
Index

- absolute value, 60
- accumulation point, 117, 123
- additive inverse, 48
- \aleph_0 , 64
- algebraic number, 69
- alternating series, 232
- antiderivative, 214
- Archimedean property, 57
- Axioms for Addition, 48
- Axioms for Multiplication, 48

- Banach Fixed Point Theorem, 169
- belongs, 10
- Bernoulli's Inequality, 53
- biconditional, 7
- bijection, 23
- bijective, 3
- binomial
 - coefficient, 52
 - formula, 52
- Bolzano–Weierstrass theorem, 97
- bounded
 - above, 55
 - below, 55
 - in a metric space, 138
 - set, 55
- Bézout's identity, 38

- Cantor function, 252
- Cantor set, 130, 134
- Cantor–Schröder–Bernstein theorem, 36
- $\text{card}(A)$, 33
- cardinality, 33
- cardinality \aleph_0 , 64

- Cartesian product, 3, 20
- Cassini identity, 38
- Cauchy complete, 141
- Cauchy product, 273
- Cauchy property, 104, 141
- Cauchy sequence, 104, 141
- Cauchy, uniform, 244
- Cauchy–Schwartz inequality, 113
- closed ball, 120
- closed interval, 60
- closed set, 116
- closed, metric space, 122
- closed, relative, 122
- closure, 118, 123
- closure point, 117, 123
- codomain, 3, 22
- common refinement partition, 200
- compact, 126
- compact metric space, 139
- complement, 2, 15
- complete, 141
- completeness property, 57
- complex addition, 75
- complex conjugate, 75
- complex exponential, 281
- complex multiplication, 75
- complex number, 74
- composition, 3, 25
- conclusion, 6
- conditional, 6
- conjunction, 5
- connected, 136
- connected component, 143

- connected metric space, 141
 contains, 10
 continuous, 145
 at a point, 145
 on a set, 145
 uniformly, 164
 continuously differentiable, 250
 contraction, 168
 contrapositive, 7
 converge, 80
 converge absolutely, 234
 converge conditionally, 234
 convergence, metric spaces, 122
 converges pointwise, 241
 converse, 7
 convex function, 190
 corresponding points, 198
 cosine, 270
 countable, 62
 countable family, 66
 countable open cover, 126
 countably infinite, 62
 cover, 126
 cut corresponding to a rational number, 72
 cut property, 103
 Darboux's Intermediate Value Theorem, 189
 De Morgan's laws, 17, 70
 decreasing
 function, 152
 sequence, 89
 Dedekind cut, 71
 dense, 59
 dense in a metric space, 142
 derivative, 176
 higher order, 190
 derived set, 117
 differentiable at c , 176
 Dini's theorem, 254
 direct method of proof, 12
 Dirichlet's function, modified, 152
 Dirichlet's function, 146
 disconnected, 136
 discontinuous at a point, 145
 discontinuous on a set, 145
 disjoint, 15, 66
 disjunction, 5
 Distributive Axiom, 49
 diverge to infinity, 88
 divergent series, 223
 diverges, 81
 domain, 3, 22
 Dominated Convergence Theorem, 252
 dot product, 110
 dyadic intervals, 95
 dyadic rational, 95
 element, 10
 empty set, 2, 11
 ε -close to L , 79
 epsilon close, 145
 equivalence class, 22
 equivalence relation, 22
 equivalent metric, 140
 Euler's constant, 267
 Euler's constant e , 106
 Euler's identity, 282
 existential quantifier, 8
 exponential function, 267
 Fibonacci sequence, 34
 field, 49
 finer partition, 200
 finite set, 33
 fixed point, 157
 Fixed Point Theorem,
 Banach, 169
 for all, \forall , 8
 function, 22
 continuous, 145
 differentiable, 176
 Lipschitz, 152
 monotone, 152
 monotone decreasing, 152
 monotone increasing, 152
 period, 256
 Riemann integrable, 200
 strictly increasing, 152
 fundamental period, 256
 Fundamental Theorem of Arithmetic, 36
 Fundamental Theorem of Calculus, 214
 gamma function, 272
 geometric series, 99
 geometric sum, 53
 global maximum, 155
 global minimum, 155
 greatest common divisor, 35
 harmonic series, 231
 Hausdorff metric, 170
 Heine–Borel theorem, 127

- higher order derivatives, 190
- hypothesis, 6

- image, 3
- imaginary part, 75
- implication, 6
- implies, 6
- includes, 10
- increasing
 - function, 152
 - monotone increasing function, 152
 - sequence, 89
- induction, 29
- inductive hypothesis, 29
- infimum, 56
- infinite set, 33
- injective, 3, 23
- integers, 2
- interior, 119, 123
- intersection, 2, 15, 66
- interval, 60
- inverse
 - left, 27
 - right, 27
- inverse function, 24
- inverse image, 3, 25
- inverse image of a set, 25
- invertible, 3, 24
- irrational, 57
- is in, 10
- isolated point, 117
- isometry, 168
- iterate
 - n th, 169
 - second, 169

