

Contents

Preface	ix
Acknowledgments	xi
Chapter 1 Background	1
1.1 Equivalence relations and the pigeonhole principle	1
1.2 Algebra and finite fields	8
1.3 Basic inequalities	16
1.4 Notation	17
1.5 Exercises: Chapter 1	18
Chapter 2 The distance problem	21
2.1 Introduction to the distance problem	21
2.2 Falconer distance problem	25
2.3 Finite field distance problem	28
2.4 Exercises: Chapter 2	32
Chapter 3 The Iosevich-Rudnev bound	35
3.1 Counting-method	35
3.2 The L^2 -method	41
3.3 Finite field spherical averages	42
3.4 Size and decay estimates for spheres	52
3.5 Finite field counterexample	57
3.6 Relations to the Falconer problem	59
3.7 Exercises: Chapter 3	60
Chapter 4 Wolff's exponent	63
4.1 Introduction	63
4.2 Proof of L^2 estimate for $\nu(t)$	65
	vii

4.3	Restriction and extension theory	70
4.4	Exercises: Chapter 4	73
Chapter 5	Rings and generalized distances	75
5.1	Distances in finite rings	75
5.2	Distances between two sets	91
5.3	Generalized distances	94
5.4	Pinned distances	98
5.5	Exercises: Chapter 5	103
Chapter 6	Configurations and group actions	105
6.1	Finite configurations	105
6.2	The Elekes-Sharir framework	111
6.3	Triangles: The “7/4” bound	113
6.4	Triangles: The “8/5” bound	117
6.5	Distance graph	120
6.6	Exercises: Chapter 6	126
Chapter 7	Combinatorics in finite fields	129
7.1	Incidence theory	129
7.2	Sum-product phenomena	137
7.3	Kakeya conjecture	146
7.4	Waring’s theorem	152
7.5	Roth’s theorem and the cap-set problem	155
7.6	The spectral gap theorem	161
7.7	Exercises: Chapter 7	166
Bibliography		169
Index		179