Preface

This text arises from ten lectures given by the first-named author at the NSF/CBMS conference "The Interface between Convex Geometry and Harmonic Analysis" held on July 29–August 3, 2006 at Kansas State University in Manhattan, KS. The main topic of these lectures is the Fourier analytic approach to the geometry of convex bodies developed over the last few years. The idea of this approach is to express certain geometric properties of convex bodies in terms of the Fourier transform and then apply methods of harmonic analysis to solve geometric problems. The Fourier approach has led to several important results, including an analytic solution of the Busemann-Petty problem on sections of convex bodies, characterizations of intersection bodies and their connections with the theory of L_p-spaces, extremal sections and projections of certain classes of bodies, and a unified approach to sections and projections of convex bodies.

The main features of the Fourier approach to convexity were reflected in the book "Fourier Analysis in Convex Geometry" written by the first-named author and published in 2005 in the Mathematical Surveys and Monographs series of the American Mathematical Society. That book includes rigorous proofs of all main results that appeared before the year 2005, as well as short descriptions of the main tools from convexity, Fourier analysis, integral geometry and probability used in the proofs of these results. The main purpose of the current book is different; it exposes in a short form the main ideas of the Fourier approach to geometry so that interested researchers and students can quickly learn the subject and start working on related problems. Beyond that, we include here several interesting new results that have appeared after the book [**K9**], in particular the solution of the Busemann-Petty problem in non-Euclidean spaces, non-equivalence of several generalizations of intersection bodies, new methods of constructing non-intersection bodies, and a continuous path between intersection and polar projection bodies leading to some insights about the mysterious duality between sections and projections of convex bodies. The last chapter includes several open problems and discussions of related results.

The structure of the book is as follows. Every chapter starts with a section including the actual lecture given at the conference. We recommend that beginners start by reading the first section of each chapter only. This way the reader gets to know the main definitions, concepts, and results. The details of proofs are sometimes not given – we prefer to expose the main underlying ideas – but in every such place we give references to the book [**K9**] or to one of the secondary sections of this book. After reading the first sections of all chapters, the reader will be prepared for other sections of this book and for related papers not included in the text.

Finally, we would like to thank the organizers of the conference at Kansas State - David Auckly and Dmitri Ryabogin - for the wonderful job of putting together a meeting that included experienced researchers, young researchers, graduate students, and undergraduate students. The additional sessions that they organized for the students were very helpful. The first-named author acknowledges the support of the U.S. National Science Foundation through the grants DMS-0455696 and DMS-0652571. The second-named author acknowledges the support of the European Network PHD, FP6 Marie Curie Actions, RTN, Contract MCRN-511953, and the U.S. National Science Foundation, grant DMS-0455696.

Alexander Koldobsky and Vladyslav Yaskin