Preface

The Unity of Combinatorics, or *TUoC*, began as Richard K. Guy's plenary lecture at the combinatorics session of an international conference in 1994. Richard wrote up the lecture as a 30-page paper that appeared in the 1995 session proceedings "Combinatorics Advances". *TUoC* was framed as an outline for a series of expository lectures on the various topics in the paper. It was a quick but fascinating survey of many different combinatorial problems and their relations to one another.

Some years later, Don Albers, Publications Director of the Mathematical Association of America (the MAA) approached Richard with a proposal that the MAA publish an expanded version of *TUoC* in their Carus Monograph Series. Some time later, Don and Richard—now in his 90s—decided to take on a younger coauthor to help get the project moving to completion. That younger author turned out to be the 70ish Ezra (Bud) Brown, a number theorist, combinatorialist, and expository writer of mathematics. Their combined efforts resulted in the book you are now reading.

We have divided Richard's original paper into chapters, expanded the exposition, added explanations, examples, and references, built up background material as needed, and augmented the original paper with additional connections as seemed appropriate. Eleven chapters consist of previously published articles or excerpts of articles relevant to the topics outlined in *TUoC*; here is a list of their titles and (authors).

- Chapter 4: "Catwalks, Sandsteps, and Pascal pyramids" (Richard K. Guy)
- Chapter 5: "Unique rook circuits" (Richard K. Guy and Mark Paulhus)

- Chapter 9: "The groups $PSL(2,7)$ and $GL(3,2)$ and why they are isomorphic" (Ezra Brown and Nicholas Loehr)
- Chapter 11: "Kirkman's schoolgirls, fields, spreads, and hats" (Ezra Brown and Keith Mellinger)
- Chapter 12: "$(7,3,1)$ and combinatorics" (Ezra Brown)
- Chapter 13: "$(7,3,1)$ and normed algebras" (Ezra Brown)
- Chapter 14: "$(7,3,1)$ and matroids" (David Neel and Nancy Neudauer)
- Chapter 16: "The $(11,5,2)$ biplane, codes, designs, and groups" (Ezra Brown)
- Chapter 17: "Rick's Tricky Six Puzzle: more than meets the eye" (Alex Fink and Richard K. Guy)
- Chapter 18: "$S(5,8,24)$" (Ian Anderson)
- Chapter 19: "The Miracle Octad Generator" (Robert Curtis)

Chapters 1, 2, 3, 6, 7, 8, 10, and 15 are Guy–Brown collaborations. Thus, we have modeled the structure on John H. Conway and Neil J. A. Sloane's classic reference work, "Sphere Packings, Lattices and Groups" (see [**46**]).

We begin with sequences, including Langford sequences, Beatty sequences, the Fibonacci numbers, Pascal's triangle, and the Catalan numbers. Topics from graph theory include colorings, packings, embeddings, Kirchoff's current graphs, and perfect squared squares. Topics from combinatorial designs include finite projective planes, block designs, Steiner systems, Kirkman's resolvable designs, difference sets, and symmetries of combinatorial designs. Other topics include Penrose tilings, combinatorial game theory, matroids, error-correcting codes, matrices with interesting combinatorial properties, and finite geometries. Along the way, we identify and describe some of the many connections between and among these topics.

Chapter 15, which treats such curious combinatorial games as Mock Turtles, Turning Corners, Moebius, and Mogul is a bridge between the

topics treated in Richard's original paper and the more advanced Chapter 17 on Rick's Tricky Six Puzzle by Alex Fink and Richard. In Chapter 15, we make mention of binary Golay codes, Mathieu groups, and the Leech lattice of dimension 24. Chapter 16 on the $(11, 5, 2)$ biplane is another way to connect all of the above three objects, as well as the ternary Golay codes and the Steiner systems $S(4, 5, 11)$ and $S(5, 6, 12)$.

We conclude with two chapters on that most unusual of all combinatorial designs, the Steiner system $S(5, 8, 24)$. This is a collection of octads (eight-element subsets of a 24-element set Ω) such that every five-element subset of Ω is contained in exactly one octad. Chapter 18 begins with a question: based only on this definition, what can we learn about the internal structure of such an $S(5, 8, 24)$? (Quite a lot, as it turns out.) Chapter 19 is Rob Curtis's beautiful exposition of the Miracle Octad Generator, an object constructed from an $S(5, 8, 24)$ that will take a five-element subset of the 24-element set Ω and find the unique octad containing that five-element subset.

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