Index

Artin conjecture
  over function fields, 12
  over number fields, 8
Artin L-functions
  for finite graphs, 61
  for finite quotients of $\mathcal{B}_3(F)$, 79
  identity on Artin L-functions for finite graphs, 62
  identity on Artin L-functions for finite quotients of $\mathcal{B}_3(F)$, 79
  over function fields, 12
  over number fields, 7

building $\mathcal{B}_3(F)$ attached to $PGL_3(F)$
  the type of a directed edge in, 74
building $\Delta(F)$ attached to $SL_4(F)$, 80
  apartments of, 80
  chambers of, 80
  directed chamber operators L_I and L'_I, 81
  directed chambers of, 81
  nonspecial vertices of, 80
  special vertices of, 80
  spin type chambers in, 83
  standard type chambers in, 83
  type i edge adjacency operator L_{P_i} on type i edges in, 81
  type 1 edges of, 81
  type 2 edges of, 81
  vertex adjacency operators A_1 and A_2 on special vertices of, 81
building $\mathcal{B}_3(F)$ attached to $PGL_3(F)$
  apartments of, 74
  the type of a directed path in, 75
building $\mathcal{B}_3(F)$ attached to $PGL_3(F)$, 74
  adjacency operator of directed chambers as an Iwahori Hecke operator, 77
  directed chambers of, 77
  galleries in, 76
  out-neighbors of a directed chamber of, 77
  type 1 edge adjacency operator as a parahoric Hecke operator, 76
  type 1 neighbors of a type 1 edge in, 75
  type 2 edge adjacency operator as a parahoric Hecke operator, 76
building $\mathcal{B}_n(F)$ attached to $PGL_n(F)$, 69
  1-skeleton as a Cayley graph, 73
  1-skeleton of, 69
  adjacent vertices/edges in, 69
  asymptotic behavior of the spectrum of A_ℓ on finite quotients of, 70
  chambers of, 69
  regular, 70
  the Hecke operators A_ℓ on vertices of, 69, 73
  the spectrum of A_ℓ, 70
  the type of a vertex of, 69
  vertices of, 69

Cayley graphs, 49
central simple division algebra, 71, 72
Chebotarev density theorem
  for 2-dimensional complexes, 85
  for function fields, 11
  for graph covers, 66
  for number fields, 7
  in general setup, 28
class number
  formula, 55
  of a function field, 11
  of a graph, 54
  of a number field, 4
  of a quaternion algebra, 54
closed geodesic cycles
  equivalent, 19, 36, 75
  in a compact quotient of upper half-plane, 19
  in a finite graph, 36
  in a finite quotient of $\Delta(F)$ of given type, 81
  in a finite quotient of $\mathcal{B}_3(F)$ of given type, 75
  primitive, 20, 36, 75
closed geodesic galleries
  in a finite quotient of $\Delta(F)$ of given type, 83

in a finite quotient of $\mathcal{B}_3(F)$ of given type, 76
closed points, 9
cohomology theory
cochain complex, 39
cochain map, 39
cohomology group, 39
common interlacing, 59
conjecture
Bilu and Linial, 58
Ramanujan, 53, 71, 72
Selberg, 22
covering radius
for a biregular bipartite tree, 57
for a regular tree, 45

decomposition subgroups of a Galois group
for graph covers, 65
for number fields, 6
Dirichlet
character, 2
density, 5, 66
Dirichlet's theorem on primes in arithmetic progressions
analogue in geometry, 32, 33
over $\mathbb{Q}$, 3

edge zeta function of type i for a finite quotient of $\mathcal{B}_3(F)$, 75
Eichler-Shimura relation, 12
eigenvalues/spectrum of a biregular bipartite graph
asymptotic behavior for largest nontrivial, 56
bounds for, 56
eigenvalues/spectrum of a regular graph
asymptotic behavior of largest nontrivial, 43
asymptotic behavior of smallest nontrivial, 44
bounds for, 43
probabilistic distribution of nontrivial, 46
trivial, 43
elements of $SL_2(\mathbb{R})$
classification of, 15–17
primitive, 18
expanders, 47
expanding constant, 47

