PREFACE

Back in 1947 Richard W. Hamming had access to a computer only on weekends. Some three decades later he recalled his frustration over its perverse behavior:

> Two weekends in a row I came in and found that all my stuff had been dumped and nothing was done. . . . And so I said, 'Damn it, if the machine can detect an error, why can't it locate the position of the error and correct it?' [**56**, Tape 2]

That question initiated the development of error-correcting codes. The offending computer, a mechanical relay model at Bell Telephone Laboratories, always came to an angry stop and switched to the next program whenever it detected an error. This behavior impelled Hamming, a pure mathematician with an applied bent, to devise the first error-correcting code.

We shall follow the devious trail that wends its way through a quarter-century of mathematics, starting with Hamming's work, which led almost immediately to that of M. J. E. Golay. The latter sparked, some twelve years later, giant steps in the packing of congruent spheres by John Leech, which, in turn, branched off through the work of J. H. Conway, into the field of simple groups. By tracing some of the twists, turns, switchbacks and dead ends of this path, we hope to provide a small window on the history of mathematics of the twentieth century. How historians ultimately will treat this golden age of mathematical creativity we cannot even guess. But one claim we can make on the basis of this study is certain: The record will not be neat and tidy. It will not divide naturally and easily into chapters with such convenient labels as "Combinatorics" or "Geometry" or "Algebra." The more the history of these times is artificially divided into such categories, the more it will mislead the inquirers who ask, "How did it all happen?"

This century presents a paradox to the historian. The high affairs of state, the spectacular in sports, and the splash of celebrities are recorded on film and videotape, in books, newspapers and magazines, in detail unparalleled in the past. However, the electronic revolution that has helped make such thorough documentation possible, renders the doings of the less conspicuous more ephemeral. A telephone conversation leaves no trace, while letters used to be saved and published in many a correspondence or Briefwechsel. Who, anymore, spends time on a diary? Even the ease of jet travel helps undermine the written record. Granted, journals do publish the research, but high page costs encourage a terse expository style. There is little room to recount the origin of the paper or to reveal what the "theorem is all about." Survey articles, while attempting to compensate for this omission, can hardly provide this background, since their starting point is usually the published literature. As George Pólya said in a recently published interview [2, pp. 16, 17]:

> ...Among the old mathematicians, I was most influenced by Euler and mostly because Euler did something that no other great mathematician of his stature did. He explained how he found his results and I was deeply interested in that. It has to do with my interest in problem solving. ...And as I came to mathematics and learned something of it, I thought: Well, it is so, I see, the proof seems to be conclusive, but how can people find such results? My difficulty in mathematics: How was it discovered?

Mathematicians are not given to writing and certainly seldom to autobiographical writing. Later centuries may know as little of the leading mathematicians of this age as we know of Shakespeare. They may be able to uncover as little of how the triumphs of this age were achieved as we can of the manner of constructing the pyramids of ancient Egypt. The trail we trace in this book would have disappeared in another thirty or forty years. Fortunately, the mathematicians who followed this route through their contributions are still very much alive. One gladly participated in many hours of taped interviews. Another, who abhorred the telephone, was generous with his written replies to my letters. A third, who is notorious for never replying to a letter, was quite willing to talk at length on the transatlantic phone. Indeed, their enthusiasm and their belief in the merit of this project helped sustain my efforts over the two years it took to clarify the history and the mathematics.

But even with all this encouragement and help, I was not able to paint a complete picture. There is still disagreement on some of the historical facts. On others, memories could not fill all the gaps. In even one case, antipathy for the entire project, indeed on the history of mathematics in any form, closed the door on the corroboration of certain details.

I was myself many times astonished by the quirks and flukes, the coincidences that lay behind the discoveries. One mathematician was goaded by a contrary machine. A physicist was inspired by a rather obscure example tucked away in the middle of a paper. Another mathematician believed on the basis of quite flimsy evidence that a certain group of symmetries would contain a large simple group. And yet another, who studied this group and determined its order, was so certain that it was simple he didn't even bother to stop and verify that it was.

