
Contents

Preface	ix
To Students	xii
Acknowledgements	xiii
Chapter 1. Language, Logic, and Proof	1
1.1. Language and logic	1
1.2. Proof	16
Chapter 2. Techniques of Proof	25
2.1. More direct proofs	25
2.2. Indirect proofs: Proofs by contradiction and contrapositive	38
2.3. Two important theorems	44
2.4. Proofs of statements involving mixed quantifiers	47
Chapter 3. Induction	53
3.1. Principle of Mathematical Induction	53
3.2. Strong induction	61
Chapter 4. Sets	67
4.1. The language of sets	67
4.2. Operations on sets	74
4.3. Arbitrary unions and intersections	83
4.4. Axiomatic set theory	89
Chapter 5. Functions	93
5.1. Definitions	93
5.2. Function composition	104
5.3. One-to-one and onto functions	107

5.4. Invertible functions	114
5.5. Functions and sets	121
Chapter 6. An Introduction to Number Theory	131
6.1. The Division Algorithm and the Well-Ordering Principle	131
6.2. Greatest common divisors and the Euclidean Algorithm	135
6.3. Relatively prime integers and the Fundamental Theorem of Arithmetic	141
6.4. Congruences	145
6.5. Congruence classes	149
Chapter 7. Equivalence Relations and Partitions	155
7.1. Introduction	155
7.2. Equivalence relations	157
7.3. Partitions	162
Chapter 8. Finite and Infinite Sets	167
8.1. Introduction	167
8.2. Finite sets	170
8.3. Infinite sets	179
8.4. What next?	188
Chapter 9. Foundations of Analysis	191
9.1. Introduction	191
9.2. The Completeness Axiom	193
9.3. The Archimedean Property and its consequences	202
9.4. What next?	207
Appendix. Writing Mathematics	209
Bibliography	211
Index	213