

# Index

- $(uH^2)^\perp$ , 112
- $A$ , 117
- $A^*$ , 62
- $BH^2$ , 109
- $C(A)$ , 163
- $C_\varphi$ , 236
- $C_{M_{a,\phi}} : \mathcal{B} \rightarrow \mathcal{B}$ , 192
- $H^2$ , 106
- $H^2(\mathbb{D})$ , 105
- $H_A + iK_A$ , 161
- $I_\ell$ , 148
- $I_{\ell,m}$ , 148
- $J_3$ , 164
- $J_n$ , 103
- $K_u$ , 112
- $L^2(\mathbb{T})$ , 104
- $M_\ell$ , 162
- $M_{a,\phi}$ , 177
- $P_+$ , 113
- $P_-$ , 113
- $P_u$ , 113
- $S$ , 108
- $S^\perp$ , 71
- $S^*$ , 111
- $S_u$ , 113
- $U_\lambda$ , 119
- $W(A)$ , 14
- $X_{p,q}$ , 148
- $\hat{f}(n)$ , 105
- $\langle\langle x \rangle\rangle$ , 90
- $\langle x, y \rangle$ , 13
- $[x]$ , 98
- $\mathbb{C}$ , 14
- $\mathbb{C}^*$ , 134
- $\mathbb{C}^n$ , 13
- $\mathbb{P}^2(\mathbb{C})$ , 163
- $\mathbb{P}^2(\mathbb{R})$ , 48
- $\mathbb{R}^*$ , 134
- $\mathbb{T}$ , 17
- $\mathbf{H}^+$ , 134
- $\mathbf{H}^-$ , 134
- $\mathcal{B}$ , 192
- $\mathcal{S}_n$ , 103
- $\|A\|$ , 63, 76
- $\|x\|$ , 13
- $\omega_+$ , 242
- $\omega_-$ , 242
- $\overleftrightarrow{uv}$ , 54, 179
- $\psi_c$ , 169, 177
- $\sigma(E)$ , 123, 232
- $\tilde{k}_a$ , 107
- $\text{tr}(A)$ , 16, 234
- $\varphi_a$ , 114
- $e_j$ , standard basis  $L^2(\mathbb{T})$ , 104
- $e_j$ , standard basis  $\mathbb{C}^n$ , 63
- $f^{(n)}$ , 92
- $f_r$ , 105
- $k_a$ , 107
- $w(A)$ , 232

- $w(T)$ , 223
- adjoint, 16
- affine part, 59
- algebraic curve, 51
- applets ☺, x
  - Blaschke product tools, 29–33, 121, 159, 170, 177, 209, 251
  - experimental tools, 138
  - geometry tools, 8, 53, 56, 217
- astroid, 204
- Bedford, Frank, 88
- Bell, Eric Temple, 1
- Benford's Law, 88
- Beurling's theorem, 111
- Blaschke
  - curve, 129, 174
  - ellipse
    - $(n, p)$ -ellipse, 184
    - 3-ellipse, 33, 43, 60, 82
  - product, 19, 23, 28
    - characterization, 19, 26
    - composition, 159, 169, 171
    - decomposable, 169
    - degree, 19
    - degree-2, 35
    - degree-3, 37–44, 82
    - degree-4, 169–174
    - finite, 19
    - infinite, 111
    - normalized, 23
    - properties, 28, 37–38
    - unicritical, 253
- Blaschke, Wilhelm, 19
- Borcea, J., 209
- boundary generating curve, 163
- Braikenridge–MacLaurin theorem, 57
- Brianchon's theorem, 55
- Brianchon, Charles, 57
- Cauchy kernel, 107
- Cauchy–Bunyakovsky–Schwarz inequality, 66, 79, 108
- chain of circles between  $\mathcal{C}_1$  and  $\mathcal{C}_2$ , 213
- Chapple, William, 45
- Chapple–Euler formula, 43–46, 175, 197
- circular point, 242
- circumscribing a conic, 55
- class  $\mathcal{S}_n$ , 103, 162
- Colosseum, 3
- companion matrix, 245
- complete orthonormal basis, 105
- composition operator, 192, 236
- compressed shift operator, 113
- compression, 71
  - matrix, 117
- conic, 51
  - associated matrix, 51
  - degenerate, 52
- constructing an ellipse
  - envelope method, 201
  - folding, 7–10
  - pin and string method, 201
- contraction, 76
- curve
  - algebraic, 51
  - class, 163, 241
  - degree, 163, 241
  - dual, 51, 163, 241