flow, 29
Anosov, 29
topological mixing, 32
Frobenius conjugacy classes
for finite 2-dimensional complexes, 84
for finite quotients of $\mathcal{B}_3(F)$, 79
for graph covers, 65
for number fields, 7

gallery zeta function of a given type for a finite quotient of $\mathcal{B}_3(F)$, 76, 77
graph
adjacency matrix of, 37
biregular bipartite, 56
degree matrix of, 38
degree of a vertex in, 37
diameter of, 44
(directed) edge adjacency matrix of, 38
eigenvalues/spectrum of, 37
matching polynomial of, 59
regular, 37
spanning tree of, 54
group of symplectic similitudes $GSp_4(F)$, 81

inertia subgroup, 6

Jacobian of a curve, 11
Jacquet-Langlands correspondence, 53, 54, 71

L-functions
associated to triples $(\mathfrak{P}, N, \Gamma)$, 27
Dirichlet, 2
general setup, 27
Langlands, 78, 79, 82
nice package of, 28
package of, 28
Laplacian operator, 21, 25
determinant of, 24
properties of, 25
spectrum of, 25

nonwandering point, 31

$(p+1)$-regular tree
as $PGL_2(\mathbb{Q}_p)/PGL_2(\mathbb{Z}_p)$, 50
as a Cayley graph, 52
Hecke operator as the vertex adjacency operator on, 51
Iwahori-Hecke operator as the directed edge adjacency operator on, 51
Perron-Frobenius theorem, 65, 84
poles of the zeta function of a graph, 65
poles of the zeta function of a regular graph, 45, 54, 55
prime geodesic theorem
for 2-dimensional complexes, 84
for compact quotients of the upper half-plane, 23
for graphs, 65
in general setup, 28
prime ideal theorem, 4
prime number theorem
for curves over finite fields, 11
for rational prime numbers, 2
primes
in a finite graph, 36

in a finite quotient of the upper half-plane, 20
in geometry, 31
of given dimension and given type in a finite quotient of $\Delta(F)$, 81, 83
of given dimension and given type in a finite quotient of $\mathcal{B}_3(F)$, 75, 76
ramified in a number field, 5
splitting completely in a graph cover, 65
splitting completely in a number field extension, 5
unramified in a number field, 5

quaternion algebra
definite, 17
Hamiltonian, 51
indefinite, 17
split over a field, 17

Ramanujan complexes as finite quotients of $\mathcal{B}_n$ for $n \geq 3$
basic approach for, 71
condition in terms of other operators, 77, 78
condition in terms of the Hecke operators, 70
explicit constructions of, 71–73
Ramanujan graphs
biregular bipartite, 57
construction of biregular bipartite, 57
explicit constructions of regular, 51–53
nice features of, 48
regular, 45
regular planar examples of, 49, 50
Riemann Hypothesis
for curves over finite fields, 11
for Dedekind zeta functions, 4
for Dirichlet L-functions, 3
for finite quotients of $\mathcal{B}_3(F)$, 77
for finite regular graphs, 45
for Selberg zeta function, 22
for the Riemann zeta function, 1
for varieties over finite fields, 13
Riemann surfaces
as quotients of the upper half-plane, 17
spectrum of, 22

signing function, 58
spectral gap, 47
strong approximation theorem, 53, 71
symplectic group $Sp_4(F)$, 80

unramified covers of a graph, 62
characterization of, 68
degree of, 62
explicit construction for two-fold covers, 58
Galois cover, 63
Galois group of, 63
intermediate cover of, 63

zeros of
the Riemann zeta function, 1
the Selberg zeta function, 22
zeta function
Dedekind, 4
for a curve over a finite field, 10
for a finite quotient of $\Delta(F)$, 81, 82
for a finite quotient of $\mathcal{B}_3(F)$, 75, 76
for a finite undirected graph, 36–38, 56
for a variety over a finite field, 9, 13
for the reduction of a modular curve, 12
Ihara, 35
Riemann, 1
Ruelle, 23
Selberg, 21
zeta identity
for a finite graph, 39
for a finite quotient of $\Delta(F)$, 81, 82
for a finite quotient of $\mathcal{B}_3(F)$, 77