

Preface

This third edition is an introduction to partial differential equations for students who have finished calculus through ordinary differential equations. The book provides physical motivation, mathematical method, and physical application. Although the first and last are the *raison d'être* for the mathematics, I have chosen to stress the systematic solution algorithms, based on the methods of separation of variables and Fourier series and integrals. My goal is to achieve a lucid and mathematically correct approach without becoming excessively involved in analysis *per se*. For example, I have stressed the interpretation of various solutions in terms of asymptotic behavior (for the heat equation) and geometry (for the wave equation).

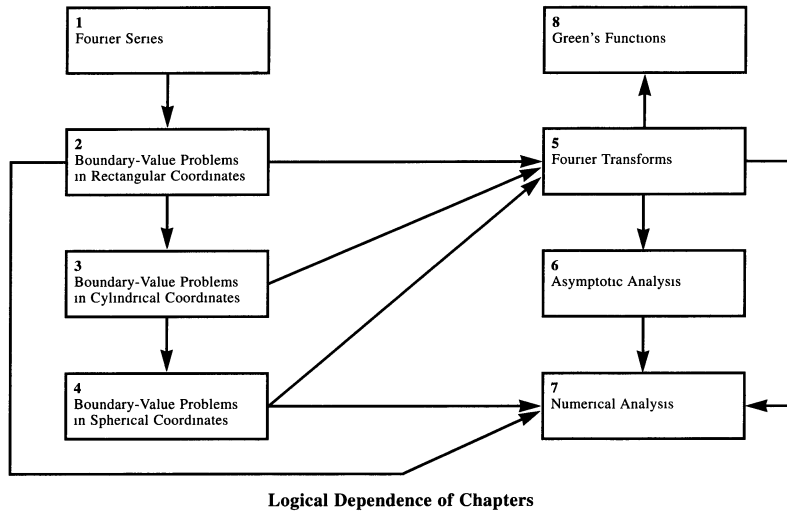
This new edition builds upon the solid strengths of the previous editions and provides a more patient development of the core concepts. Chapters 0 and 1 have been reorganized and refined to provide more complete examples that will help students master the content. For example, the Sturm-Liouville theory has been rewritten and placed at the end of Chapter 1 just before it is used in Chapter 2. The coverage of infinite series and ordinary differential equations, formerly in Chapter 0, has been moved to appendixes. In addition, we have integrated the applications of Mathematica into the text because computer-assisted methods have become increasingly important in recent years. The previous edition of this text made Mathematica applications available for the first time in a book at this level, and this edition continues this coverage. Each section of the book contains numerous worked examples and a set of exercises. These exercises have been kept to a uniform level of difficulty, and solutions to nearly 450 of the 700 exercises in the text have been provided.

Chapter 0 is a brief introduction to the entire subject of partial differential equations and some technical material that is used frequently throughout the book. Chapters 1 to 4 contain the basic material on Fourier series and boundary-value problems in rectangular, cylindrical, and spherical coordinates. Bessel and Legendre functions are developed in Chapters 3 and 4 for those instructors who want a self-contained development of this material. Instructors who do not wish to use the material on boundary-value problems should cover only Secs. 3.1 and 4.1 in Chapters 3 and 4. These sections contain several interesting boundary-value problems that can be solved without the use of Bessel or Legendre functions.

Chapter 5 develops Fourier transforms and applies them to solve problems in unbounded regions. This material, which may be treated immediately following Chapter 2 if desired, uses real-variable methods. The student is referred to a subsequent course for complex-variable methods.

The student who has finished all the material through Chapter 5 will have a good working knowledge of the classical methods of solution. To complement these basic techniques, I have added chapters on asymptotic analysis (Chapter 6),

numerical analysis (Chapter 7), and Green's functions (Chapter 8) for instructors who may have additional time or wish to omit some of the earlier material. The accompanying flowchart plots various paths through the book.



Chapters 1 and 2 form the heart of the book. They begin with the theory of Fourier series, including a complete discussion of convergence, Parseval's theorem, and the Gibbs phenomenon. We work with the class of piecewise smooth functions, which are infinitely differentiable except at a finite number of points, where all derivatives have left and right limits. Despite the generous dose of theory, it is expected that the student will learn to compute Fourier coefficients and to use Parseval's theorem to estimate the mean square error in approximating a function by the partial sum of its Fourier series. Chapter 1 concludes with Sturm-Liouville theory, which will be used in Chapter 2 and repeatedly throughout the book.

Chapter 2 takes up the systematic study of the wave equation and the heat equation. It begins with steady-state and time-periodic solutions of the heat equation in Sec. 2.1, including applications to heat transfer and to geophysics, and follows with the study of initial-value problems in Secs. 2.2 and 2.3, which are treated by a five-stage method. This systematic breakdown allows the student to separate the steady-state solution from the transient solution (found by the separation-of-variables algorithm) and to verify the uniqueness and asymptotic behavior of the solution as well as to compute the relaxation time. I have found that students can easily appreciate and understand this method, which combines mathematical precision and clear physical interpretation. The five-stage method

is used throughout the book, in Secs. 2.5, 3.4, and 4.1. Chapter 2 also includes the wave equation for the vibrating string (Sec. 2.4), solved both by the Fourier series and by the d'Alembert formula. Both methods have advantages and disadvantages, which are discussed in detail. My derivations of both the wave equation and the heat equation are from a three-dimensional viewpoint, which I feel is less artificial and more elegant than many treatments that begin with a one-dimensional formulation.

