

---

# Contents

|  |    |
|--|----|
| Preface  | xi |
| Note to the Reader   | xv |
| Chapter 1. Preliminaries   | 1  |
| 1.1. Vector Spaces   | 1  |
| 1.2. Bases and Coordinates                                       | 3  |
| 1.3. Linear Transformations                                      | 3  |
| 1.4. Matrices  | 4  |
| 1.5. The Matrix of a Linear Transformation                       | 5  |
| 1.6. Change of Basis and Similarity                              | 6  |
| 1.7. Transposes  | 8  |
| 1.8. Special Types of Matrices                                   | 8  |
| 1.9. Submatrices, Partitioned Matrices, and Block Multiplication | 9  |
| 1.10. Invariant Subspaces  | 10 |
| 1.11. Determinants   | 11 |
| 1.12. Tensor Products  | 13 |
| Exercises  | 14 |
| Chapter 2. Inner Product Spaces and Orthogonality                | 17 |
| 2.1. The Inner Product   | 17 |
| 2.2. Length, Orthogonality, and Projection onto a Line           | 18 |
| 2.3. Inner Products in $\mathbb{C}^n$                            | 21 |
| 2.4. Orthogonal Complements and Projection onto a Subspace       | 23 |
| 2.5. Hilbert Spaces and Fourier Series                           | 27 |
| 2.6. Unitary Transformations                                     | 31 |

---

|  |     |
|--|-----|
| 2.7. The Gram–Schmidt Process and QR Factorization                           | 33  |
| 2.8. Linear Functionals and the Dual Space                                   | 35  |
| Exercises  | 36  |
| Chapter 3. Eigenvalues, Eigenvectors, Diagonalization, and Triangularization | 39  |
| 3.1. Eigenvalues   | 39  |
| 3.2. Algebraic and Geometric Multiplicity                                    | 40  |
| 3.3. Diagonalizability   | 41  |
| 3.4. A Triangularization Theorem   | 44  |
| 3.5. The Geršgorin Circle Theorem  | 45  |
| 3.6. More about the Characteristic Polynomial                                | 46  |
| 3.7. Eigenvalues of $AB$ and $BA$  | 48  |
| Exercises  | 48  |
| Chapter 4. The Jordan and Weyr Canonical Forms                               | 51  |
| 4.1. A Theorem of Sylvester and Reduction to Block Diagonal Form             | 53  |
| 4.2. Nilpotent Matrices  | 57  |
| 4.3. The Jordan Form of a General Matrix                                     | 63  |
| 4.4. The Cayley–Hamilton Theorem and the Minimal Polynomial                  | 64  |
| 4.5. Weyr Normal Form  | 67  |
| Exercises  | 74  |
| Chapter 5. Unitary Similarity and Normal Matrices                            | 77  |
| 5.1. Unitary Similarity  | 77  |
| 5.2. Normal Matrices—the Spectral Theorem                                    | 78  |
| 5.3. More about Normal Matrices  | 81  |
| 5.4. Conditions for Unitary Similarity                                       | 84  |
| Exercises  | 86  |
| Chapter 6. Hermitian Matrices  | 89  |
| 6.1. Conjugate Bilinear Forms  | 89  |
| 6.2. Properties of Hermitian Matrices and Inertia                            | 91  |
| 6.3. The Rayleigh–Ritz Ratio and the Courant–Fischer Theorem                 | 94  |
| 6.4. Cauchy’s Interlacing Theorem and Other Eigenvalue Inequalities          | 97  |
| 6.5. Positive Definite Matrices  | 99  |
| 6.6. Simultaneous Row and Column Operations                                  | 102 |
| 6.7. Hadamard’s Determinant Inequality                                       | 105 |
| 6.8. Polar Factorization and Singular Value Decomposition                    | 106 |
| Exercises  | 109 |
| Chapter 7. Vector and Matrix Norms   | 113 |
| 7.1. Vector Norms  | 113 |

