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Teaching Mathematics in
Colleges and Universities:
Case Studies for
Today's Classroom

Faculty Edition

Solomon Friedberg

Avner Ash
Elizabeth Brown
Deborah Hughes Hallett
Reva Kasman
Margaret Kenney
Lisa A. Mantini
William McCallum
Jeremy Teitelbaum
Lee Zia



American Mathematical Society
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Introduction

Progress in mathematics frequently occurs by first studying particular examples and then generalizing the patterns which have been observed into far-reaching Theorems. Similarly, in teaching mathematics one frequently employs examples to motivate a general principle or to illustrate its use. This volume employs the same idea in the context of learning *how* to teach: by analyzing particular teaching situations one may develop broadly applicable teaching skills useful for the professional mathematician. These teaching situations are the Case Studies of the title. Just as a good mathematician seeks to understand the details of a particular problem but also to put it in a broader context, the examples presented are chosen to offer a serious set of detailed teaching issues but also to afford analysis from a broad perspective.

Why use examples to develop teaching skills, rather than simply giving general principles? One reason is that it is difficult to learn teaching solely from such principles. Just as ‘doing the exercises’ is an integral part of learning mathematics (if the exercises are well-conceived rather than busy-work), these Case Studies may be regarded as teaching exercises, and can play a similar role in gaining teaching expertise. A second is that no two people have the exact same idea of what good teaching actually is— in contrast to mathematics, there is frequently no one right answer. Even highly regarded teachers possess different skills and achieve different outcomes; one may enable the better students to perform at a very high level, while another shows the weaker students that, for the first time in their lives, they can do mathematics. Similarly, there is no one right answer to the Case Studies presented here. In other words, principles of good teaching are personal, and the goal here is for each person to critically develop such principles, but not to arrive at the same set of them. Finally, in teaching every day is different. To be a successful teacher, it is important to be able to analyze and deal with classroom situations as they develop. The Case Studies prepared by this project present a broad range of teaching scenarios, and give participants the opportunity to think them through. Doing so will help prepare for the next, once again different, classroom experience.

One aspect of good teaching is technical: write legibly, use the board effectively, speak audibly. These Cases do not address these issues. Rather, their focus is on more conceptual issues, in the broad areas of mathematical content as perceived by the students, of pedagogy, and of faculty-TA relations. For example, how does one help students to truly master the big ideas, such as the derivative, the integral, and the relation between them?

Manage a classroom of students with a wide range of background knowledge and of ability? Balance teaching and other responsibilities, such as completing one's dissertation?

Finally, just what is a Case Study, and what does one do with it? A Case is an excerpt from a teaching situation, described from the perspective of various students and of the instructor. The Case raises a variety of pedagogical and communication issues, to be explored and analyzed in group discussion, for example by a group of graduate teaching assistants together with a faculty facilitator. Group consideration of a Case offers the advantage of drawing upon collective experience and diverse perspectives, and allows different issues, ideas, and strategies to be considered and discussed. The methodology of Case Studies is widely used in this way in areas such as business and law, and also in teacher development, both university-level and precollegiate, in diverse subjects from the humanities to accounting. Alternatively, a reader working independently may take each Case as an exercise, thinking about the situation, asking what the different issues are, what he or she would do next or would have done differently, what can be learned.

We hope that the consideration of these mathematics Cases, in either a group or an individual setting, will be thought-provoking, and will help each reader to develop high-quality teaching skills for use in his or her own classroom.