

Preface

This book is meant to be a succinct, self-contained, but thorough introduction to the state-of-the-art of the so-called Erdős-Falconer Distance Problem, also known as the finite field distance problem. This problem has the fortune of being new, easily understood, and yet it remains unsolved, making this topic ideal for study by advanced undergraduates, graduate students, and research mathematicians alike.

The book is structured as follows. In Chapter 1 we start with background material such as a review of properties of finite fields and some basic ideas in combinatorics. In Chapter 2 we will outline the original distance problem by Erdős, a continuous version of the problem by Falconer, and a finite field distance problem which will be the core topic of the book. Chapter 3 will be spent proving the first explicit result on the Erdős-Falconer distance problem in great detail. Chapter 4 will introduce the topic of restriction theory from harmonic analysis, and we will discuss the role that it plays in proving results on the size of distance sets. Chapter 5 will include many of the generalizations of the distance problem including an extension of the problem to finite rings. Chapter 6 will outline a group-theoretic approach to the distance problem pioneered by Elekes and Sharir ([43]) and ultimately used by Guth and Katz ([70]) to establish the Erdős distance problem in the plane. We will discuss what implications

these ideas will have in regards to the finite field distance problem. The final chapter (Chapter 7) will be devoted to other topics in the area including incidence theory, sum-product phenomena, the Kakeya conjecture, the cap set problem, and Waring's problem. The goal of this final chapter is to introduce the non-expert to a handful of research problems in the area, some of which are still active areas of research, while others only having been resolved very recently. We provide enough background in finite fields to allow the reader to understand the material we discuss here. Otherwise, we provide only a cursory introduction to each topic, leaving many resources and references for the motivated reader.