
Index

- Addition and extended addition rule, 17
Aigner, M., 236, 248
Algebraic structures:
 with one composition, 261
 isomorphic, 266
 over a commutative ring with identity,
 263, 264
 substructures of, 264
 with two compositions, 262, 263
Alon, N., 66
Andrews, G., 109, 116
Apostol, T., 251, 254
Arithmetic (number-theoretic) function,
 97
Arrow impossibility theorem, 190

Bell numbers:
 and moments of a Poisson distribution,
 83
 and partitions of a set (resp.,
 equivalence relations), 27, 71
 exponential generating function for, 71
 recurrence for, 71
Berge, C., 159
Bertrand's ballot problem, 138
Bijective (combinatorial) proof, 2, 8
Binomial coefficient:
 and lattice paths, 35
 as enumerator of subsets of a given
 cardinality, 31
 recurrence, 32
 table of (Pascal's triangle), 32
Binomial inversion principle, 37

Binomial poset:
 definition, 240
 factorial function of, 240
 incidence coefficient of, 242
 reduced incidence algebra of, 243
Bogart, K., 39, 45, 195, 205
Bonferroni inequalities, 43
Burnside's lemma, 144

Canfield, E., 203, 205
Catalan numbers:
 and Dyck words (subdiagonal lattice
 paths), 137
 and parenthesizations of a word, 136
 and triangulations of a polygon, 138
 recurrence, generating function, and
 closed form for, 136
Cauchy product, 210
Cauchy's formula, 81
Cayley, A., 59, 66
Characteristic function of a set, 20, 39
Chi function (of an interval in a locally
 finite poset), 232
Cigler, J., 183
Circular words, 100–101
Combinatorial factorization of a
 polynomial, 4
Complete symmetric function, 92
Composition of a positive integer:
 generating functions for, 6
 pictorial representation and
 enumeration of, 2
 under restrictions on its parts, 6

- weak, 5
- Comtet's theorem, 90, 93
- Conjunctive normal form (of a boolean function), 157
- Cycle index of a permutation, 146
- Cycle numbers:
 - and restricted ordered occupancy, 78
 - and permutations, 79
 - as connection constants, 80
 - as signless Stirling numbers of the first kind, 79
 - as weighted Stirling numbers, 89
 - recurrence and table, 78
 - recurrence and table, 79
- Davis, R., 183, 186, 248
- De Bruijn's generalization of Polya's theorem, 155, 159
- Dedekind's problem, 198
- Dilworth's antichain decomposition theorem, 195
- Dilworth's chain decomposition theorem, 196
- Dilworth's lemma, 195
- Dirichlet product, 97, 210
- Disjunctive normal form (of a boolean function), 157
- Distribution polynomial:
 - for integer partitions, 179
 - for statistics on discrete structures, 175
 - for the inversion statistic on integer sequences, 178
 - for the inversion statistic on permutations, 176
- Dobinski's formula, 71, 81
- Doubilet, P., 227, 248
- Doyle, P., 40, 44
- Equinumerous (equipollent, equipotent) sets, 14
- Equivalence relation, 27
- Erdős, P., 24, 27
- Erickson, M., 65, 66
- Erikson, K., 109, 116
- Euler ϕ -function, 98, 99
- Eulerian derivative, 182
- Eulerian number, 85
- Exponential formula, 88
- Falling factorial polynomial, 20
- Fibonacci number:
 - and tilings, 5
 - asymptotic growth rate of, 4
 - closed form for, 4
 - combinatorial interpretation of, 3
 - of a binomial poset, 250
 - recurrence and generating function for, 3
- Finite difference:
 - and polynomial interpolation, 121
 - antidifferences, table of, 123
 - definition and basic properties of, 119
 - finite difference calculus, fundamental theorem of, 122
 - relation to the shift operator, 120
- Finitely additive measure, 19
- Ford, L., 197, 205
- Formal derivative (of a formal power series), 219
- Formal power series, 212
- Fulkerson, D., 197, 205
- Function (map, mapping, functional, transformation, operator)
 - as a distribution, 15
 - as a sequence or word, 15
 - domain partition induced by, 22
 - domain, codomain and range of, 13
 - extensional and intensional conceptions of, 13
 - graph of, 13
 - injective, surjective and bijective, 13
 - one- and two-sided inverses of, 14
 - partial, 47
 - weakly (resp., strictly) increasing, 44
- Galois numbers of a finite vector space, ff. 181
- Golden ratio, 3
- Goldman, J., 182, 183
- Graded poset, 239
- Graham, R., 21, 27, 66, 205
- Graph:
 - as an irreflexive, binary relation, 57
 - complete, 58
 - complete bipartite, 66
 - connected, 58
 - edge coloring of, 62
 - edge of, 57
 - enumeration of isomorphism classes of, 151
 - labeled and unlabeled, 58
 - vertex, vertex adjacency, degree of a vertex, 57
- Greatest lower bound (infimum), 204

