

Contents

Preface	vii
Chapter 1. Preliminaries	1
1. Riemann surfaces and line bundles	1
2. Introduction of first-order deformations	2
3. Hyperbolic geometry	4
4. Standard cusps and collars	6
5. Uniformization, $PSL(2; \mathbb{R})$ representation spaces and Mumford compactness	7
6. Collars converging to cusp pairs, version 1.0	8
7. Holomorphic plumbing fixture - collars converging to cusps, version 2.0	9
8. Further readings	11
Chapter 2. Teichmüller Space and Horizontal Strip Deformations	13
1. Definition of Teichmüller space	13
2. Deformations of concentric annuli and horizontal strips	15
3. Variational formulas for a horizontal strip	17
4. Plumbing family tangents and cotangents	20
5. Further readings	21
Chapter 3. Geodesic-Lengths, Twists and Symplectic Geometry	23
1. Basics of geodesic-lengths and twists	23
2. Twist derivatives and Riera's formula	25
3. Hessian of geodesic-length	27
4. Fenchel-Nielsen coordinates are canonical	29
5. Further readings	30
Chapter 4. Geometry of the Augmented Teichmüller Space, Part 1	33
1. Augmented Teichmüller space	33
2. Second order Masur type expansions	34
3. Model metric comparison	38
4. Teichmüller metric	39
5. Further readings	40
Chapter 5. Geometry of the Augmented Teichmüller Space, Part 2	43
1. $CAT(0)$ geometry and geodesics on $\overline{\mathcal{T}}$	43
2. Properties of Bers regions	46

3. Further readings	49
Chapter 6. Geometry of the Augmented Teichmüller Space, Part 3	51
1. Measured geodesic laminations	51
2. Visual spheres	53
3. Ending laminations for geodesics in \mathcal{T}	55
4. Alexandrov tangent cone	57
5. Teichmüller-Coxeter complex	60
6. Further readings	62
Chapter 7. Deformations of hyperbolic metrics and the curvature tensor	65
1. Prescribed curvature equation	65
2. Variational formulas	67
3. Plumbing expansion - collars converging to cusps, version 3.0	69
4. Further readings	71
Chapter 8. Collar expansions and exponential-distance sums	75
1. Example sums and expansions	75
2. Collar principle and the distant sum estimate	77
3. Bounds for single and double coset sums	78
4. Further readings	81
Chapter 9. Train tracks and the Mirzakhani volume recursion	83
1. Measured geodesic laminations and train tracks	83
2. McShane-Mirzakhani length identity	86
3. Mirzakhani volume recursion	87
4. Moduli volumes, symplectic reduction and tautological classes	92
5. Virasoro constraint equations and Witten-Kontsevich theory	96
Chapter 10. Mirzakhani prime simple geodesic theorem	99
1. Prime geodesic theorems	99
2. Counting integral multi curves	102
3. Finding the scaled orbit limit measure	104
4. Multi curve constants and Thurston volume integrals	106
Bibliography	109
Index	117