

Contents

Notation	xiii
I. Background from Group Theory	1
1. Permutation Groups and Orbits	1
2. Subgroups and Factor Groups	3
3. Conjugate Classes	8
4. Abelian Groups	10
5. Solvable and Nilpotent Groups	14
6. Sylow Subgroups	17
7. Semi-direct Products	21
II. Representations and Modules	25
8. Linear Transformations	26
9. Definitions and Examples of Representations	30
10. Representations of Groups and Algebras	38
11. Modules	50
12. Tensor Products	59
13. Composition Series	76
14. Indecomposable Modules	81
15. Completely Reducible Modules	86
III. Algebraic Number Theory	91
16. Modules over Principal Ideal Domains	91
17. Algebraic Integers	102
18. Ideals	107
19. Valuations; P -adic Numbers	115
20. Norms of Ideals; Ideal Classes	123
21. Cyclotomic Fields	135
22. Modules over Dedekind Domains	144
IV. Semi-simple Rings and Group Algebras	157
23. Preliminary Remarks	157
24. The Radical of a Ring with Minimum Condition	159
25. Semi-simple Rings and Completely Reducible Modules	163
26. The Structure of Simple Rings	173
27. Theorems of Burnside, Frobenius, and Schur	179
28. Irreducible Representations of the Symmetric Group ..	190

29. Extension of the Ground Field	198
V. Group Characters	207
30. Introduction	207
31. Orthogonality Relations	217
32. Simple Applications of the Orthogonality Relations....	224
33. Central Idempotents	233
34. Burnside's Criterion for Solvable Groups	239
35. The Frobenius-Wielandt theorem on the Existence of Normal Subgroups in a Group	241
36. Theorems of Jordan, Burnside, and Schur on Linear Groups.....	250
37. Units in a Group Ring.....	262
VI. Induced Characters	265
38. Introduction	265
39. Rational Characters	279
40. Brauer's Theorem on Induced Characters	283
41. Applications	292
42. The Generalized Induction Theorem	301
VII. Induced Representations	313
43. Induced Representations and Modules	314
44. The Tensor Product Theorem and the Intertwining Number Theorem	323
45. Irreducibility and Equivalence of Induced Modules	328
46. Examples: The Tetrahedral and Octahedral Groups ..	329
47. Applications: Representations of Metacyclic Groups ..	333
48. A Second Application: Multiplicity-free Representations	340
49. The Restriction of Irreducible Modules to Normal Subgroups	342
50. Imprimitve Modules.....	346
51. Projective Representations	348
52. Applications	355
53. Schur's Theory of Projective Representations	358
VIII. Non-Semi-Simple Rings	367
54. Principal Indecomposable Modules	367
55. The Classification of the Principal Indecomposable Modules into Blocks	377
56. Projective Modules.....	380

57. Injective Modules	384
58. Quasi-Frobenius Rings	393
59. Modules over Quasi-Frobenius Rings	403
IX. Frobenius Algebras	409
60. Injective Modules for a Finite-Dimensional Algebra ..	409
61. Frobenius and Quasi-Frobenius Algebras	413
62. Projective and Injective Modules for a Frobenius Algebra	420
63. Relative Projective and Injective Modules	426
64. Group Algebras of Finite Representation Type.....	431
65. The Vertex and Source of an Indecomposable Module	435
66. Centralizers of Modules over Symmetric Algebras	440
67. Irreducible Tensor Representations of $GL(V)$	449
X. Splitting Fields and Separable Algebras	453
68. Splitting Fields for Simple Algebras and Division Algebras	453
69. Separable Extensions of the Base Field	459
70. The Schur Index.....	463
71. Separable Algebras.....	480
72. The Wedderburn-Malcev Theorem	485
XI. Integral Representations.....	493
73. Introduction	494
74. The Cyclic Group of Prime Order	506
75. Modules over Orders.....	515
76. P -Integral Equivalence	531
77. Projective Modules: Local Theory.....	542
78. Projective Modules: Global Theory	550
79. The Jordan-Zassenhaus Theorem	558
80. Order Ideals	563
81. Genus	567
XII. Modular Representations	583
82. Introduction	584
83. Cartan Invariants and Decomposition Numbers	590
84. Orthogonality Relations	598
85. Blocks	604
86. The Defect of a Block	611
87. Defect Groups	618
88. Block Theory for Groups with Normal P -Subgroups ..	627

89. Block Distribution of Classes.....	635
90. Miscellaneous Topics.....	638
A. Generalized Decomposition Numbers	638
B. Conjugate Characters	641
C. The Number of Characters Belonging to a Block..	643
D. Numerical Bounds	645
91. Examples	646
92. Literature on Applications to Group Theory	650
A. Groups of a Given Order	651
B. Characterizations of Simple Groups	652
C. Criteria for Existence of Normal Subgroups	654
Bibliography	655
Index	673