- Gross, O., 53, 54
- Hall, P., 233–234
- Harary, F., 62, 66, 154, 159
- Harmonic numbers, 126
- Harrison, M., 159
- Hausdorff maximality principle, 194, 205
- Incidence algebra (of a locally finite poset), 228
- Indeterminate, 211
- Irrelevance of alternatives (for a social welfare function), 190
- Jordan–Dedekind chain condition, 239
- Kaplansky, I., 40, 44
- Kelley, J., 205, 259
- Kirchhoff, G., 62, 66
- Kitchen, J., 251, 254
- Knuth, D., 21, 27, 181, 183
- Kurtz, D., 203, 205
- Lagrange interpolation theorem, 140
- Lah numbers:
 - recurrence, closed form, and table, 78
- Lah numbers:
 - and ordered occupancy, 77
 - as connection constants, 78
 - as weighted Stirling numbers, 88
 - recurrence, closed form, and table, 77
- Lancaster’s theorem, 93
- Lattice:
 - algebraic, 204
 - order-theoretic, 204
 - sublattice, 204
- Least upper bound (supremum), 203
- Legendre’s theorem, 211
- Linear difference equation:
 - and rational generating functions, 132
 - for periodic and polynomial functions, 135
 - homogeneous, with constant coefficients, 127
 - in operator form, 127
 - solution using its characteristic polynomial, 128
- Liu, C., 130, 140
- Logarithmic concavity (of a real sequence), 200
- Lubell, D., 197, 205
- Lucas, E.:
 - and a congruence for binomial coefficients, 114
 - and the problème des ménages, 40, 44
- Marriage theorem, 199
- Matching, 198
- McCluskey, E., 158, 159
- Meshalkin, L., 205
- Method of linear functionals, 182
- Metric:
 - discrete, 212
 - ultrametric, 214
- Möbius function:
 - of a positive integer, 98
 - of an interval in a locally finite poset, 233
- Möbius inversion principle:
 - binomial inversion as a special case, 99
 - for arithmetic functions, 98
 - for bivariate functions on a locally finite poset, 235
 - for univariate functions on a locally finite poset, 236
- Modular binomial lattice:
 - characteristic of, 247
 - definition of, 247
- Mulay, S., 181, 183, 259
- Multinomial coefficients:
 - abbreviated notation for, 51
 - and distributions with prescribed occupancy numbers, 49
 - and ordered partitions of a set, 50
 - as enumerators of words with prescribed letter frequencies, 50
 - recurrence for, 51
- Multiplication rule, 20
- Newton’s inequality, 203
- Niven, I., 213, 223
- O’Hara, K., 183
- Orbit (of a permutation group), 144
- Ordered direct sum decomposition of a vector space, 173
- Ordered partitions of a set:
 - and preferential rankings (weak orders), 52
 - asymptotic formula for, 53
 - recurrence and exponential generating functions for, 49–50
 - infinite series for, 53
- p-order of an integer, 112

- Palmer, E., 147, 159
- Partially ordered set (poset):
- antichain in, 194
 - chain in, 194
 - comparability of two elements of, 192
 - covering relation between two elements of, 193
 - dimension of, 194
 - duality principle for, 193
 - graded, 239
 - length (resp., width) of, 194
 - maximal chain in, 194
 - minimal (resp., maximal) element of, 192, 193
 - monotone boolean function on, 198
 - order ideal in, 198
 - smallest (resp., largest) element of, 192, 193
 - subposet (induced subposet) of, 192
 - weak subposet of, 192
- Partition of a set:
- definition, 22
 - and equivalence relations, 27
 - enumeration of, by number of blocks, 69
- Partition of an integer:
- as a distribution of unlabeled balls among unlabeled boxes, 101
 - as a multiset of positive integers, 101
 - Ferrers diagram of, 102
 - generating functions for, 105–106
 - pentagonal number theorem, 107
 - recurrence and table, 102
 - self-conjugate, 104
- Patashnik, O., 21, 27
- Pattern inventory of a permutation group, 149
- Permutation group:
- as a subgroup of the symmetric group, 143
 - Burnside's lemma, 144
 - orbits induced by, 144
 - permutational equivalence of, 161
- Permutation:
- as a bijective self-map, 15
 - as a word, 15
 - Cauchy's formula, 81, 147
 - cycle decomposition of, 79
 - enumeration by number of cycles, 79
- Pervin, W., 259
- Pigeonhole principle:
- elementary form, 17
 - for functions, 23
 - for relations, 29
- Polya's first and second theorems, 146–149
- Power sum:
- and Bernoulli numbers, 111
 - definition, 109
 - recurrences for, 109, 110
- Prüfer code, 60
- Principal dual order ideal (of a poset), 236
- Principal order ideal (of a poset), 236
- Probabilistic method (for determining bounds on Ramsey numbers), 65
- Problème des rencontres:
- and the hat-check problem, 38
 - solution by binomial inversion, 38
- q-binomial (Gaussian) coefficient, 165
- q-binomial inversion principle, 169
- q-factorials of the first and second kinds, 163–164
- q-integer, 163
- q-multinomial coefficients of the first and second kind, 172
- q-Vandermonde identity, 171
- Quasi-order (preorder):
- as a partially ordered partition, 207
 - as a reflexive, transitive relation, 188
 - connection with topologies, 189
- Quine, W., 158, 159
- Ramsey, F., 62, 66
- Rank function (of a poset), 203
- Rational generating functions (fundamental theorem of), 132
- Reciprocal polynomial, 4
- Relation:
- covering, in a partially ordered set, 193
 - domain and range of, 24
 - dual, complement, symmetric and asymmetric part of, 26
 - graph of, 24
 - Intensional and extensional conception of, 24
 - matrix representation of, 26
 - pigeonhole principle for, 30
 - symmetric complement of, 191
 - types of (reflexive, symmetric, asymmetric, antisymmetric, transitive, complete, negatively transitive), 25
- Restricted growth function, 72
- Rising factorial polynomial, 21

