Preface

This is a textbook primarily intended for students with approximately a year's background in complex variable theory. The material has been collected from lecture courses given over a long period of years, mostly at Harvard University. The book emphasizes classic and semiclassic results which the author feels every student of complex analysis should know before embarking on independent research. The selection of topics is rather arbitrary, but reflects the author's preference for the geometric approach. There is no attempt to cover recent advances in more specialized directions.

Most conformal invariants can be described in terms of extremal properties. Conformal invariants and extremal problems are therefore intimately linked and form together the central theme of this book. An obvious reason for publishing these lectures is the fact that much of the material has never appeared in textbook form. In particular this is true of the theory of extremal length, instigated by Arne Beurling, which should really be the subject of a monograph of its own, preferably by Beurling himself. Another topic that has received only scant attention in the textbook literature is Schiffer's variational method, which I have tried to cover as carefully and as thoroughly as I know how. I hope very much that this account will prove readable. I have also included a new proof of $|a_4| \leq 4$ which appeared earlier in a *Festschrift* for M. A. Lavrentiev (in Russian).

The last two chapters, on Riemann surfaces, stand somewhat apart from the rest of the book. They are motivated by the need for a quicker approach to the uniformization theorem than can be obtained from Leo Sario's and my book "Riemann Surfaces."

Some early lectures of mine at Oklahoma A. and M. College had been transcribed by R. Osserman and M. Gerstenhaber, as was a lecture at Harvard University on extremal methods by E. Schlesinger. These writeups were of great help in assembling the present version. I also express my gratitude to F. Gehring without whose encouragement I would not have gone ahead with publication.

There is some overlap with Makoto Ohtsuka's book "Dirichlet Problem, Extremal Length and Prime Ends" (Van Nostrand, 1970) which is partly based on my lectures at Harvard University and in Japan.

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