Preface

This book began with two upper level mathematics courses I taught at Swarthmore College: a second course in linear algebra and a course in combinatorial matrix theory. In each case, I had expected to use an existing text, but then found these did not quite fit my plans for the course. Consequently, I wrote up complete notes for the classes. Since the material on nonnegative matrices belonged in both courses and required a fair amount of graph theory, it made sense to me to combine all of these chapters into one book. Additional chapters on topics not covered in those courses were added, and here is the total.

I started with topics I view as core linear algebra for a second course: Jordan canonical form, normal matrices and the spectral theorem, Hermitian matrices, the Perron–Frobenius theorem. I wanted the Jordan canonical form theory to include a discussion of the Weyr characteristic and Weyr normal form. For the Perron–Frobenius theorem, I wanted to follow Wielandt's approach in [**Wiel67**] and use directed graphs to deal with imprimitive matrices; hence the need for a chapter on directed graphs. For the combinatorial matrix theory course, I chose some favorite topics included in Herbert Ryser's beautiful courses at Caltech: block designs, Hadamard matrices, and elegant theorems about matrices of zeros and ones. But I also wanted the book to include McCoy's theorem about Property P, the Motzkin–Taussky theorem about Hermitian matrices with Property L, the field of values, and other topics. In addition to linear algebra and matrix theory *per se*, I wanted to display linear algebra interacting with other parts of mathematics; hence a brief section on Hilbert spaces with the formulas for the Fourier coefficients, the inclusion of a proof of the Bruck–Ryser–Chowla Theorem which uses matrix theory and elementary number theory, the chapter on error-correcting codes, and the introduction to linear dynamical systems.

Do we need another linear algebra book? Aren't there already dozens (perhaps hundreds) of texts written for the typical undergraduate linear algebra course? Yes, but most of these are for first courses, usually taken in the first or second year. There are fewer texts for advanced courses. There are well-known classics, such as Gantmacher [**Gan59**], Halmos [**Hal87**], Mal'cev [**Mal63**], Hoffman and Kunze [**HK71**], Bellman [**Bell70**]. More recently, we have the excellent book by Horn and Johnson [**HJ85**], now expanded in the second edition [**HJ13**]. But, much as I admire these books, they weren't quite what I wanted for my courses aimed at upper level undergraduates. In some cases I wanted different topics, in others I wanted a different approach to the proofs. So perhaps there is room for another linear algebra book.

This book is not addressed to experts in the field nor to the would-be matrix theory specialist. My hope is that it will be useful for those (both students and professionals) who have taken a first linear algebra course but need to know more. A typical linear algebra course taken by mathematics, science, engineering, and economics majors gets through systems of linear equations, vector spaces, inner products, determinants, eigenvalues and eigenvectors, and maybe diagonalization of symmetric matrices. But often in mathematics and applications, one needs more advanced results, such as the spectral theorem, Jordan canonical form, the singular value decomposition.

Linear algebra plays a key role in so many parts of mathematics: linear differential equations, Fourier series, group representations, Lie algebras, functional analysis, multivariate statistics, etc. If the two courses mentioned in the first paragraph are the parents of this book, the aunts and uncles are courses I taught in differential equations, partial differential equations, number theory, abstract algebra, error-correcting codes, functional analysis, and mathematical modeling, all of which used linear algebra in a central way. Thank you to all the chairs of the Swarthmore College Mathematics Department for letting me teach such a variety of courses. In the case of the modeling course, a special thank you to Thomas Hunter for gently persuading me to take this on when no one else was eager to do it.

One more disclaimer: this is not a book on numerical linear algebra with discussion of efficient and stable algorithms for actually computing eigenvalues, normal forms, etc. I occasionally comment on the issues involved and the desirability of working with unitary change of basis, if possible, but my acquaintance with this side of the subject is very limited.

This book does not contain new results. It may contain some proofs not typically seen or perhaps not readily available elsewhere. In the proof of the Jordan canonical form, the argument for the last nitty-gritty part comes from a lecture Halmos gave at the 1993 ILAS conference. He explained that he had never been satisfied with the argument in his book and always thought there should be a more conceptual approach and that he had finally found it. My apologies if the account I give here is more complicated than necessary—my notes from the talk had a sketch of a proof and then I needed to fill in details. A former colleague, Jim Wiseman, showed me the shear argument with the spherical coordinates used in the proof that the numerical range of a 2×2 matrix is an ellipse. From Hans Schneider I learned about the connection between the eigenvalue structure for an irreducible nonnegative matrix and the cycle lengths of its directed graph, and the proof in this book starts with the graph and uses it to obtain the usual result about the eigenvalues.

I would like to thank my wonderful teachers, even though many are no longer alive to receive this thanks. Olga Taussky Todd lured me into matrix theory with her course at Caltech and graciously accepted me as her doctoral student. In addition to my direct debt to her, she told me to take Herbert Ryser's matrix theory course. Since I had already taken Ryser's combinatorics course, this meant I had the good fortune to experience two years of mathematics presented with the utmost elegance, clarity, and precision. Much of the material for the chapters on zero-one matrices, Hadamard matrices, and designs comes from my notes from these courses and from Ryser's book [**Rys63**]. After Caltech, I spent a year at the University of Wisconsin, where Hans Schneider kindly mentored me and gave me the chance to coteach a graduate matrix theory course with him. From Hans Schneider I learned of the Weyr characteristic, the connection between directed graphs and the Perron–Frobenius Theorem, and acquired the Wielandt notes [**Wiel67**]. I want to thank Charles Johnson, who hosted me for a semester of leave at the University of Maryland in 1984. Many thanks to Roger Horn, for inviting me to write the survey article [**Sha91**] on unitary similarity—this was how I came to relearn the Weyr normal form and to appreciate the power of Sylvester's theorem. I also thank him for inviting me to write the *American Mathematical Monthly* article [**Sha99**] on the Weyr normal form and his patient corrections of my many errors misusing "which" and "that". Alas, I fear I still have not mastered this.

Going back further to undergraduate years at Kenyon College, thank you to all of my college mathematics teachers. Daniel Finkbeiner introduced me to the beautiful world of abstract mathematics in a first linear algebra course, followed by a second linear algebra course and more. Thanks to Robert Fesq for his Moore method abstract algebra course—I came to this course thinking seriously of being a math major, but this was the experience that sealed the deal. Thanks to Stephen Slack, both for a wonderful freshman course and then a Moore method course in topology. And thanks to Robert McLeod, both for his courses, and for generously giving his time to supervise me in an independent reading course my second year. And thanks also to Wendell Lindstrom for his beautiful course in abstract algebra. Finally, I was fortunate to have excellent math teachers in the Philadelphia public schools; I mention here Mr. Kramer (sorry I don't know his first name) of Northeast High School, for his tenth grade math class and twelfth grade calculus class.