

CONTENTS

| | |
|---|-----|
| 0. Introduction | 1 |
| 0.1. Review of opers on complex algebraic curves | 2 |
| 0.2. What about opers in positive characteristic? | 3 |
| 0.3. Counting problem of dormant opers | 4 |
| 0.4. Comparison with arguments due to Mochizuki, Joshi, and Pauly ... | 5 |
| 0.5. Structure and main theorems of the manuscript | 6 |
| 0.6. Acknowledgements | 15 |
| 0.7. Notation and Conventions | 15 |
| 1. Logarithmic connections over log schemes | 17 |
| 1.1. The adjoint representation and the Maurer-Cartan form | 17 |
| 1.2. Principal \mathbb{G} -bundles and their associated Lie algebroids | 22 |
| 1.3. Logarithmic connections and curvature | 27 |
| 1.4. Explicit descriptions of gauge transformations | 33 |
| 1.5. The moduli stack of pointed stable curves | 38 |
| 1.6. Local pointed curves and monodromy | 42 |
| 2. Opers on a family of pointed stable curves | 47 |
| 2.1. (\mathfrak{g}, \hbar) -opers on a log curve | 47 |
| 2.2. Local description and automorphisms of a (\mathfrak{g}, \hbar) -oper | 50 |
| 2.3. The case of $\mathfrak{g} = \mathfrak{sl}_2$ | 55 |
| 2.4. The \mathbb{G} -bundle ${}^\dagger\mathcal{E}_{\mathbb{G}}$ and the vector bundle ${}^\dagger\mathcal{V}_{\mathfrak{g}}$ | 56 |
| 2.5. \mathcal{A}_1 -normality and a torsor structure on the sheaf of (\mathfrak{g}, \hbar) -opers | 61 |
| 2.6. (\mathfrak{g}, \hbar) -opers on pointed stable curves | 65 |
| 2.7. The adjoint quotient of \mathfrak{g} | 69 |
| 2.8. Radii of (\mathfrak{g}, \hbar) -opers | 71 |
| 2.9. The moduli space of (\mathfrak{g}, \hbar) -opers of prescribed radii | 74 |
| 2.10. The universal moduli stack | 77 |
| 3. Opers in positive characteristic | 79 |
| 3.1. Frobenius twists and relative Frobenius morphisms | 79 |
| 3.2. Lie algebroids in positive characteristic | 82 |
| 3.3. p -curvature on an \hbar -flat \mathbb{G} -bundle | 85 |
| 3.4. Dormant/ p -nilpotent (\mathfrak{g}, \hbar) -opers and their moduli spaces | 89 |
| 3.5. The Hitchin-Mochizuki morphism | 93 |
| 3.6. Varying the parameter \hbar | 100 |
| 3.7. The finiteness of the Hitchin-Mochizuki morphism | 103 |
| 3.8. The universal moduli stacks of dormant/ p -nilpotent (\mathfrak{g}, \hbar) -opers | 106 |

| | |
|--|-----|
| 4. Flat vector bundles and differential operators | 111 |
| 4.1. Logarithmic \hbar -connections on vector bundles | 111 |
| 4.2. Logarithmic differential operators with parameter \hbar | 115 |
| 4.3. Comparison of log connections | 117 |
| 4.4. (GL_n, \hbar) -opers and comparison with (\mathfrak{sl}_n, \hbar) -opers | 122 |
| 4.5. (n, \hbar, Θ) -projective connections and $(\mathrm{GL}_n, \hbar, \Theta)$ -opers | 128 |
| 4.6. (n, \hbar) -theta characteristics | 132 |
| 4.7. Radii of $(\mathrm{GL}_n, \hbar, \vartheta)$ -opers and (n, \hbar, ϑ) -projective connections | 138 |
| 4.8. Change of (n, \hbar) -theta characteristic | 141 |
| 4.9. A canonical PGL_n -bundle underlying (\mathfrak{sl}_n, \hbar) -opers | 144 |
| 4.10. Dormant (GL_n, \hbar) -opers | 149 |
| 4.11. Comparison of the moduli functors | 154 |
| 4.12. Hypergeometric differential operators | 158 |
| 5. Duality of opers | 163 |
| 5.1. $(\mathfrak{so}_{2l+1}, \hbar)$ -opers and $(\mathfrak{sp}_{2m}, \hbar)$ -opers | 163 |
| 5.2. $(\mathrm{GO}_{2l+1}, \hbar, \vartheta)$ -opers and $(\mathrm{GSp}_{2m}, \hbar, \vartheta)$ -opers | 165 |
| 5.3. Dual opers | 170 |
| 5.4. Isomorphisms of moduli spaces induced by duality | 173 |
| 6. Local deformation of opers | 177 |
| 6.1. Deformation spaces of a curve and an \hbar -flat \mathbb{G} -bundle | 177 |
| 6.2. Cohomology of complexes associated to a (\mathfrak{g}, \hbar) -oper | 184 |
| 6.3. Infinitesimal deformations of a (\mathfrak{g}, \hbar) -oper | 188 |
| 6.4. Flat vector bundles in positive characteristic | 193 |
| 6.5. Infinitesimal deformations of a dormant (\mathfrak{g}, \hbar) -oper | 200 |
| 7. The pseudo-fusion ring for dormant opers | 207 |
| 7.1. Semi-graphs and clutching data | 207 |
| 7.2. Gluing \hbar -flat \mathbb{G} -bundles | 210 |
| 7.3. Factorization of (\mathfrak{g}, \hbar) -opers | 214 |
| 7.4. Pseudo-fusion rules and pseudo-fusion rings | 222 |
| 7.5. The ring structure of the fusion ring | 224 |
| 7.6. Factorization property of a fusion rule | 232 |
| 7.7. The pseudo-fusion ring associated to dormant opers | 237 |
| 7.8. The pseudo-fusion rule for \mathfrak{sl}_2 | 240 |
| 8. Generic étaleness of the moduli space of dormant opers | 247 |
| 8.1. Formally local description of a flat bundle | 248 |
| 8.2. Deformation of a flat bundle | 251 |
| 8.3. Deformation of a dormant GL_n -oper | 254 |
| 8.4. Dormant opers on a 3-pointed projective line | 259 |
| 8.5. The generic étaleness of the moduli stack of dormant opers | 264 |
| 9. Comparison with Quot schemes | 267 |
| 9.1. Quot schemes | 267 |
| 9.2. Comparison with the moduli space of dormant $(\mathrm{GL}_n, \vartheta)$ -opers | 268 |
| 9.3. The relation between the Quot schemes | 273 |

| | |
|--|-----|
| 9.4. Counting maximal subbundles | 275 |
| 9.5. Joshi's conjecture | 279 |
| Bibliography | 283 |
| Index | 291 |