
Index

In the first part of this index, mathematical symbols and expressions are listed in their order of appearance in the book.

- \wedge , 2
- \vee , 3
- \neg , 3
- \Rightarrow , 5
- \Leftrightarrow , 6
- \mathbb{N} , 9, 10
- \forall , 9
- \exists , 9
- \in , 9, 67
- \notin , 9, 67
- \mathbb{R} , 9, 10, 197
- \mathbb{Z} , 10
- \mathbb{Q} , 10
- \mathbb{Z}^+ , 11
- \mathbb{Q}^+ , 11
- \mathbb{R}^+ , 11
- $|$, 25
- \dagger , 25
- $\exists!$, 35
- $\sum_{k=1}^n a_k$, 54
- $n!$, 57
- $\binom{n}{k}$, 61
- $\{a_n\}_{n=1}^{\infty}$, 61
- \emptyset , 69
- \subseteq , 69
- \subsetneq , 69
- \subset , 71
- \cup , 74
- \cap , 74
- $B - A$, 74
- $B \setminus A$, 74
- \overline{A} , 74
- \times , 77
- A^n , 77
- $\mathcal{P}()$, 79
- $\bigcup_{i=1}^n A_i$, 83
- $\bigcap_{i=1}^n A_i$, 83
- $\bigcup_{i=1}^{\infty} A_i$, 85
- $\bigcap_{i=1}^{\infty} A_i$, 85
- $\bigcup_{i \in I} A_i$, 87
- $\bigcap_{i \in I} A_i$, 87
- ZF, 91
- ZFC, 91
- AC, 91
- dom, 94, 156
- $f : X \rightarrow Y$, 94
- ran, 95
- im, 95
- I_X , 96
- P, 103
- 1-1, 108
- $f : X \xrightarrow{1-1} Y$, 108
- $f : X \xrightarrow[\text{onto}]{} Y$, 108
- 1-1 correspondence, 108
- f^{-1} , 117
- $f[A]$, 121
- $f^{-1}[B]$, 122
- $\mathbb{Z}^{\geq 0}$, 132
- (a, b) , 136
- $\equiv \pmod{m}$, 145
- $[]_m$, 149
- \mathbb{Z}_m , 149
- $+_m$, 151

- $-m$, 151
- \cdot_m , 151
- \mathbb{Z}_m^* , 153
- \mathbb{Z}^* , 156
- $[\]$, 158
- X/\sim , 158
- \mathbb{N}_n , 168
- \approx , 168
- $|X|$, 168
- \vdash , 171
- CH, 190
- $\max S$, 193
- $\min S$, 193
- $\sup S$, 195
- $\inf S$, 195

- absolute value function, 33
- addition principle, 174
- algebraic, 187
- and, 2
- antecedent, 6
- Archimedean Property, 202
- associative, 23, 27, 106
- axiom, 22
- Axiom of Choice, 91

- Banach-Tarski paradox, 91
- base case, 54, 57
- Basic Properties of Integers, 23
- Basic Properties of Real Numbers, 27
- Bernoulli's Inequality, 60
- biconditional, 6
- bijective, bijection, 108
- binary operation, 102
- binomial coefficient, 61
- binomial theorem, 61
- bounded, 193
 - above, 193
 - below, 193
- cancellation, 23
- Cantor-Schröder-Bernstein Theorem, 188
- cardinality, 168, 188
 - well-defined, 171
- Cartesian product, 77
- circular argument, 45
- closed formula, 62
- closure, 23, 27
- codomain, 94
- coefficient, 96
- commutative, 23, 27
- complement, 74
- Completeness Axiom, 197
- conclusion, 6
- conditional, 5
- congruence class mod m , 149
- congruent mod m , 145
- conjunct, 2
- conjunction, 2
- consequent, 6
- Continuum Hypothesis, 189
- contradiction, 14, 39
- contrapositive, 8
- converse, 8
- coprime, 141
- countable, 179
- counterexample, 10, 29
- cover, 163

- decimal digits, 204
- decimal expansion, 204
 - nonterminating, 204
 - terminating, 204
- decreasing, 103
- definitions
 - special status, 18
- degree, 96
- DeMorgan's Laws, 4, 77
- dense, 207
- denumerable, 179
- difference, 74
- disjoint, 75
 - pairwise, 173
- disjunct, 3
- disjunction, 3
- disprove, 29
- distributive, 23, 27
- divides, 25
- Division Algorithm, 131
- divisor, 25
 - greatest common, 136
- domain
 - function, 94
 - implicit, natural, 98
 - relation, 156
- dovetail, 180, 181

- element, 9, 67
- empty set, 69
- enumerable, 179
- equality
 - of functions, 96
 - of ordered n -tuples, 78

