

Preface

Dynamics in all its variations is the study of change. In the usual physical context, change takes place within time. The objects of geometry are static and if there is any change, it is “in the eye of the beholder”. In fractal geometry this point takes on meaning, particularly in the form of changing degree of magnification and “zooming in” on an object. This suggests developing dynamical concepts appropriate to this framework.

In these notes, based on a series of lectures delivered at Kent State University in 2011, we show that ergodic theoretic concepts can be applied to the process of changing magnification to give insight to phenomena peculiar to fractals. An important step is showing how fractal dimension can be interpreted in terms of ergodic averages in an appropriate measure preserving system. The familiar phenomenon of self similarity appears as the analogue of periodicity in classical dynamics. We don't pursue the full implications of recurrence in the geometric context, but some examples of the related Ramsey type questions are considered.

The theory developed here and the major ideas originated in the papers [F] and [F']. It will develop that there is a close connection between dimension theory and rates of growth of trees. This is exploited in [FW] where analogs of Szemerédi's theorem are demonstrated in the context of trees.

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