- real part, 163
  - tangent line, 51
- cyclic quadrilateral, 221
- Dégot, J., 207
- degenerate, 52
- Denjoy–Wolff point, 251
- Denjoy–Wolff theorem, 251
- Descartes’s circle theorem, 221
- Descartes, René, 85
- diagonal, 55
- direct sum
  - of matrices, 166
- disk automorphism, 23, 134, 177
  - elliptic, 178
    - canonical, 182
    - characterization, 178
    - convex order, 182
    - order, 182
  - hyperbolic, 178
  - parabolic, 178
- dual, 51, 241–242
- dual curve, 163
- dual equation, 241
- duality, 47, 57
- eigenvalue, 14, 232
  - Hermitian matrix, 161
- eigenvector, 14
- ellipse, 3
  - area enclosed, 5
  - construction, 8–10
  - diameter, 3
  - equation, 4
  - focus, 3
  - major axis, 3
  - minor axis, 4
  - perimeter, 5–7
- Ellipse, President’s Park South, 3
- elliptical range theorem, 16, 62, 68–71, 164, 173, 222
- envelope, 202, 204
- Euclidean norm, 13
- Euler, Leonhard, 45
- Fatou’s radial limit theorem, 106
- field of values, 61
- first digit, 88
- focus, 242
  - ellipse, 3
  - real, 242
  - singular, 243
- Fourier coefficient, 105
- Frantz’s theorem, 179
- frequency, 90
- Fuss’s theorem, 175, 197
- Fuss, Nicolaus, 45
- Gauss–Lucas theorem, 206
- Gelfand’s questions, 90, 98
- general position
  - of lines, 55
  - of points, 53
- Gergonne, Joseph, 57
- glissette, 224
- half-plane
  - lower, 134
  - upper, 134
- Halmos, Paul, vii, 125
- Hardy space, 105
- Hermitian matrix
  - eigenvalue, 161
  - numerical range, 161

- hexagon
  - diagonal, 55
  - inscribed in conic, 53
  - opposite sides, 53
  - opposite vertices, 55
  - principal diagonal, 55
  - side, 53
  - simple, 53
- hexagrammum mysticum theorem, 57
- Hilbert space, 76
- homogeneous polynomial, 51
- hyperbolic
  - convex hull, 209
  - convex set, 209
- hyperbolic geometry, 208
- Illiev, L., 206
- inner function, 109
- inner product
  - $L^2(\mathbb{T})$ , 104
  - standard, 13
- inscribed in a conic, 53
- interpolation, 134, 237–241
- invariant subspace, 109, 111
- inverse of a point, 216
- inversion, 215–221
- iterate, 92, 182, 251
- Jensen's theorem, 206
- Jordan block, 78, 103, 131, 164, 168, 222
- Kepler, Johannes, 6
- King, Jonathan, 91
- Kippenhahn curve, 163
- Kippenhahn's theorem, 163
- Kippenhahn, Rudolf, 123, 159–160
- Lagrange form, 140
- Lebesgue space, 104
- Lebesgue, Henri, 104
- line at infinity, 48
- linear fractional transformation, 134
- Möbius transformations, 134
- Maclaurin expansion, 7
- Maclaurin, C., 7
- major axis, 3
- Mandart inellipse, 212
- Marden, M., 210
- matrix
  - adjoint, 16
  - associated to conic, 51
  - Cartesian decomposition, 161
  - companion, 245
  - direct sum, 166
  - Hermitian, 160, 222
  - imaginary part, 161
  - nilpotent, 222
  - nonnegative, 228
  - norm, 63, 76
  - normal, 62
  - normal, numerical range, 68, 75
  - positive, 228
  - positive definite, 222
  - real part, 161
  - reducible, 124, 246
  - self-adjoint, 62, 160
  - similar, 63
  - trace, 16, 69, 234
  - unitarily equivalent, 63
  - unitary, 62, 75
  - unitary decomposition, 123

