
Index

- abelian group, 384
- absolute value, 216
- addition principle
 - for sets, 310
- additive identity, 381
- additive inverse, 381
- affine combination, 148
- affine subspace, 220
- ambient set, 209
- antipodal map, 231
- argument
 - of a complex number, 403
- arithmetic sum, 50
- associative, 213, 381, 382
- associative division algebra, 409
- asymptotic formula, 75
- axiom of choice, 245

- base case, 52
- base ten, 163
- Bernoulli number, 71
- biconditional, 132
- bijective, 237
- bilinear form, 216
- binary operation, 380
- Binet's formula, 87
 - matrix proof, 88
- binomial coefficient, 324, 330
 - factorial formula, 331
 - recursive formula, 326
- Binomial Theorem, 334
- bunny
 - funny, 117
 - regular, 117

- cancellation property, 398
- cardinality, 242, 309
- cartesian product, 214
 - infinite number of, 215
 - n -fold, 215
 - of indexed family of sets, 245
- Cauchy–Schwarz inequality, 218
- ceiling function, 233
- characteristic 0, 410
- characteristic p , 410
- characteristic function, 346
- Chinese Remainder Theorem, 279
- choice function, 245
- circle, 222
- codomain, 225
- common divisor, 30
- commutative, 213, 381, 382
- commutative diagram, 246
- complement, 209
- complex
 - addition, 401
 - conjugate, 402
 - multiplication, 401
- complex numbers, 401
- composite number, 6
- composition
 - of functions, 230
- conditional statement, 128
- congruence
 - and coprimeness, 266
 - and division, 267

- compatible with addition and multiplication, 261
- multiplication table, 263
- congruence class, 362
 - addition and multiplication of, 366
 - addition and multiplication tables, 368
 - inverse of, 369
 - inverses, 369
 - invertible, 369
 - properties, 363
 - set of, 366
- congruence equation
 - linear, 270
- congruent
 - modulo m , 257
- conjugate matrices, 367
- conjunction, 122
- connective
 - logical, 120
- contradiction, 126
- contraposition, 134
- contrapositive, 11, 134
- converse, 130
- coprime, 33, 176
- coset
 - of a subgroup, 389
 - right, 390
- counterexample, 10
- cousin prime, 42

- De Morgan laws, 212
- decimal part
 - of a number, 233
- decrypt, 295
- definition
 - recursive, 91
- determinant
 - of a matrix, 387
- difference
 - of sets, 314
- differential geometry, 233
- Diophantine equation
 - linear, 176
- disjunction, 120
- distributive, 213
- distributive property, 382
- divisible by 3
 - criterion, 256
- divisible by 9
 - criterion, 256
- Division Theorem, 26, 162
- divisor, 6, 25
 - non-trivial, 6
 - trivial, 6
- domain, 225
- domain of discourse, 209
- domino effect, 56
- dot product
 - Euclidean, 216

- element, 204
- empty set, 206
- encrypt, 295
- epimorphism
 - of groups, 394
- equal
 - functions, 229
 - sets, 204
- equivalence class, 375
- equivalence relation, 259, 375
- Euclid's formula, 147, 148, 160
- Euclid's Theorem, 37, 171
- Euclid–Euler Theorem, 298
- Euclidean algorithm, 173
- Euclidean dot product, 216
- Euclidean space, 215
- Euler's
 - criterion, 415
 - product formula, 292
 - Theorem, 293
 - totient function, 289
- Euler's Theorem
 - proof of, 393
- even integer, 12
- exclusive or, 121
- existential quantifier, 151
- extension
 - of a function, 230
- extreme-value theorem, 406

- factorial, 94
- falsidical paradox, 60
- Faulhaber's formula, 71
- Fermat's Last Theorem, 188
- Fermat's Little Theorem, 284
 - proof using congruence classes, 373
- Fibonacci number, 83
 - Binet's formula, 87
 - generating function, 227
- field, 400
 - homomorphism, 404
 - isomorphic, 405
 - isomorphism, 405