The trail of mathematical discovery is strewn with gambles, hunches, oddities of character, luck, idiosyncrasies of background, and fortunate encounters. Chaos and spontaneity, not austere order, mark the way much of mathematics is done. The elegance of the printed article conceals the circumstances that precipitated the results. The history of mathematics is a story of people and their guesses, misfortune and struggle, not merely a list of theorems and their proofs.

In this book, we have taken a somewhat novel approach. We describe the mathematics in complete detail and its origin and evolution as well. Where the journal article is concise, because it was written with a narrow audience in mind, we have supplied the missing steps. Thus, any mathematician with even a casual acquaintance with vector spaces and groups should be able to follow each mathematical step. In fact, I wrote each page so that an upper-division student could understand it.

Along the way we will see the interplay between applied and pure mathematics, the importance of direct personal contact, and of conferences and travel. Implicitly, we can deduce the danger of isolation, of being outside the opportunity of chance contact, of missing the stimulation that comes from browsing through new books. And we will see how mathematics is discovered and lost, only to be rediscovered.

While the reader may draw many a moral from our tale, I hope that the story is of interest for its own sake. Moreover, I hope that it may inspire others, participants or observers, to preserve the true and complete record of our mathematical times. I have followed only one thread in the intricate tapestry. Many others must be carefully scrutinized before we can weave the complex design of the whole. The eventual record will serve not only the purposes of history, but also mathematics itself, providing students now and in future generations a deeper insight into the mathematics that is their heritage.

The book consists of three chapters, each describing one topic: error-correcting codes, sphere packings and simple groups. Their connection was the basis of our interest.

Chapter 1 begins with an introduction to coding and leads directly into the early, unpublished work of Hamming. This, in turn, is followed by the introduction of Golay, whose work was inspired by that of Hamming through an example cited by C. E. Shannon in his classical treatise on information theory. The priority controversy between Hamming and Golay which ensued because of Hamming's delay in publishing due to patent considerations (Bell Laboratories was actually able to patent his mathematical code!) is discussed in the last section.

Chapter 2 describes the bridge between the Golay codes and a surprisingly dense sphere packing in twenty-four dimensional Euclidean space E^{24} discovered by John Leech. This packing actually evolved in two steps, the second being the trigger for the discovery of new large simple groups, the topic of Chapter 3. The history of all the influences that led to Leech's discovery remains obscured. Certainly, Golay's work played a major role. But Leech also referenced L. J. Paige and E. H. Spanier whose work, while reminiscent of that of Golay, actually dealt with some well-known simple groups, those of E. Mathieu which had been discovered nearly one hundred years earlier.

Leech's lattice in E^{24} is the focal point of Chapter 3. Leech wanted to know its symmetries. Unable to answer this question and convinced that the answer would "repay investigation" **[81]**, Leech tried to interest others in his question. Finally, J. H. Conway, informed about the lattice by John McKay and challenged to work on it by John Thompson, answered the question. The result was the addition of three new simple groups to the growing list of new sporadic simple groups.

Two of the appendices may be of special interest to the reader. Appendix 1 summarizes the densest known sphere packings, and Appendix 6 contains the now complete list of sporadic simple groups.

Much credit for the oral history material in this book goes to those who actually made the history. Without their coop-

eration, the task could not have been completed. Professor John Leech, besides carefully reviewing the chapter on his work, offered valuable comments on the other two chapters. The University of California at Davis and Walla Walla College in College Place, Washington, provided the necessary financial assistance. I am deeply indebted to Professors C. R. Borges, G. D. Chakerian and especially to S. K. Stein for their inspiration, encouragement, suggestions and careful reading of the manuscript, and to Bonnie-Jean McNiel who braved a mass of ever-changing notation to type it.

THOMAS M. THOMPSON

College Place, Washington
February, 1983