Preface

Probability theory is based on the notion of independence. The law of large numbers and the central limit theorem describe the asymptotics of independent variables. However, in many instances one needs to deal with correlated variables, for instance in statistical mechanics. A tremendous effort was carried out to deal with such questions, for instance by developing large deviations theory. However, such general techniques often concern systems whose interaction is of the same order as entropy. These lecture notes are concerned with strongly interacting systems where the interaction overcomes the entropy. Examples of such situations are given by the eigenvalues of random matrices or the uniform tiling of a given domain. We will discuss a technique to deal with such systems: the asymptotic analysis of Dyson-Schwinger (or loop) equations. More specifically, we shall show how to use these techniques to derive the law of large numbers and the central limit theorems. The Dyson-Schwinger equations first showed up in physics. In random matrix theory, they were used to formally compute matrix integrals by solving the topological recursion in the work of Ambjorn, Eynard, and many others. Johansson was the first to use them to rigorously derive the central limit theorem for the empirical measure of the eigenvalues of Gaussian random matrices. Since this seminal work, Dyson-Schwinger equations have been used to derive central limit theorems in many more cases. When I was asked to give the Minerva Lecture Series at Columbia, I thought it was the right time to collect a few of them to highlight the general scheme of this approach. I unfortunately mostly took the time to discuss my own work in this direction, even though I had origianlly planned to cover more related topics such as the local laws derived by Erdös-Yau et al. or more general Coulomb gases as studied by Leblé and Serfaty. I hope, however, that these lecture notes will motivate the reader to read and find more applications to the asymptotic analysis of Dyson-Schwinger equations.

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