# Preface to the AMS/Chelsea Edition

We were led to write this book after having given joint graduate courses on functions of several complex variables at Princeton in the early 1960's. At that time the subject was in a very active and transitional period, with a good deal of new material beyond what had been covered in the classical books then available, especially Behnke and Thullen's *Theorie der Funktionen Mehrer Komplexer Veränderlichen* of 1933 and Bochner and Martin's *Several Complex Variables* of 1948. Lars Hörmander's book *An Introduction to Complex Analysis in Several Variables* appeared very shortly after our book, and focused on the approach through partial differential equations. In the subsequent years the subject has expanded vastly, with remarkable achievements in a number of directions – integral formulas, properties and applications of the $\overline{\partial}$ operator, detailed properties of holomorphic varieties both algebraic and topological, further detailed properties of sheaves and holomorphic mappings, and so on; and there have been a great many excellent books in the intervening years, covering the whole range of new results and techniques as well as providing some very useful introductions to the topic from a variety of points of view. In view of all these further sources of information about the subject, it is perhaps natural to ask why this book should be reissued at this point, particularly since we are not attempting to rework the book to correct a perhaps surprising number of errors and misprints. We may hope that a review of the state of the subject a half-century ago might be of some historical interest, and in addition that a short survey, focusing on the problems and techniques flourishing at the time of its writing, might serve as another useful introduction to the subject and a preparation for embarking on more detailed reading of the extensive literature that has arisen.

We wish to thank the American Mathematical Society, and particularly Edward Dunne, for their interest in reissuing the book and willingness to take the original as it stands. They are providing a web site, to which we would like to encourage any readers there may be to submit corrections and modifications, for the use of others who might like to take up a beautiful and very active topic in mathematics. The URL for this website is:

http://www.ams.org/bookpages/chel-368/.

This website includes a bibliography of comprehensive texts on several complex variables. Readers are encouraged to suggest entries that do not yet appear on this list.

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# PREFACE

The general theory of analytic functions of several complex variables was formulated considerably later than the more familiar theory of analytic functions of a single complex variable. Some of the principal function-theoretic problems were attacked and a basic foundation for the subject was laid late in the nineteenth century by Weierstrass, and around the turn of the century by Cousin, Hartogs and Poincaré. Certain central problems, either trivial in one variable or peculiar to several variables, were left open. Significant work in many directions was achieved by Bergman, Behnke, Bochner and others, in papers appearing from about the mid-1920's until the present time. The peculiarities of several complex variables were well exposed and the central difficulties clearly stated by the time of the appearance of the book of Behnke and Thullen, but the main problems were still there. Then K. Oka brought into the subject a brilliant collection of new ideas based primarily in the earlier work of H. Cartan and, in a series of papers written between 1936 and 1953, systematically eliminated these problems. But Oka's work had a far wider scope, and it was H. Cartan who realized this and developed the algebraic basis in the theory. This was essentially put into its present form in the seminars of Cartan in Paris (1951–52 and 1953–54) and the vastly useful tools provided by sheaf theory were first systematically employed there. The deep and extensive work of Grauert and Remmert on complex analytic spaces was built upon this foundation, and the same is of course true for the impressive works of many others during the last decade and at the present time.

The intention of the present volume is to provide an extensive introduction to the Oka-Cartan theory and some of its applications, and to the general theory of analytic spaces. We have neither attempted to write an encyclopedia of the subject of analytic functions of several complex variables, nor even tried to cover everything that is known today in the two areas of principal emphasis. Many fascinating aspects of this broad and active field that might have been encompassed by a book of the same title have been omitted almost entirely; the reader must look elsewhere for the differential-geometric and algebraic-geometric sides of the subject, for the theory of automorphic functions and complex symmetric spaces, for the Bergman kernel function, and for applications to mathematical physics. An attempt has been made to append a rather complete bibliography of books and papers in the two areas on which this introduction concentrates, so that the reader can pursue these topics further at will.

This book has been written with the prospective student of several complex variables in mind. In fact, the main reason for writing this book has been the untenable lack of an adequate introduction to one of the most active mathematical fields of the day. Further, there have been many recent results which cast a new light on much of the introductory material, and these results should properly be exposed early in the development of the subject. We have tried both to arrange this book so that the fundamental techniques will be exposed as soon as possible, and at the same time to give a firm foundation for their use. Of course, as a very active field, several complex variables is still in a state of flux. There are many different approaches that an introduction such as this could take, and one's choice of the ideal organization of the material varies from year to year. Indeed, were we to rewrite this book from scratch starting today it would probably turn out to be a quite different book.

The prerequisites for reading this book are, essentially, a good undergraduate training in analysis (principally the classical theory of functions of one complex variable), algebra, and topology; references have been provided for any important bits of mathematical lore which we did not consider standard minimum equipment for beginning graduate students in mathematics or their equivalent. The book is divided into nine chapters numbered with Roman numerals; each chapter is subdivided into sections indicated by capital letters. The definitions, lemmas, theorems, jokes, etc., are numbered in one sequence within each Section; the principal formulas are numbered similarly in a separate sequence. An expression such as "Theorem III C2" indicates a reference to the second numbered entity (in this case, a theorem) in Section C of Chapter III; for references within the same Chapter the Roman numeral will be dropped, and for references within the same Section the letter will often be dropped as well. References given in other forms will be left to the reader to decipher, with our best wishes for his success.

In somewhat more detail, the outline of the contents is as follows. Chapter I is in itself an introductory course in the subject (perhaps one semester in length). It presents, in outline, the essentials of the problems and some approaches to their solutions, and, in some special cases, includes the complete solutions. The discussion also shows the necessity for developing further techniques for tackling the problems, and thus motivates the remainder of the work. Except for Sections G and H, which are optional, the contents of this Chapter are prerequisite for what follows. However, to get into the subject most rapidly, bypassing the motivational portions, the reader may pass directly to Chapter II after reading Sections A, B, and C of Chapter I; Sections D through G are not needed until Chapter VI, and reading them can be postponed until specific references are given to them. Chapter II contains the local theory of analytic functions and varieties, and is the natural sequel to Chapter I. Beyond this point, there are several paths which the reader may follow, one of which of course is the straightforward plodding through the chapters as they occur. The reader mostly interested in the sheaf-theoretic aspects, especially in Cartan's famous Theorems A and B, may proceed next to Chapters IV, VI, and VIII. The reader interested rather in complex analytic spaces and their properties may proceed directly to Chapters III and V. The sheaf-theoretic notation and terminology introduced in Chapter IV are used in Chapter V for convenience, but none of the deeper properties are really required; however the discussion in Chapter VII does require some of the results of Chapter VI. The final chapter consists of an exposition, from the point of view of the preceding material, of pseudoconvexity.

This book developed from joint courses in several complex variables given by the authors at Princeton University during the academic years 1960–61 and 1962–63. It is a deep pleasure to both of us to be able to record here the debts of gratitude we owe to those who helped make this book possible. Lutz Bungart and Robin Hartshorne wrote and organized the lecture notes for our first course; these notes formed the kernel of the present work, and their reception encouraged us to proceed with the task. Thomas Bloom, William Fulton, Michael Gilmartin, and David Prill at Princeton aided in the revision of these notes and contributed many corrections and improvements. We are deeply indebted to Errett Bishop and Kenneth Hoffman, who read our tentative drafts with many helpful and inspiring comments. Finally, our thanks go to the typists of these various drafts; Caroline Browne, Eleanor Clark, Patricia Clark, and Elizabeth Epstein; and to the staff of Prentice-Hall, Inc.

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