- finite field
 - factoring polynomials, 409
- finite set, 242
- fixed point
 - of a function, 231
- floor function, 233
- fractional part
 - of a number, 233
- free variable, 153
- function, 225
 - bijjective, 237
 - ceiling, 233
 - composition, 230
 - floor, 233
 - generating, 227
 - graph, 232
 - identity, 226
 - image, 229
 - injective, 236
 - inverse of, 240
 - invertible, 240
 - multi-valued, 403
 - rational, 228
 - restriction, 229
 - surjective, 234
- functions
 - counting, 345
- Fundamental Theorem of Algebra, 405
- general Leibniz rule, 337
- general linear group, 387
 - complex, 387
- generating function, 227
- generator
 - of a group, 384
- Goldbach's conjecture, 43
- golden ratio, 88
- googol, 172
- graph
 - of a function, 232
- greatest common divisor, 28, 30
- group, 384
 - abelian, 384
 - cyclic, 384
 - epimorphism, 394
 - examples, 384
 - finite, 384
 - generator, 384
 - homomorphism, 394
 - infinite, 384
 - isomorphism, 394
 - monomorphism, 394
 - product, 387
- harmonic sequence, 226
- homomorphism
 - of a field, 404
 - of groups, 394
- ideal
 - left, 399
 - of a ring, 398
 - right, 399
- identity
 - function, 226
- identity element, 384
- if-then statement, 5
- image
 - of a function, 229
- imaginary unit, 394, 401
- implication, 128
- inclusion-exclusion principle
 - for sets, 315
- inclusive or, 121
- index
 - of a subgroup, 390
- induction
 - formal statement of, 55
 - mathematical, 51
 - strong, 78
- inductive step, 52
 - strong, 79
- inequality
 - Cauchy–Schwarz, 218
- infinite descent
 - method of, 141
- infinite number of primes, 37, 171
 - congruent to 3 mod 4, 269
- infinite set, 242
- injective, 236
- input, 225
- integers
 - set of, 3
- integral domain, 398
- integral linear combination, 34
- intersection, 30, 208
 - of indexed family of sets, 244
- inverse
 - left, 246
 - modulo m , 266
 - modulo p , 274
 - of a function, 240
 - right, 246
- inverse element, 384

- invertible
 - function, 240
- isomorphic groups, 394
- isomorphism
 - of a field, 405
 - of groups, 394
- l'Hospital's rule, 63
- Lagrange Theorem
 - in group theory, 389
- Lagrange's Theorem
 - in number theory, 410
- least common multiple, 191
- least residue system
 - modulo m , 263
- left inverse, 246
- Legendre symbol, 414
- liar paradox, 114
- linear congruence equation, 270
 - examples, 276
 - general solution, 274
 - solving using congruence class inverses, 370
- linear Diophantine equation, 140, 176
- linear subspace, 219
- logical connective, 120
- logically equivalent, 124
- lowest terms
 - fraction, 31
- mathematical induction, 51
- matrix
 - invertible, 387
 - multiplication, 387
 - similar, 376
- maximum, 28
- Mersenne prime, 40
 - exponent is irrational, 137
- mice
 - down-and-out nerdy, 326
 - hoity-toity cool, 326
- minimal surface, 233
- minimum, 28
- modulus
 - of a complex number, 402
- modus ponens, 133
- modus tollens, 134
- monoid, 387
- monomorphism
 - of groups, 394
- Moore's law, 38
- multi-valued function, 403
- multinomial coefficient, 339
- multinomial theorem, 339
- multiple, 25
- multiplication principle
 - for sets, 312
- multiplicative identity, 382
- negation, 124
- Nicomachus's conjecture, 39
- non-empty, 206
- non-negative, 122
- non-positive, 122
- norm, 216
- normal subgroup, 390
- not, 124
- odd integer, 14
- or, 120
 - exclusive, 121
 - inclusive, 121
- order
 - of a group, 384
 - of a group element, 391
- ordered n -tuple, 215
- ordered pair, 214
- output, 225
- paradox
 - liar, 114
 - Russell's, 207
- parity, 14
- partial fractions, 228
- partition, 210, 376
 - of power set, 324
- Pascal's triangle, 330
- perfect number, 39, 297
- permutation, 395
 - even, 397
 - odd, 397
- pigeonhole principle, 243
- plane geometry proofs, 112
- Polymath8 project, 43
- Polynomial Division Theorem, 411
- power
 - of a group element, 391
- power series, 227
- power set, 322
- pre-image, 237
- prime, 4
 - cousin, 42
 - factorization, 36
 - Mersenne, 40