Following Chapter 2, there is a wide choice in the direction of the course. Those instructors who wish to give a complete treatment of boundary-value problems in cylindrical and spherical coordinates, including Bessel and Legendre functions, will want to cover all of Chapters 3 and 4. Other instructors may ignore this material completely and proceed directly to Chapter 5, on Fourier transforms. An intermediate path might be to cover Secs. 3.1 and/or 4.1, which treat (respectively) Laplace's equation in polar coordinates and spherically symmetric solutions of the heat equation in three dimensions. Neither topic requires any special functions beyond those encountered in trigonometric Fourier series.

Chapter 5 treats Fourier transforms using the complex exponential notation. This is a natural extension of the complex form of the Fourier series, which is covered in Sec. 1.5. Using the Fourier transform, I reduce the heat, Laplace, wave, and telegraph equations to ordinary differential equations with constant coefficients, which can be solved by elementary methods. In many cases, these Fourier representations of the solutions can be rewritten as explicit representations (by what is usually known as the Green function method). The method of images for solving problems on a semi-infinite axis is naturally developed here. The Green functions methods are developed more systematically in Chapter 8. After preparing the one-dimensional case, I give a self-contained treatment of the explicit representation of the solution of Poisson's equation in two and three dimensions. In addition to the traditional physical applications, the Black-Scholes model of option pricing from financial mathematics is included.

Throughout the book I emphasize the asymptotic analysis of *series* solutions of boundary-value problems. Chapter 6 gives an elementary account of asymptotic analysis of *integrals*, in particular the Fourier integral representations of the solutions obtained in Chapter 5. The methods include integration by parts, Laplace's method, and the method of stationary phase. These culminate in an asymptotic analysis of the telegraph equation, which illustrates the group velocity of a wave packet.

No introduction to partial differential equations would be complete without some discussion of approximate solutions and numerical methods. Chapter 7 gives the student some working knowledge of the finite difference solution of the heat equation and Laplace's equation in one and two space dimensions. The material on variational methods first relates differential equations to variational

problems and then outlines some direct methods that may be used to arrive at approximate solutions, including the finite element method.

This book was developed from course notes for Mathematics C91-1 in the Integrated Science Program at Northwestern University. The course has been taught to college juniors since 1977; Chapters 1 to 5 are covered in a 10-week quarter. I am indebted to my colleagues Leonard Evens, Robert Speed, Paul Auvil, Gene Birchfield, and Mark Ratner for providing valuable suggestions on the mathematics and its applications. The first draft was written in collaboration with Michael Hopkins. The typing was done by Vicki Davis and Julie Mendelson. The solutions were compiled with the assistance of Mark Scherer. Valuable technical advice was further provided by Edward Reiss and Stuart Antman.

In preparation of this new edition, I received valuable comments and suggestions from Andrew Bernoff, Joseph B. Keller, Thaddeus Ladd, Jeff Miller, Carl Prather, Robert Seeley, and Marshall Slemrod. I also acknowledge the reviewing services of the following individuals: David Bao, University of Houston; William O. Bray, University of Maine; Peter Colwell, Iowa State University; Kenneth A. Heimes, Iowa State University; Yinxi Huang, University of Memphis; Mohammad Kozemi, University of North Carolina–Charlotte; and William Mays, Gloucester Community College (NJ).

In preparation of the past edition, I received valuable comments and suggestions from James W. Brown, Charles Holland, Robert Pego, Mei-Chang Shen, Clark Robinson, Nancy Stanton, Athanassios Tzavaras, David Kapov, and Dennis Kosterman. For the second edition, I also acknowledge the reviewing services of the following individuals: William O. Bray, University of Maine; William E. Fitzgibbon, University of Houston; Peter J. Gingo, University of Akron; Mohammad Kozemi, University of North Carolina–Charlotte; Gilbert N. Lewis, Michigan Technical University; Geoffrey Martin, University of Toledo; Norman Meyers, University of Minnesota–Minneapolis; Allen C. Pipkin, Brown University; R. E. Showalter, University of Texas–Austin; and Grant V. Welland, University of Missouri–St. Louis.

In the preparation of the first edition I was encouraged by John Corrigan of the McGraw-Hill College Division. Preparing the second edition of this text, I benefited from the editorial services of Karen M. Hughes and Richard Wallis. Most recently, for this new edition with Waveland Press, it has been a pleasure to work with Jan Fisher and the staff of Publication Services. The current printing was completed with the editorial assistance of Miron Bekker, Harry R. Hughes, Monica Sharpnack, Nancy Stanton, and Alphonse Sterling.

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