- L'Hôpital's rule, 188
- Lagrange remainder, 191
- least element, 31
- left inverse, 27
- left limit, 163
- left-sided derivative, 177
- lim inf, 100
- lim sup, 100
- limit, 158
- limit infimum, of sets, 70
- limit point, 117, 123
- limit supremum, of sets, 70
- Lipschitz, 152
- little "oh", 193
- local extremum, 183
- local maximum, 183

- local minimum, 183
- logarithmic function, 216
- logical connectives, 4
- lower bound, 55

- mathematical induction, 29
- maximum, 155
- maximum element, 39
- Mean Value Theorem, 184, 186
- measure zero, 133
- member, 2, 10
- mesh, 199
- metric, 110
- metric completion, 172
- metric space, 113
- metric spaces, convergence, 122
- metric, equivalent, 140
- metric, topologically equivalent, 139
- middle-thirds Cantor set, 130, 131
- minimum, 155
- modulus, 75
- monotone
 - function, 152
- monotone decreasing
 - function, 152
 - sequence, 89
- monotone increasing
 - sequence, 89
- monotone sequence property, 93
- monotone sequence theorem, 92
- monotonic
 - sequence, 89
- multiplicative inverse, 48

- n th degree Taylor polynomial, 190
- n th order derivative, 190
- natural logarithmic function, 216
- natural numbers, 2, 12
- necessary, 6
- negation, 4
- nested intervals, 93
- Nested Intervals Theorem, 93
- nondecreasing
 - sequence, 89
- nonnegative integers, 12
- norm, 110

- one-to-one, 3, 23
- one-to-one correspondence, 23
- onto, 3, 23
- open ball, 120
- open ball in \mathbb{R} , 115

- open cover, 126
- open in a metric space, 120
- open interval, 60
- open relative to a subspace, 121
- open set, 115
- order, 52
- Order Axioms, 50
- order complete, 57
- ordered pair, 3, 20

- pairwise disjoint, 66
- partition, 197
- Peano axioms, 30
- perfect set, 131
- period, 256
 - fundamental, 256
- periodic function, 256
- π , 271
- positive set, 50
- power series, 259
- power set, 2
- prime, 3
- prime number, 35
- proof by contradiction, 13
- proof by contrapositive, 13
- proper subset, 11

- radius of convergence, 260
- rational numbers, 2
- real numbers, 47, 57, 71
- real part, 75
- real root of polynomial, 154
- rearrangement of a series, 237
- reflexive relation, 21
- relation, 21
- restriction, function, 24
- Riemann sum, upper, 202
- Riemann integrable, 200
- Riemann integral, 200
 - improper, 221
- Riemann sum, 198
- Riemann sum, lower, 202
- right inverse, 27
- right limit, 163
- right-sided derivative, 177
- Rolle's theorem, 184
- root, 61

- separable, 142
- sequence, 64
 - decreasing, 89
 - increasing, 89
 - monotone decreasing, 89
 - monotone increasing, 89
 - monotonic, 89
 - strictly decreasing, 89
 - strictly increasing, 89
- sequence of partial sums, 223
- sequence of functions, 241
- sequence of partial sums, 99
- sequentially compact, 125
- sequentially compact metric space, 138
- series, 223
- series, rearrangement of, 237
- set difference, 14
- set membership, 10
- set minus, 14
- sine, 270
- statement, 4
- strictly decreasing
 - sequence, 89
- strictly increasing
 - function, 152
 - sequence, 89
- Strong Nested Intervals Theorem, 94
- subsequence, 90
- subsequential limit points, 100
- subset, 2, 10
- sufficient, 6
- sum of a series, 223
- superset, 10
- supremum, 55
- surjective, 3, 23
- symmetric difference, 2
- symmetric relation, 21

- Taylor polynomial, 190
- Taylor series, 265
- Taylor series about c , 265
- Taylor's theorem
 - integral remainder, 219
 - Lagrange remainder, 191
- term of a sequence, 64
- there exists, \exists , 8
- Thomae's function, 152
- topologically equivalent metric, 139
- totally bounded, 142
- totally disconnected, 143
- totally disconnected subset of \mathbb{R} , 134
- transitive relation, 22
- triangle inequality, 110
- trigonometric cosine, 270
- trigonometric sine, 270

ultrametric, 113
uncountable, 63
uniform convergence, 243
uniform partition, 198
uniformly Cauchy, 244
uniformly continuous, 164
union, 2, 14, 66
universal quantifier, 8
upper bound, 55

Weierstrass M -Test, 255
well-ordering principle, 31