Introduction

One reason why combinatorics has been slow to become accepted as part of mainstream mathematics is the common belief that it consists of a bag of tricks in many areas:

> combinatorial number theory (partitions, integer sequences), combinatorial set theory, Ramsey theory, partially ordered sets, lattices (in the poset, geometric, and number-theoretic senses), error-correcting codes, combinatorial designs (latin squares and rectangles, projective and affine geometries, Steiner systems, Kirkman's Schoolgirls Problem), combinatorial games, enumerative combinatorics (recurrence relations, generating functions), 0–1 matrices, graph theory (including tournaments, topological properties, coloring problems, networks), recreational mathematics, scheduling, combinatorial geometry, packing and covering (in number-theoretic, set-theoretic, graph-theoretic or geometric contexts),

with little or no connection between them. We shall see that they have numerous threads weaving them together into a beautifully patterned tapestry.[1]

The reader may wonder how we intend to accomplish this. That, we can tell you in one word: examples.

Examples are the life blood of combinatorics. Careful study and exploration of examples has developed into many and varied areas of research in combinatorics—and we know that our examples can lead you

[1] Reprinted/adapted by permission from Springer Nature: Kluwer Academic Publishers, "The Unity of Combinatorics" by Richard K. Guy, in C. J. Colbourn and E. S. Mahmoodian (eds.), *Combinatorics Advances*, 129–159 © 1995. Additional selected material appears in Chapters 1, 2, 3, 6, 7, 8, 10, and 15.

to learn about, appreciate, and enjoy the combinatorial world. Thus, a child playing with six colored blocks found a pattern that opened up the active and growing world of Langford sequences. The solution to Fibonacci's silly and totally unrealistic problem about rabbits led to the world of recurrence relations and recursions. De Bruijn's example of a circular arrangement of four 0's and four 1's that contain all eight bit strings of length three exactly once led to a general study of such sequences with numerous applications in signal processing and computer science. Euler's explanation of why the Seven Bridges of Königsberg could not be traversed without repeating an edge gave birth to the vast area of graph theory.

Pictures and diagrams are useful and sometimes beautiful ways to view these examples, so we have included lots of pictures. Some of the pictures show up more than once; the Fano plane shows up eight times because of its connections with so many areas of combinatorics. We provide definitions where they are needed, and sometimes you will find that the definition of some term has been repeated. This saves you the time and effort of going to the index and then paging back through the text to find the first occurrence of the term.

We include statements of theorems, supplying references in which you can find the proofs. We do provide proofs of certain theorems that, at first glance, seem hard to believe. To cite one example, Samuel Beatty's theorem states that if α and β are irrational numbers greater than 1, then the sequences $\{\lfloor n\alpha \rfloor\}$ and $\{\lfloor n\beta \rfloor\}$ contain every positive integer without repetition if and only if $1/\alpha + 1/\beta = 1$. This is by no means obvious, and so we give a proof.

As for the level of difficulty, let us remind you of the original goal of the Carus Monograph Series. Mary Carus's original intent was that books in the Carus series be accessible "not only to mathematicians but to scientific workers and others with a modest mathematical background." H. E. Slaught, who probably encouraged Carus to make the original donation, hoped to reach "that still wider circle of thoughtful people who, having a moderate acquaintance with elementary mathematics, are quite willing and eager to extend that acquaintance indefinitely along informational lines, provided it can be done without prolonged and painful study of the mathematical treatises which abound in extreme rigor and endless detail."

Some of the chapters, sections, or subsections can be read in bed or while commuting or traveling (plane, train, bus, subway, or streetcar)—very much like reading the MAA periodical *Math Horizons*.

Some of them can be read sitting up in a chair. In this they are like papers in the *College Math Journal*.

The majority of the parts of this book are reminiscent of papers in *Mathematics Magazine*: they can be read sitting at a table or standing by a board, armed with writing implements (and possibly a computer algebra system).

A few are like articles in the *American Mathematical Monthly*: they should be read standing by a board, armed with writing implements (and possibly a computer algebra system), and the reader should prepared to do a fair amount of work and a whole lot of pacing.

Each of our topics will prompt some of our readers to ask questions and to pursue the topic, to which we say: Go For It.

This is, after all, a math book.