

# CONTENTS

<b>Introduction</b> .....	1
Overview .....	4
Acknowledgements .....	4
<b>Part I. Gelfand-Kazhdan descent and chiral differential operators</b> .....	7
<b>1. Flat vector bundles and Harish-Chandra descent</b> .....	11
1.1. Algebra of Harish-Chandra pairs .....	11
1.2. Bundles .....	12
1.3. Descent .....	14
1.4. The characteristic map .....	18
<b>2. Formal vector bundles and Gelfand-Kazhdan descent</b> .....	19
2.1. A Harish-Chandra pair for the formal disk .....	19
2.2. The coordinate bundle .....	21
2.3. The category of formal vector bundles .....	25
2.4. Gelfand-Kazhdan descent .....	28
2.5. Formal characteristic classes .....	30
<b>3. Harish-Chandra structure on CDOs</b> .....	37
3.1. Recollections on vertex algebras .....	37
3.2. The $\beta\gamma$ -vertex algebra .....	39
3.3. The classical limit .....	41
3.4. Harish-Chandra structure on $\text{CDO}_n$ .....	43
3.5. Formal automorphisms .....	46
3.6. The conformal structure for the equivariant vertex algebra .....	50
3.7. The character of a graded vertex algebra .....	52
<b>4. Extended Gelfand-Kazhdan descent</b> .....	57
4.1. The extended bundle .....	57
4.2. Extended modules .....	65
4.3. Extended descent .....	66
<b>5. Descent for vertex algebras</b> .....	69
5.1. General descent .....	69
5.2. Formal vertex algebras .....	70
5.3. Descending Gelfand-Kazhdan vertex algebras .....	71

<b>Part II. The curved <math>\beta\gamma</math> system and its factorization algebra</b> .....	77
<b>6. Overview</b> .....	79
<b>7. A brief overview of derived deformation theory and <math>L_\infty</math> algebras</b> .....	81
<b>8. The formal <math>\beta\gamma</math> system</b> .....	85
8.1. The free $\beta\gamma$ system as a BV theory .....	85
8.2. The formal $\beta\gamma$ system .....	86
8.3. The $W_n$ action on $\mathfrak{g}_n^S$ and on $\mathbb{D}\mathfrak{g}_n^S$ .....	87
8.4. A Noether current and the obstruction-deformation complex .....	89
8.5. Closed two-forms as local functionals .....	94
8.6. Holomorphic vector fields on the source .....	98
<b>9. Equivariant BV quantization of the formal <math>\beta\gamma</math> system</b> .....	101
9.1. Recollections on equivariant BV quantization .....	102
9.2. The pre-theory .....	104
9.3. The obstruction .....	110
9.4. The extended theory .....	115
9.5. The conformal anomaly .....	118
<b>10. The partition function of the equivariant theory</b> .....	123
10.1. The formal Witten class .....	123
10.2. The theory on an elliptic curve .....	125
10.3. Proof of Proposition 10.1.2 .....	126
<b>11. The factorization algebras of equivariant observables</b> .....	129
11.1. An overview of factorization algebras .....	129
11.2. A comment on functional analysis .....	131
11.3. The non-equivariant classical observables .....	133
11.4. The non-equivariant quantum observables .....	134
11.5. The $W_n$ -equivariant classical observables .....	137
11.6. The $\widetilde{W}_n$ -equivariant quantum observables .....	138
11.7. An aside on the two versions of non-equivariant observables .....	139
<b>12. Semi-strict Gelfand-Kazhdan descent</b> .....	141
12.1. Semi-strict modules .....	141
12.2. Semi-strict descent .....	143
12.3. Descent of the equivariant observables .....	144
12.4. Comparison with Costello's work .....	145
<b>13. A concrete description of the observables</b> .....	149
13.1. Polynomials, power series, and the $(W_n, \mathrm{GL}_n)$ -decomposition of observables .....	149
13.2. The classical observables supported at a point .....	153
13.3. The decomposition by conformal dimension: the rotation action on the source .....	156
13.4. The quantum observables .....	156

<b>14. Conformal structure on observables</b> .....	159
<b>Part III. Comparison of the constructions</b> .....	163
<b>15. Overview</b> .....	165
<b>16. From factorization to vertex algebras</b> .....	167
16.1. Translation and derivations .....	167
16.2. Rotation and decomposition .....	169
16.3. The theorem about $\widehat{\text{Vert}}$ .....	170
<b>17. Observables for the formal <math>\beta\gamma</math> system</b> .....	173
17.1. Some useful identifications .....	174
17.2. Quantizing observables .....	176
17.3. An example .....	177
<b>18. Local symmetries acting on observables</b> .....	179
18.1. General arguments .....	179
18.2. The action of $\widehat{W}_n$ .....	184
<b>19. The main result</b> .....	187
19.1. Remark on conformal structure .....	188
<b>20. Discussion of some physics literature</b> .....	189
20.1. General comments about nonlinear $\sigma$ -models .....	189
20.2. Anomalies and obstructions .....	190
20.3. Chiral algebras and observables .....	191
<b>Appendix</b> .....	195
<b>21. The <math>\beta\gamma</math> system as an infinite-volume limit</b> .....	197
21.1. Introduction .....	197
21.2. The ingredients .....	197
21.3. The first-order formulation of the sigma model .....	198
21.4. An involution on the space of fields .....	199
21.5. Replacing the first-order action functional .....	200
21.6. The heuristic argument .....	200
21.7. The BV argument .....	201
21.8. The infinite volume limit .....	204
21.9. The chiral splitting .....	205
<b>Bibliography</b> .....	207