
Preface

Preface to the Second Edition

In the second edition of this book, much of the material has been rewritten to clarify the presentation. It also has provided the opportunity for correcting many minor typographical errors or mistakes. Also, the definition of a chaotic attractor has been changed to include the requirement that the chaotic attractor is transitive. This is the usual definition and it eliminates some attractors that should not be called chaotic. Several new applications are included for systems of differential equations in Part 1. I would encourage readers to email me with suggestions and further corrections that are needed.

R. Clark Robinson
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Preface to the First Edition

This book is intended for an advanced undergraduate course in dynamical systems or nonlinear ordinary differential equations. There are portions that could be beneficially used for introductory master level courses. The goal is a treatment that gives examples and methods of calculation, at the same time introducing the mathematical concepts involved. Depending on the selection of material covered, an instructor could teach a course from this book that is either strictly an introduction into the concepts, that covers both the concepts on applications, or that is a more theoretically mathematical introduction to dynamical systems. Further elaboration of the variety of uses is presented in the subsequent discussion of the organization of the book.

The assumption is that the student has taken courses on calculus covering both single variable and multivariables, a course on linear algebra, and an introductory

course on differential equations. From the multivariable calculus, the material on partial derivatives is used extensively, and in a few places multiple integrals and surface integrals are used. (See Appendix A.1.) Eigenvalues and eigenvectors are the main concepts used from linear algebra, but further topics are listed in Appendix A.3. The material from the standard introductory course on differential equations is used only in Part 1; we assume that students can solve first-order equations by separation of variables, and that they know the form of solutions from second-order scalar equations. Students who have taken an introductory course on differential equations are usually familiar with linear systems with constant coefficients (at least the real-eigenvalue case), but this material is repeated in Chapter 2, where we also introduce the reader to the phase portrait. At Northwestern, some students have taken the course covering part one on differential equations without this introductory course on differential equations; they have been able to understand the new material when they have been willing to do the extra work in a few areas that is required to fill in the missing background. Finally, we have not assumed that the student has had a course on real analysis or advanced calculus. However, it is convenient to use some of the terminology from such a course, so we include an appendix with terminology on continuity and topology. (See Appendix A.)

Organization

This book presents an introduction to the concepts of dynamical systems. It is divided into two parts, which can be treated in either order: The first part treats various aspects of systems of nonlinear ordinary differential equations, and the second part treats those aspects dealing with iteration of a function. Each separate part can be used for a one-quarter course, a one-semester course, a two-quarter course, or possibly even a year course. At Northwestern University, we have courses that spend one quarter on the first part and two quarters on the second part. In a one-quarter course on differential equations, it is difficult to cover the material on chaotic attractors, even skipping many of the applications and proofs at the end of the chapters. A one-semester course on differential equations could also cover selected topics on iteration of functions from Chapters 9–11. In the course on discrete dynamical systems using Part 2, we cover most of the material on iteration of one-dimensional functions (Chapters 9–11) in one quarter. The material on iteration of functions in higher dimensions (Chapters 12–13) certainly depends on the one-dimensional material, but a one-semester course could mix in some of the higher dimensional examples with the treatment of Chapters 9–11. Finally, Chapter 14 on fractals could be treated after Chapter 12. Fractal dimensions could be integrated into the material on chaotic attractors at the end of a course on differential equations. The material on fractal dimensions or iterative function systems could be treated with a course on iteration of one-dimensional functions.

The main concepts are presented in the first sections of each chapter. These sections are followed by a section that presents some applications and then by a section that contains proofs of the more difficult results and more theoretical material. The division of material between these types of sections is somewhat arbitrary. The theorems proved at the end of the chapter are restated with their

original theorem number. The material on competitive populations and predator–prey systems is contained in one of the beginning sections of the chapters in which these topics are covered, rather than in the applications at the end of the chapters, because these topics serve to develop the main techniques presented. Also, some proofs are contained in the main sections when they are more computational and serve to make the concepts clearer. Longer and more technical proofs and further theoretical discussion are presented separately at the end of the chapter.

A course that covers the material from the primary sections, without covering the sections at the end of the chapter on applications and more theoretical material, results in a course on the concepts of dynamical systems with some motivation from applications.

The applications provide motivation and illustrate the usefulness of the concepts. None of the material from the sections on applications is necessary for treating the main sections of later chapters. Treating more of this material would result in a more applied emphasis.

Separating the harder proofs allows the instructor to determine the level of theory of the course taught using this book as the text. A more theoretic course could consider most of the proofs at the end of the chapters.

Computer Programs

This book does not explicitly cover aspects of computer programming. However, a few selected problems require computer simulations to produce phase portraits of differential equations or to iterate functions. Sample Maple worksheets, which the students can modify to help with some of the more computational problems, will be available on the webpage:

<http://www.math.northwestern.edu/~clark/dyn-sys>.

(Other material on corrections and updates of the book will also be available at this website.) There are several books available that treat dynamical systems in the context of Maple or Mathematica: two such books are [Kul02] by M. Kulenović and [Lyn01] by S. Lynch. The book [Pol04] by J. Polking and D. Arnold discusses using Matlab to solve differential equations using packages available at <http://math.rice.edu/~dfield>. The book [Nus98] by H. Nusse and J. Yorke comes with its own specialized dynamical systems package.

Acknowledgments

I would like to acknowledge some of the other books I have used to teach this material, since they have influenced my understanding of the material, especially with regard to effective ways to present material. I will not attempt to list more advanced books which have also affected my understanding. For the material on differential equations, I have used the following books: F. Brauer and J. Nohel [Bra69], M. Hirsch and S. Smale [Hir74], M. Braun [Bra73], I. Percival and D. Richards [Per82], D.W. Jordan and P. Smith [Jor87], J. Hale and H. Koçak [Hal91], and S. Strogatz [Str94]. For the material on iteration of functions, I have used the following books: the two books by R. Devaney [Dev89] and [Dev92], D. Gulick [Gul92], and K. Alligood, T. Sauer, and J. Yorke [All97].

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R. Clark Robinson
clark@math.northwestern.edu
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