---

|  |     |
|--|-----|
| 7.2. Matrix Norms  | 117 |
| Exercises  | 119 |
| Chapter 8. Some Matrix Factorizations  | 121 |
| 8.1. Singular Value Decomposition  | 121 |
| 8.2. Householder Transformations   | 127 |
| 8.3. Using Householder Transformations to Get Triangular, Hessenberg,<br>and Tridiagonal Forms | 129 |
| 8.4. Some Methods for Computing Eigenvalues  | 134 |
| 8.5. LDU Factorization   | 138 |
| Exercises  | 141 |
| Chapter 9. Field of Values   | 143 |
| 9.1. Basic Properties of the Field of Values   | 143 |
| 9.2. The Field of Values for Two-by-Two Matrices   | 145 |
| 9.3. Convexity of the Numerical Range  | 148 |
| Exercises  | 150 |
| Chapter 10. Simultaneous Triangularization   | 151 |
| 10.1. Invariant Subspaces and Block Triangularization  | 151 |
| 10.2. Simultaneous Triangularization, Property P, and Commutativity                            | 152 |
| 10.3. Algebras, Ideals, and Nilpotent Ideals   | 154 |
| 10.4. McCoy's Theorem  | 157 |
| 10.5. Property L   | 158 |
| Exercises  | 161 |
| Chapter 11. Circulant and Block Cycle Matrices   | 163 |
| 11.1. The J Matrix   | 163 |
| 11.2. Circulant Matrices   | 163 |
| 11.3. Block Cycle Matrices   | 165 |
| Exercises  | 167 |
| Chapter 12. Matrices of Zeros and Ones   | 169 |
| 12.1. Introduction: Adjacency Matrices and Incidence Matrices                                  | 169 |
| 12.2. Basic Facts about $(0, 1)$ -Matrices   | 172 |
| 12.3. The Minimax Theorem of König and Egerváry  | 173 |
| 12.4. SDRs, a Theorem of P. Hall, and Permanents   | 174 |
| 12.5. Doubly Stochastic Matrices and Birkhoff's Theorem  | 176 |
| 12.6. A Theorem of Ryser   | 180 |
| Exercises  | 182 |
| Chapter 13. Block Designs  | 185 |
| 13.1. $t$ -Designs   | 185 |

---

|   |     |
|---|-----|
| 13.2. Incidence Matrices for 2-Designs                  | 189 |
| 13.3. Finite Projective Planes                          | 191 |
| 13.4. Quadratic Forms and the Witt Cancellation Theorem | 198 |
| 13.5. The Bruck–Ryser–Chowla Theorem                    | 202 |
| Exercises   | 205 |
| Chapter 14. Hadamard Matrices                           | 207 |
| 14.1. Introduction                                      | 207 |
| 14.2. The Quadratic Residue Matrix and Paley’s Theorem  | 208 |
| 14.3. Results of Williamson                             | 212 |
| 14.4. Hadamard Matrices and Block Designs               | 216 |
| 14.5. A Determinant Inequality, Revisited               | 219 |
| Exercises   | 219 |
| Chapter 15. Graphs                                      | 221 |
| 15.1. Definitions                                       | 221 |
| 15.2. Graphs and Matrices                               | 223 |
| 15.3. Walks and Cycles                                  | 224 |
| 15.4. Graphs and Eigenvalues                            | 226 |
| 15.5. Strongly Regular Graphs                           | 227 |
| Exercises   | 232 |
| Chapter 16. Directed Graphs                             | 235 |
| 16.1. Definitions                                       | 235 |
| 16.2. Irreducibility and Strong Connectivity            | 238 |
| 16.3. Index of Imprimitivity                            | 242 |
| 16.4. Primitive Graphs                                  | 245 |
| Exercises   | 247 |
| Chapter 17. Nonnegative Matrices                        | 249 |
| 17.1. Introduction                                      | 249 |
| 17.2. Preliminaries                                     | 250 |
| 17.3. Proof of Perron’s Theorem                         | 254 |
| 17.4. Nonnegative Matrices                              | 258 |
| 17.5. Irreducible Matrices                              | 259 |
| 17.6. Primitive and Imprimitve Matrices                 | 260 |
| Exercises   | 262 |
| Chapter 18. Error-Correcting Codes                      | 265 |
| 18.1. Introduction                                      | 265 |
| 18.2. The Hamming Code                                  | 266 |
| 18.3. Linear Codes: Parity Check and Generator Matrices | 267 |

---

|   |     |
|---|-----|
| 18.4. The Hamming Distance  | 269 |
| 18.5. Perfect Codes and the Generalized Hamming Code                                  | 271 |
| 18.6. Decoding  | 273 |
| 18.7. Codes and Designs   | 274 |
| 18.8. Hadamard Codes  | 276 |
| Exercises   | 277 |
| Chapter 19. Linear Dynamical Systems  | 279 |
| 19.1. Introduction  | 279 |
| 19.2. A Population Cohort Model   | 281 |
| 19.3. First-Order, Constant Coefficient, Linear Differential and Difference Equations | 283 |
| 19.4. Constant Coefficient, Homogeneous Systems                                       | 285 |
| 19.5. Constant Coefficient, Nonhomogeneous Systems; Equilibrium Points                | 288 |
| 19.6. Nonnegative Systems   | 292 |
| 19.7. Markov Chains   | 295 |
| Exercises   | 300 |
| Bibliography  | 303 |
| Index   | 311 |