- Rota, G.-C., 19, 27, 81, 182, 183, 227, 248, 249
- Rothschild, B., 62, 66
- Schoenfield, J., 215, 223
- Schur's lemma, 65
- Semigroup algebra, 209
- Sen, A., 190, 205
- Shattuck, M., 181, 183
- Sieve formula (principle of inclusion and exclusion):
 abstract form (inversion formula for set functions), 41
 basic form, 19
 complementary form, 19
 noninductive proof of, 39
- Snake oil method, 123
- Social welfare function, 190
- Spanning subset (of a vector space), 183
- Spanning tree (of a connected graph), 62
- Spencer, J., 64, 66, 81
- Sperner poset, 203
- Sperner's theorem, 197
- Spiegel, M., 122, 140
- Stanley, R., 11, 27, 131, 227, 248, 249
- Stirling numbers of the first kind:
 and elementary symmetric functions, 75
 as connection constants, 75
 recurrence and table, 76
 signless, 79
- Stirling numbers of the second kind:
 and restricted growth functions, 72
 and set partitions with prescribed number of blocks, 69
 as connection constants, 74
 exponential generating function for, 70
 recurrence for, 70
 table of (Stirling's triangle), 70
- Stone, H., 159
- Strict order, 191
- Strong convergence (of a complex sequence), 213
- Strong logarithmic concavity (of a real sequence), 200
- Strong pointwise convergence:
 of a sequence in C^N , 213
 of an infinite series in C^N , 213
 of sequences and series in $C^{\text{Int}(P)}$, 229
- Summability:
 of a complex sequence, 216, 251
 of a double sequence in C^N , 217
 of a sequence in $C^{\text{Int}(P)}$, 230
 of a sequence in C^N , 216
- Sylvester, J., 104, 105
- System of distinct representatives (SDR), 198–199
- Szekeres, A., 24, 27
- Szpilrajn's theorem, 194, 205
- Top-down summation, 7
- Topology:
 base of, 255
 definition of, 255
 discrete, 255
 indiscrete, 255
 metric, 256
 of pointwise convergence, 257
 product, 257
 separability axioms for, 257
- Total (linear) order, 187
- Touchard, J., 39, 44
- Tournament, 29
- Tree:
 as connected graph with no cycles, 59
 Cayley's formula for the number of, 59
 Kirchhoff's matrix tree theorem, 62
 Prüfer code of, 60, 61
- Trotter, W., 195, 205
- Twenty-fold way, 77, 78
- Unimodal sequence of real numbers, 167, 200
- Vacuous implication, principle of, 12
- Vandermonde, A.-T.:
 binomial coefficient identity, 33
 determinant factorization theorem, 129
- Velleman, D., 14, 27, 194, 205
- Wade, W., 36, 251, 254
- Wagner, C., 109, 117, 180, 183
- Warner, S., 210, 223, 261, 266
- Weak order:
 as a reflexive, transitive, complete relation, 188
 connection with ordered partitions, 189
- Weak Pareto property (of a social welfare function), 190
- Weight function on the positive integers:
 definition, 87
 weighted Stirling and Bell numbers, 88
- Well-ordering, 187
- West, D., 59, 66
- Wilf, H., 44, 46, 123, 140
- Worpitsky's identity, 85

Zeilberger, D., 183

Zeta function:

of a positive integer, 98

of an interval in a locally finite poset,
231