

EDITOR'S PREFACE

Because the present translation of Felix Hausdorff's famous *Mengenlehre* is the work of more than one hand, and because during the considerable length of time in which this translation was in preparation, other duties, both personal and professional, have arisen to prevent the translators from seeing the work of translation through to completion, it has devolved upon me to make such changes, in terminology as well as language, as consistency in style would require.

A. G.

PREFACE TO THE SECOND ENGLISH EDITION

Professor R. L. Goodstein has been kind enough to write two appendixes for the present edition: Appendix E, on the contradictions in Naive Set Theory, and Appendix F, on the Axiom of Choice. The Editor wishes to thank Professor Goodstein for pointing out that such supplementation is desirable and for his kindness in acceding to the Editor's request that he write the Appendixes.

Thanks are also due to Professor Smbat Abian and Mr. Coley C. Mills, Jr. for the correction of a number of errata in the First Edition.

A. G.

FROM THE PREFACE TO THE SECOND EDITION

The present book has as its purpose an exposition of the most important theorems of the Theory of Sets, along with complete proofs, so that the reader should not find it necessary to go outside this book for supplementary details while, on the other hand, the book should enable him to undertake a more detailed study of the voluminous literature on the subject. The book does not presuppose any mathematical knowledge beyond the differential and integral calculus, but it does require a certain maturity in abstract reasoning; qualified college seniors and first year graduate students should have no difficulty in making the material their own. More difficult topics, at the end of individual chapters, may be skipped on a first reading. The reader who is interested only in acquiring the simplest facts of point-set theory can, after a quick perusal of the first two chapters, immediately attack the sixth. The mathematician will, I hope, find in this book some things that will be new to him, at least as regards formal presentation and, in particular, as regards the strengthening of theorems, the simplification of proofs, and the removal of unnecessary hypotheses.

The selection of material must of necessity be somewhat subjective, since the subject is so extensive and is still growing almost from day to day. Unavoidably, various expectations regarding the book, including some of the author's own, could not be realized. A textbook, after all, can not aim at the completeness of a comprehensive report. In this case, there was the additional restriction that the new edition had to be substantially curtailed in length as compared to the first edition (*Grundzüge der Mengenlehre*, Leipzig 1914 [Repr., Chelsea, 1955]); this would have necessitated a revision down to the smallest detail, and I found a complete rewriting of the book preferable. I thought it might be easiest to sacrifice, of the topics treated in the first edition, most of the theory of ordered sets, a subject that stands somewhat by itself, as well as the introduction to Lebesgue's theory of integration, which does not lack for exposition elsewhere. What is more to be regretted is the abandonment, owing to the necessity of saving space, of the *topological* point of view in point-set theory, which seems to have attracted many people to the first edition of this book; in this new edition, I have restricted myself to the simpler

theory of *metric* spaces and have given only a quick survey (§ 40) of topological spaces, which is a rather inadequate substitute. Finally, I have cut down the generality in the other direction as well, by omitting the special theory of Euclidean space (e.g., the Jordan Curve Theorem for Plane Curves); that is, roughly speaking, I have omitted all of the material that is based on approximate polygons and polyhedra. Thus, the reader will find a large number of theorems concerning Euclidean space, but only such theorems as hold for Euclidean space considered as a special case of a separable or a complete or a locally connected space, or the like.

As against this omission there are some additions, such as a more complete treatment of Borel Sets and Suslin Sets (discovered in 1917) as well as of the Baire functions. Also, continuous mappings and homeomorphisms are treated in more detail than they were in the first edition. As to a discussion of the so-called paradoxes and of questions regarding the foundations of the subject, I have decided to omit this from the present edition as well.

PREFACE TO THE THIRD EDITION

Set Theory has continued its steady and active development, and this in itself might have made desirable an actual revision of this book. However, circumstances have prevented my doing this. Accordingly, the first nine chapters are an almost unchanged reprint of the second edition; but in order to do at least partial justice to the progress achieved by the subject in the meantime, I have added a new — tenth — chapter covering two subjects that seem to be especially worthy of detailed exposition, and I have also touched upon three further topics in short supplements — without proofs, however. Had it not been for lack of space, the number of additional topics could have been considerably extended.