- pair, 42
 - twin, 42
- prime ideal, 399
- principal ideal domain, 399
- principal value of the argument, 403
- private key, 295
- product
 - cartesian, 214
- product group, 387
- projective space
 - real, 224
- proof
 - flawed, 108
- proper subset, 205
- propositional logic, 133
- public key, 295
- Pythagorean theorem, 112
- Pythagorean triple, 145, 188
 - primitive, 146
- quadratic reciprocity
 - law of, 418
 - supplements to the law of, 422
- quadratic residue, 280, 412
 - non-trivial, 412
 - remainder, 412
- quadratic residues
 - set of non-zero, 280
- quantifier
 - existential, 151
 - universal, 152
- quotient, 26, 161
- quotient function, 166
- quotient ring, 399
- quotient set, 376
- range, 229
- rational function, 228
- rational number, 31
- real projective space, 224
- recursion theorem, 92
- recursive definition, 91
- reduced residue system, 293
- reflexive, 258
- relation, 374
 - equivalence, 259, 375
 - reflexive, 375
 - symmetric, 375
 - transitive, 375
- remainder, 26, 161
- remainder function, 166
- residue
 - quadratic, 412
- restriction
 - of a function, 229
- Riemann sum, 74
- right inverse, 246
 - of a group element, 387
- ring, 397
 - addition, 397
 - multiplication, 397
 - of integers, 382
- root of unity, 394
- RSA
 - algorithm, 37
 - factoring challenge, 38
- RSA algorithm, 294
- Rubik's cube, 397
- rule of inference, 133
- Russell set, 208
- Russell's Paradox, 207
- scalar multiplication, 215
- Scherk's surface, 233
- Schneider's formula, 293
- Schroedingers cat
 - Schrödinger's cat, 123
- self-referential, 114
- semiprime, 143, 297
- sequence, 226
- series
 - power, 227
- set, 204
 - difference, 314
 - empty, 206
 - equality, 204
 - finite, 242
 - infinite, 242
 - of congruence classes, 366
- set difference, 210
- set of remainders, 166
- sieve of Eratosthenes, 21
- soap bubble, 233
- sphere, 221
 - unit, 222
- square root
 - of a prime is irrational, 142
 - of two is irrational, 141
- squaring function, 226
- stereographic projection, 222, 223
- Stirling's formula, 69
- strong induction, 78
- subfield, 405
- subgroup, 387

- non-trivial, 388
- subring, 398
- subset, 204
 - proper, 205
- subspace
 - affine, 220
 - linear, 219
- Sudoku, 397
- sum of positive divisors function, 296
- summation formula
 - visualizing, 69
- surjective, 234
- symmetric, 216
- symmetric group, 395
- symmetric relation, 259

- tautology, 125
- tic-tac-toe, 354
- tiling numbers, 81
- torus, 377
- toy model, 107
- transitive relation, 259
- transposition, 397
- truth set, 154
- truth table, 120, 123
- truth teller and liar riddle, 115
- truth value, 114
- twin prime, 42, 299

- union, 208
 - of indexed family of sets, 244
- universal quantifier, 152
- universal set, 209

- Vajda–Everman identity, 90
- vector addition, 215
- vector space, 215
 - axioms, 216
- Venn diagram, 315