- maximum modulus theorem, 26
- measure, 95–97
- Mersenne, Marin, 57
- minor axis, 4
- model space, 112
- negative pedal curve, 227
- Nelson, Wayne James, 87
- nephroid, 205
- nested form, 240
- Newcomb, Simon, 88
- Newton form, 238
- nilpotent, 222
- nonnegative matrix, 228
- norm
  - Euclidean, 13
  - matrix, 63
  - vector, 13
- normalized Blaschke product, 23
- numerical radius
  - matrix, 224, 232
  - operator, 223
- numerical range, 14, 61
  - circular disk, 174
  - elliptical disk, 159, 168, 171
  - Hermitian matrix, 161, 222
  - normal matrix, 75
  - properties, 63, 68
- operator
  - quasi-nilpotent, 223
  - bounded, 76
  - contraction, 77
  - nilpotent, 222
  - norm, 77
  - shift, 108
- opposite sides, 53
- opposite vertices, 55
- orthogonal complement, 71
- orthogonal projection, 71
- oval, 165
- parallel postulate, 208
- Parseval's identity, 105
- Pascal line, 57
- Pascal's theorem, 53
- Pascal, Blaise, 57
- pedal curve, 227
- pedal point, 227
- Perron–Frobenius theorem, 228
- phase portrait, 30–32
- Poincaré model, 208
- points at infinity, 48
- polygonal chain
  - inscribed in conic, 53
- polynomial
  - homogenous, 51
- Poncelet
  - curve, 121, 130
  - ellipse
    - 3-ellipse, 60, 82
    - 4-ellipse, 175
    - convex, 175
  - property, 121
- Poncelet's theorem, 92
  - alternate version, 157
  - false statements, 158
  - for triangles, 18, 57
  - general, 156
- Poncelet, Jean-Victor, 47
- porism, 213
- positive matrix, 228
- principal diagonal, 55

- projection, 71, 113
- projective geometry, 47
- projective coordinates, 48
- projective space, 48
- Ptolemy's theorem, 221
  
- quasi-nilpotent, 223
  
- radial limit, 106
- radical axis, 219
- rational function, 135
  - degree, 135
- reference circle, 216
- reproducing kernel, 107
  - normalized, 107, 114
  
- Saratov notebook, 47
- Schmeisser, G., 207
- Schur's theorem, 64
- Sendov's conjecture, 206
- shift
  - backward, 111
  - forward, 109
- shift operator, 108
  - adjoint, 111
  - compressed, 113
- side of a polygonal chain, 53
- Siebeck's theorem
  - general, 243
  - triangles, 211
- Soddy's formula, 221
- spectral radius, 231
- spectral theorem
  - for normal matrices, 67
- spectrum, 123, 232
- standard form, 117
- standard inner product, 13
  
- Steiner inellipse, 39, 211
- Steiner's porism, 213
- Steiner's theorem, 210
- Stone–Weierstrass theorem, 26
- strongly real of positive type, 135, 239
- support line, 124, 162–164
  
- Takenaka–Malmquist basis, 114
- tangent line, 51
- tangential equation, 241
- Toeplitz, Otto, 61
- Toeplitz–Hausdorff theorem, 61–62, 71–73
- trace, 16, 234
- Traité des propriétés projectives des figures, 48
  
- unit circle, 17
- unitary 1-dilation, 76–82, 119
- unitary dilation, 76
  
- vector, 49
  - cross product, 49
  - dot product, 49
  - stochastic, 229
  
- Walsh's two-circle theorem, 206
- Weyl's theorem, 98
  
- zero inclusion question, 236