Preface

This textbook in geometry differs in several essential respects from other current texts covering the same subject-matter. The nature of these differences is readily apparent to any experienced teacher who briefly scrutinizes the content of the book. In this connection, however, a word of caution is in order. Teachers of experience, from their very familiarity with other texts, may tend at first sight to be misled by the novelty of this presentation and *may see in it a difficulty which is apparent rather than real.* They should be quick to realize, however, that their students—unhampered by experience—will not be so bothered. *Actual classroom experience with an experimental edition of these materials over a period of several years has conclusively demonstrated their teachability.* From these materials students acquire a mastery of geometry that is noticeably superior to the mastery gained from the traditional handling of the subject. Moreover, students trained according to the principles of this book need have no fear of college entrance examinations in geometry.

The traditional approach to demonstrative geometry involves careful study of certain theorems which the beginner is eager to accept without proof and which he might properly be led to take for granted as assumptions or postulates. Such an approach obscures at the very outset the meaning of "proof" and "demonstration." The employment of superposition in the proof of some of these theorems is even more demoralizing. This method of proof is so out of harmony with the larger aim of geometry instruction that despite its validity its use is commonly restricted to those few cases for which no better method can be found. For fundamental postulates of our geometry, therefore, we have chosen certain propositions of such broad import that the method of superposition will not be needed. We utilize only five fundamental postulates. They are stated and discussed in Chapter 2.

For a rigorous mathematical presentation of the postulates which we have employed, see the article "A Set of Postulates for Plane Geometry, Based on Scale and Protractor" published in the *Annals of Mathematics*, Vol. XXXIII, April, 1932. Naturally it has been advisable in an elementary course to slur over or ignore some of the subtler mathematical details, for these are not suitable material for the mind of the student at this juncture. Nevertheless, wherever the presentation involves a substantial incompleteness, its nature is indicated so far as possible in an accompanying footnote.

Another bugbear to beginners in geometry is "the incommensurable case." Euclid could not ignore the diagonal of the unit square and other similar lines, though he had no numbers with which to designate their lengths. By means of inequalities and an exceedingly shrewd definition of proportion he was able, however, to handle these incommensurable cases. Within the last century this treatment of incommensurables—or an equivalent statement—has been taken as the definition of irrational numbers in general. Hence our geometry accepts the fundamental properties of real numbers, including the irrationals, and so avoids explicit mention of the incommensurable case. For the teacher's convenience we present a brief discussion of the fundamental properties of the system of real numbers in the section *Laws of Number*, pages 284-288.

Taking for granted these fundamental properties of number also leads to many other simplifications and gives us a tool of the greatest power and significance. Among other things it enables us to combine the ideas "equal triangles" and "similar triangles" in general statements in which equality is but a special case of similarity, where the factor of proportionality is 1. And, further, we can base the treatment of parallels on similarity. This procedure, although the reverse of Euclid's, is logically equivalent to it. Thus our geometry, though differing in many important respects from Euclid's, is still Euclidean; the differences reflect the progress of mathematics since Euclid's time.

These changes are all by way of simplification and condensation. As a result, we have a two-dimensional geometry built on only five fundamental postulates, seven basic theorems, and nineteen other theorems, together with seven on loci. The increase of knowledge and the growing demands of civilization make it more important than ever before that our instruction be as compact and profitable for the student as possible.

Incorporation of the system of real numbers in three of the five fundamental assumptions of this geometry gives these assumptions great breadth and power. They lead us at once to the heart of geometry. That is why a geometry that is built on so strong a base is so simple and compact. It is because of the underlying power, simplicity, and compactness of this geometry that we call it *basic* geometry.

In a course in demonstrative geometry our prime concern is to make the student articulate about the sort of thing that hitherto he has been doing quite unconsciously. We wish to make him critical of his own, and others', reasoning. Then we would have him turn this training to account in situations quite apart from geometry. We want him to see the need for assumptions, definitions, and undefined terms behind every body of logic; to distinguish between good and bad arguments; to see and state relations correctly and draw proper conclusions from them. To this end we have inserted analyses or summaries of the reasoning employed in the proofs of nearly all the propositions and have included in Chapters 1 and 10 a detailed consideration of logical reasoning in fields outside of geometry.

This book is designed to require one year of study, although it may readily be spread over two years in schools following that plan. Essentially it is a course in plane geometry. Realizing, however, that most students will give not more than one year to the study of geometry, we have incorporated certain material from three-dimensional geometry and from modern geometry (so called), so that for all classes of students this first year of geometry may be as rich and enduring an experience as possible. The three-dimensional material is based on the student's intuition and is not intended to be logically complete.

The exercises are important for their content and for the development of the subject. None should be omitted without careful consideration. Those marked with a star are of especial importance.

The basic theorems 6, 7, 8, 11, and 13 can be deduced from the fundamental assumptions; but they will probably appear to the student to be sufficiently obvious without proof. It will be wise, therefore, to permit the student to assume these theorems at the outset. He should return to them later, when he has caught the spirit of the subject more fully and can be interested in the problem of reducing his list of assumptions to a minimum. This comes about naturally in Chapter 10, where the assumptions are reconsidered.

The brief treatment of networks and the slope and equation of a line in Chapter 4, together with the related exercises, and the brief treatment of the equation of a circle in Chapter 5 are included to show the relation of this geometry to analytic geometry. They may be omitted without injury to the logical development of the subject. The teacher may wish to deal lightly with continuous variation in Chapter 8 and to omit in Chapter 9 the sections devoted to power of a point, radical axis, inversion, and projection.

In every class in geometry it is excellent procedure to elicit suggestions from the students as to theorems worth proving and the best way of proving them. Unfortunately, however, theorems cannot always be proved in the order proposed by the students; or they can be proved in the suggested order only if certain other intermediary theorems are interpolated at the right places. Correct decisions on questions of this sort sometimes require complete grasp of the entire logical framework of the geometry. In order to avoid confusion, therefore, the authors have chosen the order of the theorems in the several exercises; they wish, nevertheless, to approve every well-considered effort on the part of teachers to elicit the coöperation of students in building the geometry. It is not impossible that a faulty procedure suggested by the students will have more educative value than a correct procedure imposed by author or teacher. This is most likely to be true when the teacher points out to the students wherein their procedure is wrong.

The value of demonstrative geometry as prevailingly taught in secondary schools is being questioned, and not without cause. It would be difficult to prove that the study of the subject necessarily leads in any large measure to those habits, attitudes, and appreciations which its proponents so eagerly claim for it. But it would be even more difficult to prove that other subjects of instruction can yield these outcomes as easily and as surely as can demonstrative geometry in the hands of an able and purposeful teacher. Teachers of demonstrative geometry are confronted with the challenge to re-shape their instruction so that it more nearly achieves the desired objectives. This textbook is offered as an instrument to that end.

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