- water pouring puzzle, 187
- well-ordering principle, 163
 - equivalence with induction, 163
- Wilson’s Theorem, 288
- without loss of generality, 18

Selected Published Titles in This Series

- 61 **Bennett Chow**, Introduction to Proof Through Number Theory, 2023
- 59 **Oscar Gonzalez**, Topics in Applied Mathematics and Modeling, 2023
- 58 **Sebastian M. Cioabă and Werner Linde**, A Bridge to Advanced Mathematics, 2023
- 57 **Meighan I. Dillon**, Linear Algebra, 2023
- 55 **Joseph H. Silverman**, Abstract Algebra, 2022
- 54 **Rustum Choksi**, Partial Differential Equations, 2022
- 53 **Louis-Pierre Arguin**, A First Course in Stochastic Calculus, 2022
- 52 **Michael E. Taylor**, Introduction to Differential Equations, Second Edition, 2022
- 51 **James R. King**, Geometry Transformed, 2021
- 50 **James P. Keener**, Biology in Time and Space, 2021
- 49 **Carl G. Wagner**, A First Course in Enumerative Combinatorics, 2020
- 48 **Róbert Freud and Edit Gyarmati**, Number Theory, 2020
- 47 **Michael E. Taylor**, Introduction to Analysis in One Variable, 2020
- 46 **Michael E. Taylor**, Introduction to Analysis in Several Variables, 2020
- 45 **Michael E. Taylor**, Linear Algebra, 2020
- 44 **Alejandro Uribe A. and Daniel A. Visscher**, Explorations in Analysis, Topology, and Dynamics, 2020
- 43 **Allan Bickle**, Fundamentals of Graph Theory, 2020
- 42 **Steven H. Weintraub**, Linear Algebra for the Young Mathematician, 2019
- 41 **William J. Terrell**, A Passage to Modern Analysis, 2019
- 40 **Heiko Knospe**, A Course in Cryptography, 2019
- 39 **Andrew D. Hwang**, Sets, Groups, and Mappings, 2019
- 38 **Mark Bridger**, Real Analysis, 2019
- 37 **Mike Mesterton-Gibbons**, An Introduction to Game-Theoretic Modelling, Third Edition, 2019
- 36 **Cesar E. Silva**, Invitation to Real Analysis, 2019
- 35 **Álvaro Lozano-Robledo**, Number Theory and Geometry, 2019
- 34 **C. Herbert Clemens**, Two-Dimensional Geometries, 2019
- 33 **Brad G. Osgood**, Lectures on the Fourier Transform and Its Applications, 2019
- 32 **John M. Erdman**, A Problems Based Course in Advanced Calculus, 2018
- 31 **Benjamin Hutz**, An Experimental Introduction to Number Theory, 2018
- 30 **Steven J. Miller**, Mathematics of Optimization: How to do Things Faster, 2017
- 29 **Tom L. Lindstrøm**, Spaces, 2017
- 28 **Randall Pruim**, Foundations and Applications of Statistics: An Introduction Using R, Second Edition, 2018
- 27 **Shahriar Shahriari**, Algebra in Action, 2017
- 26 **Tamara J. Lakins**, The Tools of Mathematical Reasoning, 2016
- 25 **Hossein Hosseini Giv**, Mathematical Analysis and Its Inherent Nature, 2016
- 24 **Helene Shapiro**, Linear Algebra and Matrices, 2015
- 23 **Sergei Ovchinnikov**, Number Systems, 2015
- 22 **Hugh L. Montgomery**, Early Fourier Analysis, 2014
- 21 **John M. Lee**, Axiomatic Geometry, 2013
- 20 **Paul J. Sally, Jr.**, Fundamentals of Mathematical Analysis, 2013
- 19 **R. Clark Robinson**, An Introduction to Dynamical Systems: Continuous and Discrete, Second Edition, 2012

For a complete list of titles in this series, visit the
AMS Bookstore at www.ams.org/bookstore/amstextseries/.