- of ordered pairs, 78
- of sets, 71
- equinumerous, 168
- equivalence class, 158
- equivalence relation, 157
- equivalent, 6
- Euclid's Lemma, 141
- Euclidean Algorithm, 136, 139
- even, 18
- existential instantiation, 21

- factorial function, 57
- family of sets, 84
- Fibonacci numbers, 60
- finite, 168
- free variable, 2
- function, 94
 - composite, composition, 104
 - decreasing, 103
 - equality, 96
 - increasing, 103
 - inverse, 114
 - invertible, 114
 - polynomial, 96
 - rational, 98
 - real-valued, 98
 - well-defined, 152, 162
- Fundamental Theorem of Arithmetic,
 - 45
 - Existence, 64
 - Uniqueness, 142

- gcd, 136
- Given-Goal diagram, 19
- graph, 95
- group, 152

- hypothesis, 6

- identity function, 96
- if, 6
- if . . . , then, 5
- iff, 6
- image, 94, 95
 - of a set under a function, 121
- implication, 5
- implies, 5
- increasing, 103
- index set, 84
- indexed family of sets, 84
- induction, 53

- inductive definition, 57
- inductive hypothesis, 55
- inductive step, 54, 57
- infimum, 195
- infinite, 168
- injective, injection, 108
- integers, 10
- integers mod m , 149
- intersection, 74
- interval, 201
- interval notation, 73
- inverse
 - left, 121
 - right, 121
- inverse function, 114
- inverse image
 - of a set under a function, 122
- invertible, 114
 - left-, 121
 - right-, 121
- irrational number, 40

- linear combination, 138
- logical connectives, 2, 5
- logically equivalent, 4, 14
- lower bound, 193
 - greatest, 195

- maps to, 94
- matrix, 37
 - multiplication, 38
- maximum, 193
- metric, 104
- minimum, 193
- modus ponens*, 16
- multiplication principle, 174

- natural numbers, 9
- necessary, 6
- negation, 3
- not, 3
- n -tuple, 77
- null set, 69

- odd, 18
- one-to-one, 108
- only if, 6
- onto, 108
- or, 3
 - exclusive, 3
 - inclusive, 3

- ordered field, 192
- ordered pair, 77
- partition, 162
- Pigeonhole Principle, 171
- PMI, 53
- polynomial, 96
- power set, 79
- predicate, 9
- preimage, 94
 - of a set under a function, 122
- prime, 29, 45
- Principle of Mathematical Induction, 53
 - modified, 58
- Principle of Strong Mathematical Induction, 63
- proof
 - by cases, 30
 - by contradiction, 38
 - direct, 17
 - indirect, 38
 - of the contrapositive, 41
 - uniqueness, 34
- proposition, 1
- PSMI, 63
- quantifier, 9
 - alternating, 47
 - existential, 9
 - hidden, 12
 - implied, 12
 - mixed, 47
 - modified, 12
 - universal, 9
- range, 95
- rational function, 98
- rational numbers, 10
- real numbers, 10, 197
 - algebraic, 187
 - transcendental, 187
- real-valued, 98
- recursion step, 57
- recursive definition, 57
- reflection, 104
- reflexive, 157
- relation, 155
 - binary, 155
- relatively prime, 141
- restriction, 171
- rigid motion, 103
- rule of deduction, 16
- Russell's paradox, 90
- sequence, 61
- set
 - complement, 74
 - conditional definition, 68
 - constructive definition, 68
 - difference, 74
 - empty, 69
 - equality, 71
 - indexed family, 84
 - intersection, 74
 - null, 69
 - power, 79
 - symmetric difference, 82
 - union, 74
- sigma notation, 54
- solution, 37
- statement letter, 2
- subset, 69
 - proper, 71
- sufficient, 6
- supremum, 195
- surjective, surjection, 108
- symmetric, 157
- symmetric difference, 82
- tautology, 14
- transcendental, 187
- transitive, 23, 26, 27, 157
- translation, 104
- triangle inequality, 37
- trichotomy, 5, 23, 27
- truth table, 2
- uncountable, 179
- union, 74
- uniqueness proof, 34
- universal set, 67
- universe, 2
- upper bound, 193
 - least, 195
- useful denial, 5
 - conjunction, 5
 - disjunction, 5
 - implication, 7
 - quantified statements, 11
- vacuously true, 73
- well-defined, 10, 150, 152, 162
 - $+\mathbb{Q}$, $\cdot\mathbb{Q}$, 162
 - cardinality, 171

Well-Ordering Principle, 132
without loss of generality, 127
WLOG, 127
WOP, 132
working backwards, 31