Preface

This book provides the essential elements of real analysis with which mathematicians, and those who use mathematics at more than a superficial level, may want to be familiar. It covers the foundations of differential and integral calculus in one variable, beginning with the structure of the real numbers as a complete ordered field and concluding with a chapter on Lebesgue measure and integration. Along the way, we discuss sequences and series of real numbers, the metric topology of the real numbers, continuous functions, sequences and series of functions, differentiation, and the Riemann integral.

While the material is developed "from scratch", we have written the book primarily for students of mathematics (and related fields) that have several semesters of calculus under their belts. In particular we expect that the reader has attained a basic level of "mathematical maturity" and is prepared to handle some abstraction. The book is appropriate, for example, as a text for a one or two term advanced undergraduate course in analysis, or as a supplementary text for a first year graduate course.

Needless to say, none of the material presented here is new or original, and all of it can be found in any number of well-known texts (to say nothing of an increasingly rich trove of online sources),[1] and our goal in writing this book is not to provide another compendium of this material. Rather our main purpose here is to provide an introduction to the methods of real analysis and a way of thinking about it. As such, there are many common topics that we do not cover at all. For example, while we do introduce the metric topology of $\mathbb{R}^n$ and some of its features, we don't discuss calculus in several variables.

We strongly believe that students learn more by filling in some of the details themselves, and as we progress through the book, we provide them with more opportunities to do so. Sometimes we do this explicitly, by leaving proofs or parts of proofs as exercises, and at other times, the proofs we give are complete, but "terse". Our aim is to

[1] As a consequence, we do not include references with two exceptions, as footnotes.

provide a text that sits a level between the more elementary introductions to analysis, and those that are meant for graduate-level courses in mathematics.

The text focuses primarily on the core material, with few numerical examples and no applications to other fields. If used in a course where such examples are useful or necessary, we leave it to the instructor to provide them. On the other hand, we do include sections that explore topics in real analysis that are either not commonly discussed in undergraduate texts on real analysis, or are slightly more advanced. Such sections are marked with asterisks and may be omitted without affecting the flow of the material. We include them because the topics they cover are interesting (to us), and because they provide an opportunity to practice using the analytic tools that have been developed. If this text were used as a course textbook, these sections could be used as topics for students to present in class, "seminar style", for example.

The material is divided into eight chapters, with an appendix that covers the basic elements of set theory and a construction of the real numbers. Each chapter is divided into sections, and each section is divided into subsections. Definitions, lemmas, theorems, etc., are not individually numbered and are referred to according to the subsection in which they appear, where no subsection contains more than one of each type of statement. For example, Theorem* 4.3.3 is the theorem that appears in Chapter 4, Section 4.3, Subsection* 4.3.3. Exercises are collected at the end of each section, and are numbered to refer to the chapter and section in which they appear. E.g., **ex5.4.2** is the second exercise for Section 5.4 of Chapter 5.