

CONTENTS

Chapter 1	The Concept of an Analytic Function	
§1	The complex numbers	1
§2	Point sets in the complex plane	8
§3	Functions of a complex variable	10
Chapter 2	General Properties of Rational Functions	
§1	The n -th power	23
§2	Polynomials	29
§3	Rational functions	31
Chapter 3	Linear Transformations	
§1	Basic properties of linear transformations	37
§2	Mapping problems	45
§3	Stereographic projection	52
Chapter 4	Mapping by Rational Functions of Second Order	60
Chapter 5	The Exponential Function and its Inverse. The General Power	
§1	Definition and basic properties of the exponential function	65
§2	Mapping by means of the exponential function. The logarithm	70
§3	The general power	73
Chapter 6	The Trigonometric Functions	
§1	The sine and cosine	78
§2	The tangent and the cotangent	82
§3	The mappings given by the functions $\tan z$ and $\cot z$. Their inverse functions	85
§4	The mappings given by the functions $\sin z$ and $\cos z$. The functions $\arcsin z$ and $\arccos z$	87
§5	Survey of the Riemann surfaces of the elementary functions	91
Chapter 7	Infinite Series with Complex Terms	
§1	General theorems	95
§2	Power series	100

Chapter 8	Integration in the Complex Domain. Cauchy's Theorem	
§1	Complex line integrals	108
§2	The primitive function	114
§3	Cauchy's theorem	117
§4	The general formulation of Cauchy's theorem	121
Chapter 9	Cauchy's Integral Formula and its Applications	
§1	Cauchy's formula	131
§2	The Taylor expansion of an analytic function	135
§3	Consequences of Cauchy's integral formula	138
§4	The Laurent expansion	146
§5	Isolated singularities of an analytic function	149
§6	The inverse of an analytic function	155
§7	Mapping by a rational function	162
§8	Normal Families	165
Chapter 10	The Residue Theorem and its Applications	
§1	The residue theorem	167
§2	Application of the residue theorem to the evaluation of definite integrals	169
§3	The partial-fraction expansion of $\cot \pi z$	173
§4	The argument principle	175
§5	Applications of the argument principle	177
Chapter 11	Harmonic Functions	
§1	Preliminary considerations	184
§2	Gauss's mean-value theorem. The maximum and minimum principles	191
§3	Poisson's formula	195
§4	The harmonic measure	197
§5	The Dirichlet problem	204
§6	Harnack's principle	206
Chapter 12	Analytic Continuation	
§1	The principle of analytic continuation	213
§2	The monodromy theorem	216
§3	The inverse of a rational function	217
§4	Harmonic continuation. The reflection principle	219
Chapter 13	Entire Functions	
§1	Infinite products	225
§2	Product representation of the function $w = \sin \pi z$	228
§3	The Weierstrass factorization theorem	231
§4	Jensen's formula. The growth of entire functions	236

Chapter 14 Periodic Functions

§1 Definitions of simply and doubly periodic functions 242

§2 Reduction of simply periodic functions to the exponential function 244

§3 The basic properties of doubly periodic functions 246

§4 The Weierstrass \wp -function 249

§5 The Weierstrass ζ - and σ -functions 256

§6 Representation of doubly periodic functions by means of the σ -function 259

§7 The differential equation of the function $\wp(z)$ 261

§8 Representation of doubly periodic functions as rational functions of \wp and \wp' 262

§9 Addition theorem for doubly periodic functions 264

§10 Determination of a doubly periodic function with prescribed principal parts 266

§11 Mapping by a doubly periodic function of order 2 268

§12 Elliptic integrals 272

Chapter 15 The Euler Γ -Function

§1 Definition of the Γ -function 278

§2 Stirling's formula 280

§3 The product representation of the Γ -function 285

Chapter 16 The Riemann ζ -Function

§1 Definition and the Euler product formula 289

§2 Integral representation of the ζ -function 291

§3 Analytic continuation of the ζ -function 292

§4 Riemann's functional equation 295

§5 The zeros of the ζ -function and the distribution of prime numbers 298

Chapter 17 The Theory of Conformal Mapping

§1 The Riemann mapping theorem 305

§2 Construction of the solution 308

§3 Boundary correspondence under conformal mapping. 316

§4 The connection between conformal mapping and the Dirichlet problem 325

§5 The conformal mapping of polygons 328

§6 Triangle functions 335

§7 The Picard theorem 338

Index 345