Preface to the AMS Chelsea Edition

This is a new edition of a book published in 1965. The book, based on notes of a summer course in 1963, is an introduction to the theory of higher-order elliptic boundary value problems, a theory developed in the 50s of the last century. The book contains a detailed study of basic problems of the theory, such as the problem of existence and regularity of solutions of higher-order elliptic boundary value problems. It also contains a study of spectral properties of operators associated with elliptic boundary value problems. Thus, Weyl's law on the asymptotic distribution of eigenvalues is studied in the book in a great generality.

The new edition contains few changes. A number of explanatory remarks were added. Several inaccuracies and various misprints were corrected. Some notations were changed to conform to present day standard notation. A list of references was added.

I would like to thank the American Mathematical Society for republishing this book, and in particular thank the Publisher Dr. Sergei Gelfand for his help in the preparation of this edition.

Finally, I am deeply indebted to Professor Yehuda Pinchover who has given generously of his time, helping me throughout all stages of preparing this edition for publication.

Jerusalem October, 2009 Shmuel Agmon

Preface

This book reproduces with few corrections notes of lectures given at the Summer Institute for Advanced Graduate Students held at the William Rice University from July 1, 1963, to August 24, 1963. The Summer Institute was sponsored by the National Science Foundation and was directed by Professor Jim Douglas, Jr., of Rice University.

The subject matter of these lectures is elliptic boundary value problems. In recent years considerable advances have been made in developing a general theory for such problems. It is the purpose of these lectures to present some selected topics of this theory. We consider elliptic problems only in the framework of the L^2 theory. This approach is particularly simple and elegant. The hard core of the theory is certain fundamental L^2 differential inequalities.

The discussion of most topics, with the exception of that of eigenvalue problems, follows more or less along well-known lines. The treatment of eigenvalue problems is perhaps less standard and differs in some important details from that given in the literature. This approach yields a very general form of the theorem on the asymptotic distribution of eigenvalues of elliptic operators.

Only a few references are given throughout the text. The literature on elliptic differential equations is very extensive. A comprehensive bibliography on elliptic and other differential problems is to be found in [14].

These lectures were prepared for publication by Professor B. Frank Jones, Jr., with the assistance of Dr. George W. Batten, Jr. I am greatly indebted to them both. Professor Jones also took upon himself the trouble of inserting explanatory and complementary material in several places. I am particularly grateful to him. I would also like to thank Professor Jim Douglas for his active interest in the publication of these lectures.

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