systems	352
§A. The derivative in several variables	370
§B. Convergence, compactness, and continuity	374
§C. Critical points that are saddles	378
§D. Periodic solutions of $x'' + x = \varepsilon\psi(x)$	388
§E. A dram of potential theory	395
§F. Brouwer's fixed-point theorem	399
Bibliography	403
Index	407