

CONTENTS

Acknowledgments	ix
1. Introduction	1
1.1. Introduction	1
1.2. A guide to reading this book	6
2. Some Background and Motivation	9
2.1. Reciprocity over \mathbf{Z}	9
2.2. Inner forms of $GL(2)$: conjectures	14
3. Notation	21
3.1. A summary of important notation	21
3.2. Fields and adeles	25
3.3. The hyperbolic 3-manifolds	27
3.4. Homology, cohomology, and spaces of modular forms	29
3.5. Normalization of metric and measures	34
3.6. S -arithmetic groups	35
3.7. Congruence homology	37
3.8. Eisenstein classes	45
3.9. Automorphic representations. Cohomological representations	46
3.10. Newforms and the level raising/level lowering complexes	47
4. Raising the Level: newforms and oldforms	51
4.1. Ihara's lemma	51
4.2. No newforms in characteristic zero.	56
4.3. Level raising	60
4.4. The spectral sequence computing the cohomology of S -arithmetic groups	62
4.5. $\zeta(-1)$ and the homology of PGL_2	67
5. The split case	81
5.1. Noncompact hyperbolic manifolds: height functions and homology ..	82
5.2. Noncompact hyperbolic manifolds: eigenfunctions and Eisenstein series	86
5.3. Reidemeister and analytic torsion	92
5.4. Noncompact arithmetic manifolds	98

5.5. Some results from Chapter 4 in the split case	102
5.6. Eisenstein series for arithmetic manifolds: explicit scattering matrices	106
5.7. Modular symbols, boundary torsion, and the Eisenstein regulator ...	113
5.8. Comparing Reidmeister and analytic torsion: the main theorems ...	125
5.9. Small eigenvalues	130
5.10. The proof of Theorem 5.8.3	155
6. Comparisons between Jacquet-Langlands pairs	163
6.1. Notation	163
6.2. The classical Jacquet Langlands correspondence	164
6.3. Newforms, new homology, new torsion, new regulator	164
6.4. Torsion Jacquet-Langlands, crudest form	166
6.5. Comparison of regulators and level-lowering congruences: a conjecture	167
6.6. Torsion Jacquet-Langlands, crude form: matching volume and congruence homology	174
6.7. Essential homology and the torsion quotient	176
6.8. Torsion Jacquet-Langlands, refined form: spaces of newforms	180
6.9. The general case	187
7. Numerical examples	191
7.1. The manifolds	192
7.2. No characteristic zero forms	193
7.3. Characteristic zero oldforms	194
7.4. Characteristic zero newforms and level lowering	197
7.5. Eisenstein Deformations: Theoretical Analysis	204
7.6. Eisenstein Deformations: Numerical Examples	208
7.7. Phantom classes	209
7.8. $K_2(\mathcal{O}_F)$ and $F = \mathbf{Q}(\sqrt{-491})$	213
7.9. Table	215
Bibliography	217
Index	225